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Modeling of Synchronous Machines Including Voltage Regulation

***Abstract**. Synchronous machines are intensively used as generators due to their very good voltage and frequency regulation characteristics. To study the control of the generated active and reactive power it is necessary to have a very accurate mathematical dynamic models to implement efficient simulations. This article deals with the modelling of synchronous generators (machines) including their voltage regulation systems for the purposes of voltage transient analysis. There is a principle of mathematical modelling of the synchronous machine, including its voltage regulation system described in the first part of the article. The modelling of the synchronous machine and its excitation system response to the rapid load changes, in particular to the rapid reactive power changes, is based on real measured data, according to the machine parameters and the response of recorded voltage and current changes on its terminals. Described methodology does not follow standard procedures for modelling the excitation systems and is based on the analytical solution of the differential equations. Verification of the created mathematical model is performed by comparing simulated and real measured data, which is realized in the second part of the article.*

***Streszczenie.** Maszyny synchroniczne wykorzystuje się powszechnie jako generatory ze względu na ich bardzo dobrą charakterystykę regulacji napięcia i częstotliwości. W badaniach sterowania wytwarzaniem mocy czynnej i biernej, do zaimplementowania wydajnych symulacji, niezbędne są bardzo dokładne modele matematyczne układów dynamicznych. W artykule omówiono modelowanie generatorów synchronicznych, w tym ich układów regulacji napięcia do celów analizy napięcia przejściowego. W pierwszej części opisano zasady matematycznego modelowania maszyny synchronicznej, w tym jej systemu regulacji napięcia. Modelowanie maszyny synchronicznej i reakcji jej układu wzbudzającego na szybkie zmiany obciążenia, w szczególności mocą bierną, oparto na rzeczywistych danych pomiarowych, zgodnie z parametrami maszyny i odpowiedzią zarejestrowanych zmian napięcia i prądu na jej zaciskach. Opisana metodologia jest odmienna od standardowych procedur modelowania układów wzbudzenia i bazuje na analitycznym rozwiązaniu równań różniczkowych. Weryfikację opracowanego modelu matematycznego zrealizowano przez porównanie symulowanych i rzeczywistych danych pomiarowych, co przedstawiono w drugiej części artykułu. (**Modelowanie maszyn synchronicznych z uwzględnieniem regulacji napięcia**).*

Keywords: modeling of a synchronous generator, voltage regulation, excitation systems.
Słowa kluczowe: modelowanie generatora synchronicznego, regulacja napięcia, pobudzenie systemu.

Introduction

Synchronous generators form the principal source of electric energy in power systems. The commercial birth of the synchronous generator can be dated to 1891, when the first alternate transmission of electricity from Lauffen to Frankfurt (Germany, length 175 km, 20 kV line) was realized. Many large loads are driven by synchronous motors. Synchronous compensators are used to compensate reactive power and voltage control. These devices operate on the same principle and are collectively referred to as synchronous machines [1].

The basic operating state of each power system (and also of a synchronous machine) is its steady state (electrical and mechanical) of operating system quantities. Any change in system operating parameters leads to disturbing steady state (equilibrium), resulting in a transient phenomenon. To describe the behavior of the synchronous machine in these conditions, it is important to determine its steady state, but also to characterize its dynamic behavior during transient conditions. Because of the transient conditions, regulation (the frequency and active power regulation, as well as the voltage and reactive power regulation) is an essential part of the power system [2].

Mathematical Model of a Synchronous Machine

Modeling and analysis of synchronous machines has always been a challenge. The problem has been explored since the twenties and thirties of the last century and is still the subject of various studies. There is a considerable amount of literature dealing with the issue of synchronous machines in the power engineering area. Research on synchronous machines is usually based on intuition, experience and years of practice. There are many softwares for analyzing, planning, optimizing and simulating steady states and transition conditions with already programmed elements and user interfaces that allow modeling of synchronous machines and other elements of the power system [3]-[6].

A synchronous machine is an AC rotating machine which speed under steady state condition is proportional to the frequency of the voltage and current in its armature. The machine consists of two essential elements: the field (rotor) and the armature (stator). If a synchronous machine is used as a generator, the drive (turbine) rotates the rotor on which the field winding is located. A field winding flows a DC current producing a rotating magnetic flux that induces a three-phase voltage in the stator winding (armature). If the generator is loaded, its current flows through its three-phase stator winding form a second rotating magnetic flux. The stator winding (magnetic flux caused by a stator current flowing) acts against the main magnetic flux (caused by the field current and rotation of the rotor). This phenomenon is called the reaction of the armature (stator), which is reduced by amortisseur windings located on the rotor. If the alternator is loaded, rotor speed does not change. Only a phase shift occurs between the axis of the stator rotating magnetic field (flux) and the axis of the rotor magnetic field (the stator magnetic field is behind the rotor magnetic field as a result of the armature reaction). This phase shift is defined by the internal (load) angle δ_{i} expressed in degrees or radians. In other words, the load angle δ_{i} represents a phase shift between the internal induced voltage (no-load) on the stator (from the rotor magnetic field) and the terminal voltage of the synchronous generator (from the magnetic field induced by the stator current) [7].

There are two basic rotor structures used, depending on speed: a rotor with salient (projecting) poles and a round (cylindrical) rotor. Generally, the round rotor structure is used for high speed synchronous machines, such as steam turbine generators, whereas the salient pole structure is used for low speed applications, such as hydroelectric generators. The synchronous speed is given by [7]

$$n_{\mathrm{S}} = \frac{60 f}{p}, \tag{1}$$

where: n_s - is the synchronous speed in revolutions per minute, f - is the frequency in hertz, p - is the number of pairs of field poles.

The creating of the mathematical model (equations) of the synchronous generator is based on its construction. Fig. 1 shows electrical circuits of the rotor and stator of the synchronous generator,
where: U , V , W - is an index for stator phase quantities, fd - is an index for quantities of the field winding, k - is an index for the number of amortisseur windings (1 or 2), $k\mathrm{d}$ - is an index for quantities of amortisseur windings in d-axis, $k\mathrm{q}$ - is an index for quantities of amortisseur windings in q-axis, θ - is the angle by which d-axis leads the magnetic axis of the phase U winding in degress or radians, ω - is the rotor angular velicity in radians per second [1], [8], [9].

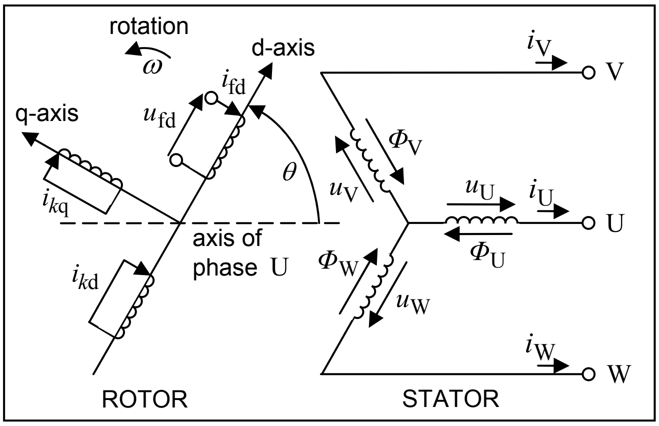

Fig. 1. Stator and rotor electrical circuits of a synchronous generator

According to Fig. 1 the positive direction of a stator winding current is assumed to be out of the machine (if a synchronous machine is used as a generator). The positive direction of field and amortisseur current is assumed to be into the machine. In Fig. 1 the rotor windings are located in two axes [1], [7]-[9]:

- in the direct d-axis, magnetically located in the centre of the north pole
- and the quadrature q-axis, 90 degrees ahead of the direct d-axis.

Since the rotor is rotating with respect to the stator, the angle θ (Fig. 1) is continuously increasing and is related to the rotor angular velocity ω and time t as follows [7]:

(2) $$\theta = \omega t + \theta_0 ,$$

where: t - is time in seconds, θ_0 - is the value of the angle θ at time $t = 0$.

In developing equations of a synchronous machine, the following assumptions are made [7], [9]:

- The stator windings are sinusoidally distributed along the air-gap as far as the mutual effects with the rotor are concerned.
- The stator slots cause no appreciable variation of the rotor inductances with rotor position.
- Magnetic hysteresis is negligible.
- Magnetic saturation effects are negligible.

According to Fig. 1, for instantaneous values of phase voltages at generator terminals [7]-[10]

(3) $$u_{\mathrm{U}} = \frac{\mathrm{d}\Phi_{\mathrm{U}}}{\mathrm{d}t} - R_{\mathrm{a}} i_{\mathrm{U}}$$

(4) $$u_{\mathrm{V}} = \frac{\mathrm{d}\Phi_{\mathrm{V}}}{\mathrm{d}t} - R_{\mathrm{a}} i_{\mathrm{V}}$$

(5) $$u_{\mathrm{W}} = \frac{\mathrm{d}\Phi_{\mathrm{W}}}{\mathrm{d}t} - R_{\mathrm{a}} i_{\mathrm{W}}$$

where: u_{U} - is the instantaneous value of the phase stator voltage in phase U in volts, u_{V} - is the instantaneous value of the phase stator voltage in phase V in volts, u_{W} - is the instantaneous value of the phase stator voltage in phase W in volts, Φ_{U} - is the magnetic stator flux linkage in phase U in webers, Φ_{V} - is the magnetic stator flux linkage in phase V in webers, Φ_{W} - is the magnetic stator flux linkage in phase W in webers, i_{U} - is the instantaneous value of the stator current in phase U in amperes, i_{V} - is the instantaneous value of the stator current in phase V in amperes, i_{W} - is the instantaneous value of the stator current in phase W in amperes, R_{a} - is the armature (stator) resistance per phase U , V , W in ohms.

Equations expressing electromagnetic circuits of the rotor of the synchronous generator have the form [7]-[10]:

(6) $$u_{\mathrm{fd}} = \frac{\mathrm{d}\Phi_{\mathrm{fd}}}{\mathrm{d}t} + R_{\mathrm{fd}} i_{\mathrm{fd}}$$

(7) $$u_{k\mathrm{d}} = 0 = \frac{\mathrm{d}\Phi_{k\mathrm{d}}}{\mathrm{d}t} + R_{k\mathrm{d}} i_{k\mathrm{d}}$$

(8) $$u_{k\mathrm{q}} = 0 = \frac{\mathrm{d}\Phi_{k\mathrm{q}}}{\mathrm{d}t} + R_{k\mathrm{q}} i_{k\mathrm{q}}$$

where: u_{fd} - is the rotor field voltage in volts, $u_{k\mathrm{d}}$ - is the rotor voltage of amortisseur winding of the number k in the d-axis in volts, $u_{k\mathrm{q}}$ - is the rotor voltage of amortisseur winding of the number k in the q-axis in volts, Φ_{fd} - is the magnetic rotor flux linkage in the field winding in webers, $\Phi_{k\mathrm{d}}$ - is the magnetic rotor flux linkage in the amortisseur winding of the number k in the d-axis in webers, $\Phi_{k\mathrm{q}}$ - is the magnetic rotor flux linkage in the amortisseur winding of the number k in the q-axis in webers, i_{fd} - is the rotor current flowing in the field winding in amperes, $i_{k\mathrm{d}}$ - is the rotor current flowing in the amortisseur winding of the number k in the d-axis in amperes, $i_{k\mathrm{q}}$ - is the rotor current flowing in the amortisseur winding of the number k in the q-axis in amperes, R_{fd} - is the (rotor) resistance of the field winding in ohms, $R_{k\mathrm{d}}$ - is the (rotor) resistance of the amortisseur winding of the number k in the d-axis in ohms, $R_{k\mathrm{q}}$ - is the (rotor) resistance of the amortisseur winding of the number k in the q-axis in ohms.

Amortisseurs or damper windings (Fig. 1) are closed circuits with no applied voltage In the dampers, currents are induced in them only when the magnetic field due to the stator windings or the field winding is changing [7].

The magnetic stator flux linkages Φ_{V}, Φ_{V}, Φ_{W} of the synchronous generator are defined by the dependence on currents and inductances of all alternator windings (according to the relation $\Phi = li$, Φ - the magnetic flux linkage, l - the instantaneous value of the inductance, i - the instantaneous value of the current). Since the synchronous generator is a rotating machine, some of its parameters are not static but depend on time (rotor position). All self and mutual stator inductances and mutual inductances between the rotor and the stator are characterized by the function of the rotor position with respect to the stator and change over time. In the case of the self-inductances of the field winding and amortisseur windings in the d-axis and the q-axis, and also the mutual inductances between these windings, their values are constant (these inductances do not change depending on the rotor position over time) [7], [8].

Because inductances in equations (3) to (5) are functions of the rotor position and are thus time-varying, the solution of these equations represents a complex and complex task in in solving machine and power system problems. The solution is dq0 transformation of stator phase variables, which greatly simplifies the calculation. Phase quantities of stator windings are transformed into the d-axis and q-axis described above. The transformation of stator currents i_{U}, i_{V}, i_{W} from the phase variables (U, V, W) to the dq0 variables (currents i_{d}, i_{q}, i_0) in amperes can be written in the following matrix form [11]:

$$\text{(9)} \qquad \begin{pmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \\ i_0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} \cos\theta & \cos(\theta-120°) & \cos(\theta+120°) \\ -\sin\theta & -\sin(\theta-120°) & -\sin(\theta+120°) \\ 0{,}5 & 0{,}5 & 0{,}5 \end{pmatrix}\begin{pmatrix} i_{\mathrm{U}} \\ i_{\mathrm{V}} \\ i_{\mathrm{W}} \end{pmatrix},$$

and the inverse transformation is given by

$$\text{(10)} \qquad \begin{pmatrix} i_{\mathrm{U}} \\ i_{\mathrm{V}} \\ i_{\mathrm{W}} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-120°) & -\sin(\theta-120°) & 1 \\ \cos(\theta+120°) & -\sin(\theta+120°) & 1 \end{pmatrix}\begin{pmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \\ i_0 \end{pmatrix}.$$

The dq0 transformation (9) can also be applied to stator magnetic fluxes and voltages or loads connected to generator terminals [11].

The dq0 transformation is characterized by properties that lead to simplifications compared to the synchronous generator model in phase values (U, V, W) [11]:

- Constant values of stator inductances and mutual inductances between the rotor and the stator.
- Omitting the rotating operator θ, respectively the rotor angular velocity ω: omitting time dependency.
- Quantities are expressed instead of instantaneous values in the time domain (sine and cosine time dependencies) in the vector form (dq0).
- Reduction the three ac quantities to two dc quantities.
- The vector form of the variable consists of d-part (real component) and q-part (imaginary component), in specific cases also zero component.

By applying the dq0 transformation to equations (3)-(5) the new set of equations will be given [7], [8]:

$$\text{(11)} \qquad u_{\mathrm{d}} = \frac{\mathrm{d}\Phi_{\mathrm{d}}}{\mathrm{d}t} - \omega\Phi_{\mathrm{q}} - R_{\mathrm{a}} i_{\mathrm{d}}$$

$$\text{(12)} \qquad u_{\mathrm{q}} = \frac{\mathrm{d}\Phi_{\mathrm{q}}}{\mathrm{d}t} + \omega\Phi_{\mathrm{d}} - R_{\mathrm{a}} i_{\mathrm{q}}$$

$$\text{(13)} \qquad u_0 = \frac{\mathrm{d}\Phi_0}{\mathrm{d}t} - R_{\mathrm{a}} i_0$$

where: u_{d} - is the d-component of the phase stator voltage in volts, u_{q} - is the q-component of the phase stator voltage in volts, u_0 - is the zero component of the phase stator voltage in volts, Φ_{d} - is the d-component of the magnetic stator flux linkage in webers, Φ_{q} - is the q-component of the magnetic stator flux linkage in webers, Φ_0 - is the zero component of the magnetic stator flux linkage in webers, i_{d} - is the d-component of the stator current in amperes, i_{q} - is the q-component of the stator current in amperes, i_0 - is the zero component of the stator current in amperes.

The quantities given in equations (3) to (13) in physical units (amperes, volts, ohms, webers, henrys, etc.) can be expressed using the per unit system which offers computational simplicity by eliminating units and expressing system quantities as dimensionless ratios. Network analysis often use per units to simplify calculations and to understand the results. In the case of a synchronous machine, there are several per unit systems, the most commonly used system of per units is the "L_{ad} base reciprocal per unit system". In the given per unit system, the interconnection between the stator and the rotor of the synchronous generator is incorporated, as it is in the case of a transformer (turns ratio). In this system, the nominal stator quantities are used as base values. When working with real values (in physical units), the rotor quantities must be converted to the stator's side. Expressions of quantities in physical units and per units cannot be combined [11].

The basic mathematical model of a synchronous machine consists of equations for stator voltages (11), (12), (13) and for rotor voltages (6), (7), (8). After applying dq0 transformation, a system of first order linear differential equations is obtained [1], [11], [12]:

$$\text{(14)} \qquad \boldsymbol{N}\frac{\mathrm{d}\boldsymbol{i}}{\mathrm{d}t} = -\boldsymbol{M}\boldsymbol{i} + \boldsymbol{u}$$

where: $\boldsymbol{M}$ - is the matrix with constant coefficients (it contains parameters of the synchronous generator or the external load connected to the generator terminals in ohms or siemens), $\boldsymbol{N}$ - is the matrix with constant coefficients (it contains parameters of the synchronous generator or the external load connected to the generator terminals in henrys or farads), $\boldsymbol{u}$ - is the matrix of input known quantities with constant coefficients, it contains the matrix of stator voltages (u_{d}, u_{q}, u_0) and rotor voltages (u_{fd}, $u_{k\mathrm{d}}$, $u_{k\mathrm{q}}$) in volts, $\boldsymbol{i}$ - is the matrix of output unknown quantities, it is formed by the matrix of stator currents (i_{d}, i_{q}, i_0) and rotor currents (i_{fd}, $i_{k\mathrm{d}}$, $i_{k\mathrm{q}}$) in amperes, $\frac{\mathrm{d}\boldsymbol{i}}{\mathrm{d}t}$ - is the matrix of derived unknown rotor and stator currents in amperes with respect to time in seconds.

The solution of the system (14) consists of determining the initial conditions: unknown currents (the matrix $\boldsymbol{i}$) at time 0. The initial conditions are defined by the steady state solution of the synchronous generator: simplification of the system (14). During steady state, time derivations do not occur in the equations (14) for describing the synchronous machine. Also, all currents flowing in amortisseur windings are equal to 0. These are closed circuits in which currents are induced only in the case of a transient event. The zero component is skipped and the angular velocity is not changed (is equal to the nominal).

After the modifications listed in [1], it is possible to integrate into the system (14) an external load connected to terminals of the synchronous generator. The external load can be used to express stator voltages (u_{d}, u_{q}, u_0), thus making the field voltage (u_{fd}) the only input. An external load may also consists of the power system composed of several elements (devices: transformers, lines, etc.). The value of the field voltage can be obtained from the steady state solution of the synchronous generator, as described in [11].

Excitation System of a Synchronous Machine

The synchronous generator is provided with two automatic (feedback) controllers for the regulation of the terminal voltage and frequency. These controllers indirectly influence the reactive power and active power outputs of the generator respectively. The main objective of the excitation system is to control the field current (or field voltage) of the synchronous machine. The field current is controlled so as to regulate the terminal voltage of the machine [7], [8].

Excitation systems of synchronous machines consist of the following basic parts [7], [13]:

- Exciter. Provides DC power to the synchronous machine field winding, constituting the power stage of the excitation system.
- Regulator. Processes and amplifies input control signals to a level and form appropriate for control of the exciter. They maintain the terminal voltage (or reactive power) at the desired level. They are most often implemented by a proportional (P) or proportional-integration (PI) algorithm.
- Power system stabilizer. Provides an additional input signal to the regulator to damp power system oscillations. Some commonly used input signals are rotor speed deviation, accelerating power, and frequency deviation.
- Limiters and protective circuits. They include a wide array of control and protective functions which ensure that the capability limits of the exciter and synchronous generator are not exceeded. Some of the commonly used functions are the filed current limiter, maximum excitation limiter, terminal voltage limiter, under excitation limiter etc.

The modelling of various excitation systems has been reported in two IEEE Committee reports [14], [15]. The first [14] deals with types of excitation systems, their nomenclature and control. The second document [15] deals with the modeling of excitation systems to study the stability of the power system. The excitation systems in both articles [14], [15] are sorted into three group:

- DC excitation systems,
- AC excitation systems,
- static excitation systems.

Modeling of excitation systems is discussed in more detail in [7], [8], [13]-[15].

Real test of island operation and the model creation

The real test of an island operation of one hydrogenerator supplied a load through four transformers and two power lines was used for the generator's field voltage and current computation. The single-line and equivalent circuit diagram of the examined real power system are presented in Fig. 2. The aim of the test was to check up the possibility of the hydrogenerator HG to supply an important load in the case of black-out occurrence.

There were two measurements provided during the test. First one was the measurement-1 of one period (0,02 seconds) RMS values and angles of voltages and currents at the generator terminals and second one was the measurement-2 of one period RMS values and angles of voltages and currents at the 6,3 kV terminals of the transformer T4. The test have been started with the HG start-up at no-load conditions. Consequently the transformer T1, Line-1, transformer T2, Line-2 and transformer T3 and T4 were connected to the generator step by step to create the network to an important load. After the network creation the startup of few motors were provided.

Tables 1 to 3 presents the electrical parameters of considered electrical devices used for the mathematical model creation. The mathematical model for the network was created based on the equivalent circuit diagram shown in Fig. 2.

The model each electrical device was created for purposes of generation of the set of differential equations (14), where matrices $\boldsymbol{M}$ and $\boldsymbol{N}$ consists of the R, L, C and G parameters corresponding to the equivalent model of individual devices. The model of the generator was created based on equations (6), (7), (8), (11) and (12), for the field winding, and amortisseur and armature windings in dq-axes. The 0-axis was not considered in the model. The model of a transformer was created as a T-section for both axes (d and q) with two branches representing the primary and secondary windings impedance $\bar{Z}_{\mathrm{kT}} = R_{\mathrm{kT}}/2 + \mathrm{j}\omega L_{\mathrm{kT}}/2$ and with the one branch representing the magnetization impedance $\bar{Z}_{\mathrm{mT}} = R_{\mathrm{mT}} + \mathrm{j}\omega L_{\mathrm{mT}}$. The model of a power line was created as a π-section for both axes (d and q) with one branch representing the series impedance $\bar{Z}_{\mathrm{Line}} = R_{\mathrm{Line}} + \mathrm{j}\omega L_{\mathrm{Line}}$ and with two branches, each representing the half of shunt admittance $\bar{Y}_{\mathrm{Line}} = G_{\mathrm{Line}} + \mathrm{j}\omega C_{\mathrm{Line}}/2$. The conductance G_{Line} in the case of each line was considered as a value close to 0. The model of load was represented by one series branch $\bar{Z}_{\mathrm{Load}} = R_{\mathrm{Load}} + \mathrm{j}\omega L_{\mathrm{Load}}$ and by one shunt admittance $\bar{Y}_{\mathrm{Load}} = G_{\mathrm{Load}} + \mathrm{j}\omega C_{\mathrm{Load}}$. There were set of first order differential equations created according to the first and the second Kirchhoff's law for the equivalent circuit diagram, which was modified to the form of equation (14). After this modification it is possible to find following parts of the model for each particular equipment as a mutually coupled submatrices in matrices $\boldsymbol{M}$ and $\boldsymbol{N}$:

$$\boldsymbol{M}_{\mathbf{HG}} = \begin{pmatrix} -R_{\mathrm{a}} & \omega L_{\mathrm{q}} & 0 & 0 & -\omega L_{\mathrm{aq}} \\ -\omega L_{\mathrm{d}} & -R_{\mathrm{a}} & \omega L_{\mathrm{ad}} & \omega L_{\mathrm{ad}} & 0 \\ 0 & 0 & R_{\mathrm{fd}} & 0 & 0 \\ 0 & 0 & 0 & R_{\mathrm{1d}} & 0 \\ 0 & 0 & 0 & 0 & R_{\mathrm{1q}} \end{pmatrix},$$

$$\boldsymbol{N}_{\mathbf{HG}} = \begin{pmatrix} -L_{\mathrm{d}} & 0 & L_{\mathrm{ad}} & L_{\mathrm{ad}} & 0 \\ 0 & -L_{\mathrm{q}} & 0 & 0 & L_{\mathrm{aq}} \\ -L_{\mathrm{ad}} & 0 & L_{\mathrm{ffd}} & L_{\mathrm{ad}} & 0 \\ -L_{\mathrm{ad}} & 0 & L_{\mathrm{ad}} & L_{\mathrm{11d}} & 0 \\ 0 & -L_{\mathrm{aq}} & 0 & 0 & L_{\mathrm{11q}} \end{pmatrix},$$

$$\boldsymbol{M}_{\mathbf{kT}} = \begin{pmatrix} -R_{\mathrm{kT}}/2 & \omega L_{\mathrm{kT}}/2 \\ -\omega L_{\mathrm{kT}}/2 & -R_{\mathrm{kT}}/2 \end{pmatrix},$$

$$\boldsymbol{N}_{\mathbf{kT}} = \begin{pmatrix} -L_{\mathrm{kT}}/2 & 0 \\ 0 & -L_{\mathrm{kT}}/2 \end{pmatrix},$$

$$\boldsymbol{M}_{\mathbf{mT}} = \begin{pmatrix} -R_{\mathrm{mT}} & \omega L_{\mathrm{mT}} \\ -\omega L_{\mathrm{mT}} & -R_{\mathrm{mT}} \end{pmatrix},$$

$$\boldsymbol{N}_{\mathbf{mT}} = \begin{pmatrix} -L_{\mathrm{mT}} & 0 \\ 0 & -L_{\mathrm{mT}} \end{pmatrix},$$

$$\boldsymbol{M}_{\mathbf{yLine}} = \begin{pmatrix} G_{\mathrm{Line}} & -\omega C_{\mathrm{Line}}/2 \\ \omega C_{\mathrm{Line}}/2 & G_{\mathrm{Line}} \end{pmatrix},$$

$$\boldsymbol{N}_{\mathbf{yLine}} = \begin{pmatrix} C_{\mathrm{Line}}/2 & 0 \\ 0 & C_{\mathrm{Line}}/2 \end{pmatrix},$$

$$\boldsymbol{M}_{\mathbf{zLine}} = \begin{pmatrix} -R_{\mathrm{Line}} & \omega L_{\mathrm{Line}} \\ -\omega L_{\mathrm{Line}} & -R_{\mathrm{Line}} \end{pmatrix},$$

$$\boldsymbol{N}_{\mathbf{zLine}} = \begin{pmatrix} -L_{\mathrm{Line}} & 0 \\ 0 & -L_{\mathrm{Line}} \end{pmatrix},$$

$$\boldsymbol{M}_{\mathbf{yLoad}} = \begin{pmatrix} G_{\mathrm{Load}} & -\omega C_{\mathrm{Load}} \\ \omega C_{\mathrm{Load}} & G_{\mathrm{Load}} \end{pmatrix},$$

$$\boldsymbol{N}_{\mathbf{yLoad}} = \begin{pmatrix} C_{\mathrm{Load}} & 0 \\ 0 & C_{\mathrm{Load}} \end{pmatrix},$$

$$\boldsymbol{M}_{\mathbf{zLoad}} = \begin{pmatrix} -R_{\mathrm{Load}} & \omega L_{\mathrm{Load}} \\ -\omega L_{\mathrm{Load}} & -R_{\mathrm{Load}} \end{pmatrix},$$

$$\boldsymbol{N}_{\mathbf{zLoad}} = \begin{pmatrix} -L_{\mathrm{Load}} & 0 \\ 0 & -L_{\mathrm{Load}} \end{pmatrix},$$

where: $\boldsymbol{M}_{\mathbf{HG}}$, $\boldsymbol{N}_{\mathbf{HG}}$ - are submatrices corresponding to the model of hydrogenerator, $\boldsymbol{M}_{\mathbf{kT}}$, $\boldsymbol{N}_{\mathbf{kT}}$ - are submatrices corresponding to the windings of transformer, $\boldsymbol{M}_{\mathbf{mT}}$, $\boldsymbol{N}_{\mathbf{mT}}$ - are submatrices corresponding to the magnetization impedance of transformer, $\boldsymbol{M}_{\mathbf{yLine}}$, $\boldsymbol{N}_{\mathbf{yLine}}$ - are submatrices corresponding to the shunt admittance of the line, $\boldsymbol{M}_{\mathbf{zLine}}$, $\boldsymbol{N}_{\mathbf{zLine}}$ - are submatrices corresponding to the series impedance of the line, $\boldsymbol{M}_{\mathbf{yLoad}}$, $\boldsymbol{N}_{\mathbf{yLoad}}$ - are submatrices corresponding to the shunt admittance of the load, $\boldsymbol{M}_{\mathbf{zLoad}}$, $\boldsymbol{N}_{\mathbf{zLoad}}$ - are submatrices corresponding to the series impedance of the load.

Table 1. Parameters of transformers (T1, T2, T3 and T4) of the examined real power system (see Fig. 2)

Transformer	T1	T2	T3	T4
rated power (MVA)	40	200	125	25
primary rated voltage (kV)	10.5	230	242	13.8
secondary rated voltage (kV)	121	121	13.8	6.3
short-circuit voltage (%)	10.23	10.75	12	12
no-load current (%)	0.299	0.05	0.056	0.094
short-circuit losses (kW)	170.6	347.7	395	127
no-load losses (kW)	46.5	41.5	45	13.5
tap number (–)	3	13	17	19
step of tap (%)	5	2	2	1.78

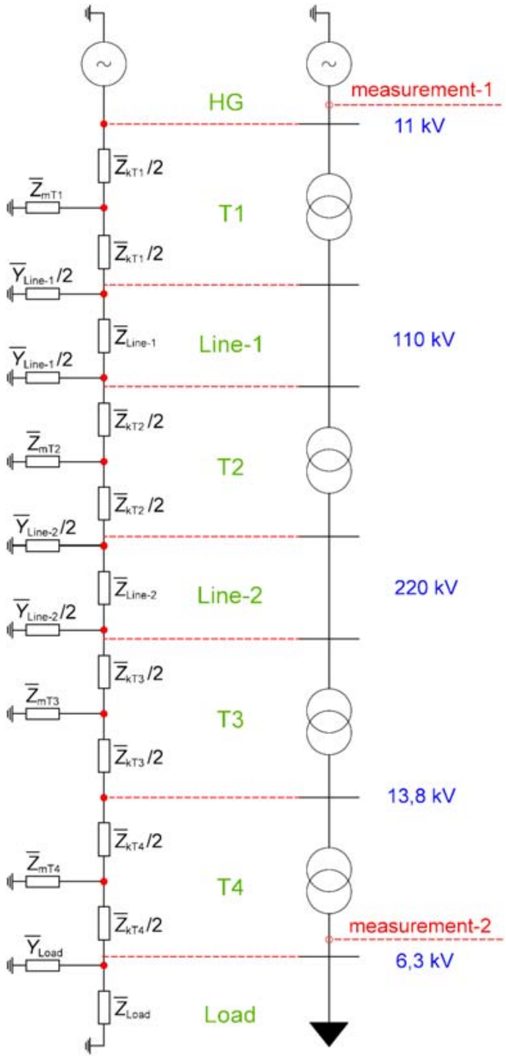

Fig. 2. Equivalent circuit and single-line diagram of the examined real power system

Table 2. Parameters of hydrogenerator of the examined real power system (see Fig. 2)

Quantity		Value
rated voltage	U_{nG} (kV)	11
rated power	S_{nG} (MVA)	35
rated current	I_{nG} (A)	183.7
rated power factor	$\cos\varphi_{\mathrm{nG}}$ (–)	0.850
synchronous reactance in d-axis	X_{d} (p. u.)	0.897
transient reactance in d-axis	X'_{d} (p. u.)	0.272
rated power factor	$\cos\varphi_{\mathrm{nG}}$ (–)	0.850
subtransient reactance in d-axis	X''_{d} (p. u.)	0.229
leakage reactance	X_{l} (p. u.)	0.207
synchronous reactance in q-axis	X_{q} (p. u.)	0.675
transient reactance in q-axis	X'_{q} (p. u.)	0.675
subtransient reactance in q-axis	X''_{q} (p. u.)	0.229
transient open-circuit time constant in d-axis	T'_{do} (s)	2.638
subtransient open-circuit time constant in d-axis	T''_{do} (s)	0.012
transient short-circuit time	T'_{d} (s)	0.80

constant in d-axis		
subtransient open circuit time constant in d-axis	T''_{d} (s)	0.01
subtransient open-circuit time constant in q-axis	T''_{qo} (s)	0.03
subtransient short-circuit time constant in q-axis	T''_{q} (s)	0.01
armature resistance	R_a (Ω)	0.0155

Table 3. Parameters of lines (Line-1 and Line-2) of the examined real power system (see Fig. 2)

rated voltage (kV)	110	220
resistance (Ω)	2.514	4.984
inductive reactance (Ω)	6.247	32.974
capacity susceptance (μS)	42.6	243.639

Results of the field voltage computation

There were following three different events simulated using the above mentioned mathematical model of the network shown in Fig. 2:

- Line-2 connection to T2 during network creation.
- Starting of 1 250 kW induction machine supplied from T4.
- Starting of 1 600 kW induction machine supplied from T4.

There were measurement of one period RMS values and angles of voltage and current realized at generator terminals and at load, realized during the real test. Obtained data were used for one period impedance change at both places. Based on the solution of the equation (14) the field voltage was computed to achieve simulated voltage at the generator terminals be the same as the measured one. In the cases of the induction motors connection two approaches were provided. The first one was the computation of the field voltage (and field current) based on the impedance change directly on the generator terminals (model of the generator without the rest of the network). The second approach was the computation of the field voltage (and field current) based on the impedance change at the 6,3 kV terminals of the transformer T4. The comparison of the results reached by the both approaches was used for the network model verification.

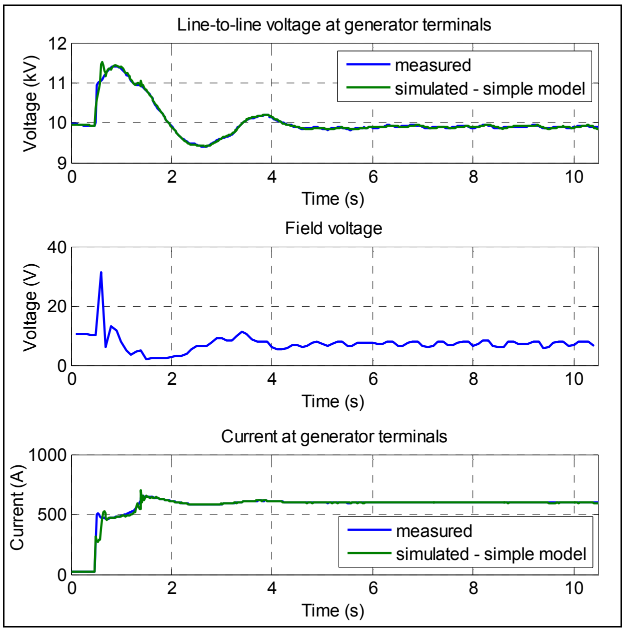

Fig. 3. Simulated and measured voltage and current at generator terminals and computed field voltage after line 2 connection

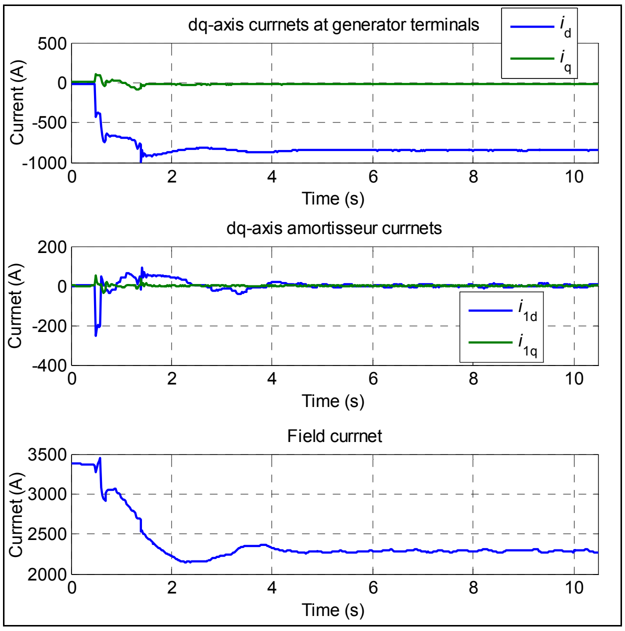

Fig. 4. Simulated dq-axis currents at generator terminals, amortisseur and computed field current in the case of the line 2 connection

The simulated and measured voltage and current at generator terminals and computed field voltage after the Line-2 connection are shown in Fig. 3. Because at this time the network have been just in the creating process yet, the load at the generator terminals were capacitive. The reason was weak active power load (only due to the active losses of the transformers T1 and T2 as well as of the Line-1) and capacitive reactive power generated by the Line-1 already connected at that time. Connection of the Line-2 reached the raises of the capacitive reactive power from approx. 300 kVAr up to 10 MVAr. Due to this capacitive reactive power raising the overvoltage occurs, which was mitigated by voltage regulation the field voltage course of which is shown in the Fig. 3. Fig.4. shows the simulated dq-axis currents at generator terminals, amortisseur and computed field current in this case.

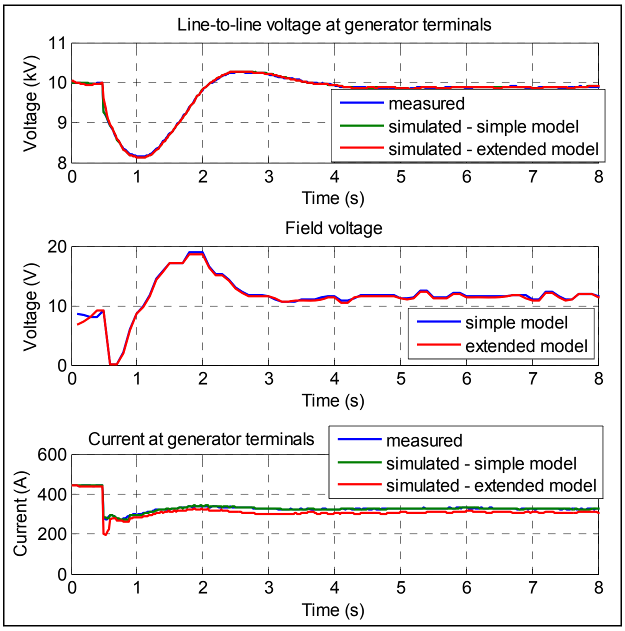

Fig. 5. Simulated and measured voltage and current at generator terminals and computed field voltage after 1 250 kW induction motor connection

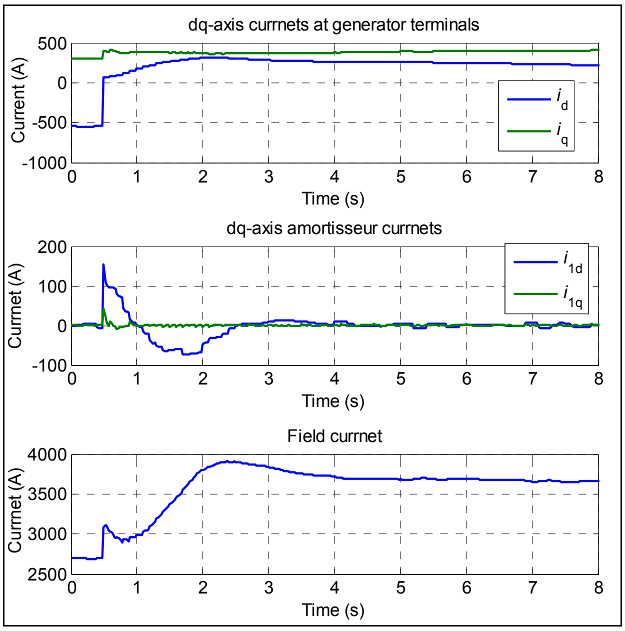

Fig. 6. Simulated dq-axis currents at generator terminals, amortisseur and computed field current in the case of the 1 250 kW induction motor connection

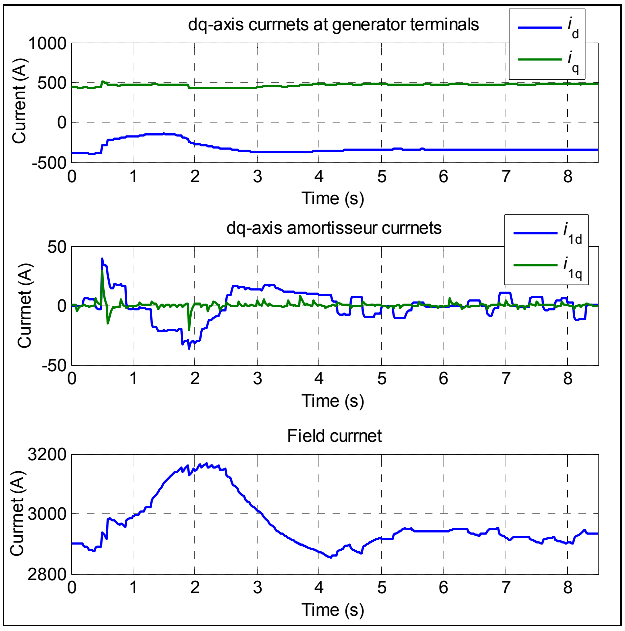

Fig. 8. Simulated dq-axis currents at generator terminals, amortisseur and computed field current in the case of the 1 600 kW induction motor connection

The simulated and measured voltage and current at generator terminals and computed field voltage after 1 250 kW induction motor connection are shown in Fig. 5. The starting of such motor leads to the voltage dip which level depends on the actual short-circuit power. In the case of island operation, where the network was supplied only from the hydrogenerator, the RMS voltage felt down to approximately 80 % of initial value. The regulation operation is clear from the field voltage course. It is clear from the beginning of the event, that the voltage regulation evaluated the situation of the sudden voltage dip as a fault (such as s short-circuit) and kept the excitation to zero.

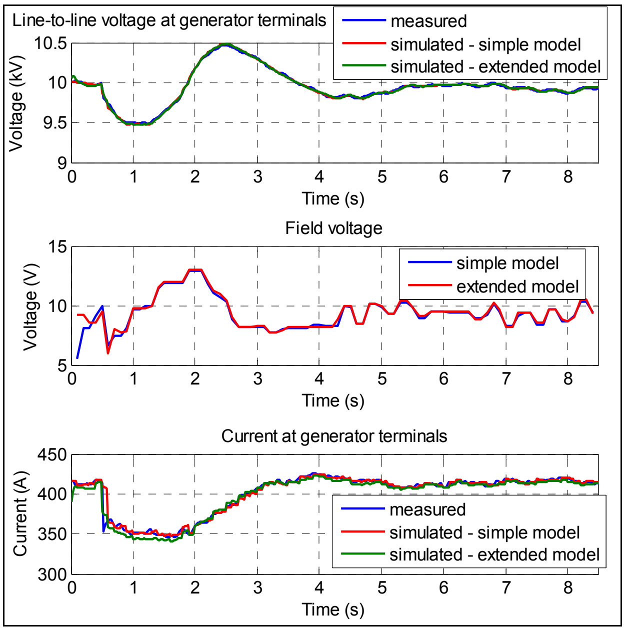

Fig. 7. Simulated and measured voltage and current at generator terminals and computed field voltage in the case of the 1 600 kW induction motor connection

Due to generation of the currents in the amortisseur (shown in Fig. 6) the falling of the voltage was slowed down. After about 100 ms, the voltage regulator began to excite and increased the voltage to the required value. It is clear from the course of current at generator terminals that the generator was operating in the underexcited mode. The starting of the induction motor represents the reactive power consumption so during the motor start up the generator was less underexcited (i.e. the current decreased at the generator terminals).

Similar voltage regulation operation can be seen in the Fig. 7 and Fig. 8, where the results of the voltage and current measurement and simulation during the 1 600 kW induction motor start up are shown. In this case the start of the motor was shorter (the time duration of the start was approx. 3,5 sec), so the voltage regulation operation after the end of motor start can be also seen in these figures.

Conclusion

The purpose of the generator's field voltage and current courses calculation based on known RMS values and angles of voltages and currents at generator terminals is to obtain relevant information for modeling the excitation system of the generator. Such approach is suitable mainly in the cases of missing of key information about modeled excitation system, such as its type, transfer function, values of constants. The principle of field voltage and current time courses calculation based on known RMS values and angles of voltages and currents at generator terminals as well as known electrical parameters of the generator is described in this paper. The calculation principle is based on the realization of the set of calculations of the set of differential equations (14), with only known parameters are R, L, C and G values of considered electrical equipment and the initial conditions of voltages and currents at the beginning of the solved time section. Since the only parameter, that can be used to find the correct solution of the differential equation system to adjust the result, is just the unknown generator's field voltage, a value of the generator's field voltage was searched at each time section so that the calculated voltage at the generator terminals was the same (with sufficient precision) with real measured values. There are results of field voltage and current calculations in case of three different events (switch on the line and start of two different induction machines) presented in this paper, based on which almost identical voltage and current courses were obtained as those obtained by real measurement. In addition, in the case of induction motors startup, the calculations of the excitation voltage time courses were realized in two ways: by using a simple network model consisting only of the model of generator and the impedance at its terminals, respectively using the model extended including transformers and power lines between generator terminals and induction motors. For both approaches, almost identical voltage and current time courses were obtained, what is indicating the correctness of the network model between generator terminals and induction motors, as well as the suitability of using of both models.

To know the voltage and current curses in time, as well as the field voltage and current time courses in the case of different events (switching on or switching off the lines or the load) is necessary for the appropriate excitation system model finding in the next step, whether in the form of transfer functions or in the form of mathematical function or algorithm.

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REFERENCES

[1] Margitová A., Modelovanie prechodných javov synchrónnych strojov, *Technická univerzita v Košiciach*, 2017, diploma thesis
[2] Džmura J., Úvod do stability prenosu elektrickej energie, *Technická univerzita v Košiciach*, 2012
[3] Park R. H., Two-Reaction Theory of Synchronous Machines - Generalized Method of Analysis - Part I and Part II, *AIEE Trans.*, 48 (1929), 716-727, 52 (1933), 352-355
[4] Concordia C., Synchronous Machines, *John Wiley & Sons*, 1951
[5] Kimbark E. W., Power System Stability, Vol. III: Synchronous Machines, *John Wiley & Sons*, 1956
[6] Machowski J., Bialek J. W., Bumby J. R., Power System Dynamics: Stability and Control, *John Wiley & Sons*, 2008
[7] Kundur P., Power System Stability and Control, *McGraw-Hill*, 1994
[8] Padiyar K. R., Power System Dynamics: Stability and Control, *BS Publications*, 2008
[9] Anderson M., Fouad A. A., Power System Control and Stability, *Iowa State University Press*, 1977
[10] Saadat H., Power System Analysis, *McGraw-Hill*, 1999
[11] Kanálik M., Margitová A., Trojfázové skratové prúdy na svorkách synchrónneho generátora určené podľa normy STN EN 60909-0 a pomocou matematického modelu synchrónneho generátora, *Elektroenergetika: International Scientific and Professional Journal on Electrical Engineering*, 11 (2018), No. 1, 35-44
[12] Barakat A., Tnani S., Champenois G., Mouni E., Analysis of Synchronous Machine Modeling for Simulation and Industrial Applications, *Elsevier*, 18 (2010), 1382-1396
[13] Murgaš J., Foltin M., Blaho M., Budiace systémy elektrární na území Slovenskej republiky, *ATP Journal*, 7 (2011)
[14] Computer Representation of Excitation Systems. *IEEE Trans.*, PAS-87 (1968), 1460-1464
[15] Excitation System Models For Power System Stability Studies.. *IEEE Trans.*, PAS-100 (1981), No. 2, 494-509