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doi:10.15199/48.2019.08.20

New definition formulas for apparent power and active current of three-phase power system

Abstract. *A new definition formula for apparent power of the three-phase system is justificated. It is the average geometric value of the currents power losses and short circuit power of the voltage source loaded by transmission line resistances. The Buchholz's apparent power formula follows from this definition as a special case if both zero sequence components of currents and voltages are nil. Under unbalanced voltage the meaning of apparent power in accordance with proposed formula may exceed Buchholz's one more than on 10%. The generalized definition formula for introduced by Professor Fryze concept of power system's active current is grounded. It is part of the short-circuit current that is equal to ratio between load power and short-circuit power of the voltage source. In such way specified active current provides up to 15% power losses gain in the transmission line compared to the original Fryze's definition in the presence of voltage zero-sequence component.*

Streszczenie. *Opracowano nową formułę określającą pozorną moc układu trójfazowego. Jest to średnia wartość geometryczna strat prądu i mocy zwarciowej źródła napięcia obciążonego rezystancją kabla. Wzór na pozorną moc Buchholza wynika z tej definicji jako szczególnego przypadku, gdy oba składniki zerowej sekwencji prądów i napięć są zerowe. Przy niezrównoważonym napięciu pozorna wartość mocy zgodnie z proponowaną formułą może przekroczyć wartość formuły Buchholza o więcej niż 10%. Przedstawiono uogólnioną formułę determinującą koncepcję prądu czynnego systemu elektroenergetycznego wprowadzoną przez Profesora Fryzego. Jest to część prądu zwarciowego, równa stosunkowi mocy obciążenia i mocy zwarciowej źródła napięcia. W ten sposób określony prąd czynny zapewnia do 15% wzrost strat mocy w linii transmisyjnej w porównaniu z pierwotną definicją Fryze'a w obecności składowej zerowej napięcia.* ***Nowa definicja mocy pozornej i prądu czynnego w układzie trójfazowym***

Keywords: active current, apparent power, active filter, zero sequence component.
Słowa kluczowe: prąd czynny, moc pozorna, filtr aktywny, składowa zerowa sekwencji

Introduction

The power theory of electrical systems has been evolving continuously for more than a century, beginning with the works of Steinmetz [1]. A significant milestone in this direction was the introduction by Professor S. Fryze of active current concept [2,3]. This concept served as a main theoretical basis for constructing active and passive filtering devices [4]. With the development of the elemental base of power semiconductor devices and signal processors, the control strategies of active filters developed in the direction of ensuring minimal power losses, unity power factor and maximum efficiency [5-9]. These strategies were based on new provisions of the theory of power, concerning the decomposition of currents and powers with giving physical meaning to the individual components. A critical review of these theoretical positions, carried out in [5-10] and many others, suggests that the main component of the current decomposition is the active current in the Fryze form, and the sum of the squares of the power components is the square of the apparent power using the Buchholz formula as product of rms values of line currents and line-to-neutral voltages.

The independence of the Fryze's active current and Buchholz's apparent power from the ratio of cable resistances is doubtful that it is correct for four-wire systems (Fig. 1) with non-zero neutral current. The determination of the apparent power of a three-phase power supply system considers the limitations at which the active power is maximized, as an allowable value of the power losses caused by the flow of consumed currents [10]. This value is proportional to the square of the rms value of the consumed line currents, which appear in the Buchholz apparent power formula, only in a three-wire power system with identical values of the active resistances of the line wires. Especially this difference is manifested in a three-phase four-wire power system, where the active resistance of the neutral wire differs from the active resistance of each line wire. That is why the current multiplier of the formula for the apparent power of a three-phase four-wire power supply system according to the standards [11, 12] is proportional to the power losses, containing different values of these resistances. Formulation of the optimal control strategy for active filter currents [13] that provides a minimal power losses in the non-sinusoidal mode contains both the ratio of the cable resistances and the zero-sequence voltage component. The need for accounting of power line longitudinal and transverse parameters, as well as the neutral conductor in the study of the system energy characteristics was also noted in [7, 9, 11].

The goal of this work is a new justification, refinement and generalization of the basic concepts of the power theory - active current and apparent power for two types of three-phase circuits, both three-wire and four-wire.

Fryze's active current of three-phase power system and Buchholz's formula of the apparent power

Let us consider the periodic non-sinusoidal mode of currents and voltages of a three-phase power supply system with a resistive model (Fig. 1) of the transmission line.

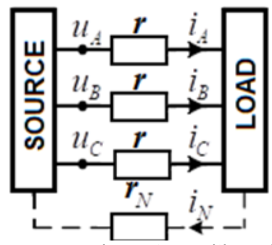

Fig. 1. Three-phase power supply system with resistive model of the transmission line

In the Fryze's archive publication [3], the active current of a three-phase circuit is defined as a three-coordinate vector of time functions

(1) $$\mathbf{i}_F(t) = (P/U^2)\mathbf{u}(t)\,,$$

where $\mathbf{u}(t) = \| u_A(t) \quad u_B(t) \quad u_C(t) \|^{\wedge}$ - the vector of instantaneous values of line-to-neutral voltages; $\wedge$ - sign of transposition; $P = \frac{1}{T}\int_0^T [u_A(t)i_A(t) + u_B(t)i_B(t) + u_C(t)i_C(t)]dt$ -

active power; T - the period of three-phase voltages source; $U^2 = \frac{1}{T}\int_0^T [u_A^2(t) + u_B^2(t) + u_C^2(t)]dt$ - the square of the rms value of the voltage vector.

Introducing the notation for the scalar product of periodic time vectors

$$\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{i}(t)dt = \mathbf{u}\circ\mathbf{i},$$

let us show that the Fryze's active current (1) has a minimum rms value among those currents that are characterized by the active power $P = \mathbf{u}\circ\mathbf{i}$ for a given voltage vector.

For vectors **u**(t) and **i**(t), satisfying this restriction, we write the inequality Cauchy -Schwartz [14]:

$$(\mathbf{u}\circ\mathbf{i})^2 \le (\mathbf{u}\circ\mathbf{u})(\mathbf{i}\circ\mathbf{i}),$$

from which it follows that

$$\mathbf{i}\circ\mathbf{i} \ge \frac{(\mathbf{u}\circ\mathbf{i})^2}{\mathbf{u}\circ\mathbf{u}} = \frac{P^2}{U^2}. \tag{2}$$

In the right-hand side of inequality (2) we have the square of the norm of the Fryze's active current according to formula (1):

$$\mathbf{i}_F\circ\mathbf{i}_F = \left(P/U^2\right)^2 \times (\mathbf{u}\circ\mathbf{u}) = P^2/U^2. \tag{3}$$

Thus, it follows from (2) and (3) that

$$\mathbf{i}\circ\mathbf{i} = \frac{1}{T}\int_0^T \mathbf{i}^\wedge(t)\mathbf{i}(t)dt \ge \frac{1}{T}\int_0^T \mathbf{i}_F^\wedge(t)\mathbf{i}_F(t)dt = \mathbf{i}_F\circ\mathbf{i}_F,$$

which was to be proved.

Equality in the expression, which follows from (2)

$$(\mathbf{u}\circ\mathbf{i})_{\max} = \sqrt{(\mathbf{u}\circ\mathbf{u})(\mathbf{i}\circ\mathbf{i})}$$

sets the maximum value of the left part - the active power at the given voltages of the three-phase source and the limits for the rms value of the linear currents. Precisely this maximum value of active power is adopted as apparent power [10]. This implies the Buchholz formula for the apparent power of three-phase systems:

$$S_B = P_{\max} = \sqrt{(\mathbf{u}\circ\mathbf{u})(\mathbf{i}\circ\mathbf{i})} = \sqrt{\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{u}(t)dt \times \frac{1}{T}\int_0^T \mathbf{i}^\wedge(t)\mathbf{i}(t)dt} = UI. \tag{4}$$

Thus, the Fryze's active current in the form (1) with the given active power and source voltages minimizes the value of the current multiplier of the apparent power according to the Buchholz formula (4).

The new formula for the apparent power of the three-phase four-wire power system and the refined value of the active current

We justify the formula for the apparent power for case three-phase four-wire power system. The power losses are given by

$$\Delta P = \frac{1}{T}\int_0^T [i_A^2(t)r + i_B^2(t)r + i_C^2(t)r + i_N^2(t)r_N]dt,$$

where the neutral current can be represented in the form

$$i_N(t) = i_A(t) + i_B(t) + i_C(t) = \mathbf{i}^\wedge(t)\mathbf{j} = \mathbf{j}^\wedge\mathbf{i}(t);\ \mathbf{j}^\wedge = \|1 \quad 1 \quad 1\|.$$

Considering this, we transform expression for power losses to the matrix-vector form as follows:

$$\Delta P = \frac{1}{T}\int_0^T [\mathbf{i}^\wedge(t)\mathbf{i}(t)r + \mathbf{i}^\wedge(t)\mathbf{j}\mathbf{j}^\wedge\mathbf{i}(t)r_N]dt = \frac{1}{T}\int_0^T \mathbf{i}^\wedge(t)\mathbf{R}\mathbf{i}(t)dt =$$

$$= \frac{1}{T}\int_0^T [\mathbf{R}^{1/2}\mathbf{i}(t)]^\wedge \mathbf{R}^{1/2}\mathbf{i}(t)dt = (\mathbf{R}^{1/2}\mathbf{i})\circ(\mathbf{R}^{1/2}\mathbf{i}),$$

where $\mathbf{R} = r\mathbf{I} + r_n\mathbf{j}\mathbf{j}^\wedge = \mathbf{R}^\wedge$ – a transmission line resistance matrix; $\mathbf{I}$ –- is an identity matrix of dimension 3 × 3.

For vectors $\mathbf{R}^{-1/2}\mathbf{u}(t)$ and $\mathbf{R}^{1/2}\mathbf{i}(t)$ which satisfy constraint $(\mathbf{R}^{-1/2}\mathbf{u})\circ(\mathbf{R}^{1/2}\mathbf{i}) = P$ we write down the Cauchy-Schwartz inequality:

$$[(\mathbf{R}^{-1/2}\mathbf{u})\circ(\mathbf{R}^{1/2}\mathbf{i})]^2 \le [(\mathbf{R}^{-1/2}\mathbf{u})\circ(\mathbf{R}^{-1/2}\mathbf{u})]\times[(\mathbf{R}^{1/2}\mathbf{i})\circ(\mathbf{R}^{1/2}\mathbf{i})].$$

This implies the inequality for the powers

$$(\mathbf{R}^{-1/2}\mathbf{u})\circ(\mathbf{R}^{1/2}\mathbf{i}) = P \le \sqrt{[(\mathbf{R}^{-1/2}\mathbf{u})\circ(\mathbf{R}^{-1/2}\mathbf{u})]\times[(\mathbf{R}^{1/2}\mathbf{i})\circ(\mathbf{R}^{1/2}\mathbf{i})]} \tag{5}$$

and the formula for the apparent power of a four-wire power system, obtained by another way in [15]:

$$S = P_{\max} = \sqrt{[(\mathbf{R}^{-1/2}\mathbf{u})\circ(\mathbf{R}^{-1/2}\mathbf{u})]\times[(\mathbf{R}^{1/2}\mathbf{i})\circ(\mathbf{R}^{1/2}\mathbf{i})]} = \sqrt{\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}^{-1}\mathbf{u}(t)dt \times \frac{1}{T}\int_0^T \mathbf{i}^\wedge(t)\mathbf{R}\mathbf{i}(t)dt}. \tag{6}$$

The apparent power (6) contains a second multiplier in the form of precisely the power losses, rather than the square rms value of the line currents and is valid for an arbitrary relationship between the active resistances of the wires. For a three-wire system with $\mathbf{R} = r\mathbf{I}$ and $\mathbf{R}^{-1}\mathbf{u}(t) = r^{-1}\mathbf{u}(t)$ formula (6) transforms into the Buchholz's formula of apparent power (4). Similarly, these formulas are equivalent in the absence of a zero-sequence component in the voltage vector, when $\mathbf{j}^T\mathbf{u}(t) = 0$. It was shown in [16] that apparent power formula (6) under the certain condition is fully equivalent to the standardized one [11] and remove the uncertainty factor in that IEEE standard. Also, as was shown in [17], it is fully consistent with the approach of the European standard [12] to determine the apparent power for the case of sinusoidal mode of the power supply system.

Equality in formula (5) takes place under the condition of proportionality of the vectors $\mathbf{R}^{1/2}\mathbf{i}(t) = k\mathbf{R}^{-1/2}\mathbf{u}(t)$, which is equivalent to

$$\mathbf{i}(t) = k\mathbf{R}^{-1}\mathbf{u}(t). \tag{7}$$

Let's clarify the physical meaning of the vector $\mathbf{i}_S(t) = \mathbf{R}^{-1}\mathbf{u}(t)$. Obviously, this is the short circuit current vector of voltage source loaded by transmission line resistances. Really, when the load terminals are closed, it satisfies the equation

$$\mathbf{u}(t) - \mathbf{R}\mathbf{i}_S(t) = \mathbf{0}.$$

Thus, the first multiplier in (6) is short circuit power P_S of the voltage source loaded by cable resistances and apparent power is the average geometric value of currents power losses and short circuit power of the voltage source loaded by transmission line resistances:

$$S = \sqrt{P_S \times \Delta P}. \tag{8}$$

This definition (8) is valid for two types of three-phase circuits, both four -wire and three-wire. In the latter case

$$\sqrt{P_S \times \Delta P} =$$

$$= \sqrt{\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t) \times (r\mathbf{I})^{-1} \times \mathbf{u}(t)dt \times \frac{1}{T}\int_0^T \mathbf{i}^\wedge(t) \times (r\mathbf{I}) \times \mathbf{i}(t)dt} = S_B.$$

When the current of a three-phase source is formed by shunt active filter (SAF), formula (7) implements the strategy of achieving a unity power factor with minimum energy losses [18]. The value of the proportionality coefficient is determined from the condition of the zero active power of the SAF in the form

$$k = \frac{P}{\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}^{-1}\mathbf{u}(t)dt} = \frac{P}{P_S}.$$

Substitution of this value into formula (7) gives the refined value of the active current in the case of a four-wire three-phase power supply system that minimizes power losses in transmission line:

(9) $$\mathbf{i}_A(t) = \frac{P}{\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}^{-1}\mathbf{u}(t)dt}\mathbf{R}^{-1}\mathbf{u}(t) = \frac{P}{P_S}\mathbf{i}_S(t).$$

In the absence of voltage zero-sequence component or in the case of a three-wire transmission line, expressions (1) and (9) are equivalent. The expression in the denominator of formula (9) and the first multiplier of the apparent power in formula (6) is the short-circuit power of the three-phase voltage source loaded by cable resistances. Therefore, the active current of an arbitrary three-phase power supply system is part of the source short-circuit current vector equal to the ratio of the load power to the short-circuit power of the voltage source.

Using the ratio for the line resistance matrix, we simplify the expression for the short circuit current vector:

(10) $$\mathbf{i}_S(t) = \mathbf{R}^{-1}\mathbf{u}(t) = (r\mathbf{I} + r_N\mathbf{j}\mathbf{j}^\wedge)^{-1}\mathbf{u}(t) =$$

$$= r^{-1}\left[\mathbf{I} - \frac{r_N}{r(1 + r_N\mathbf{j}^T\mathbf{j}/r}\mathbf{j}\mathbf{j}^\wedge\right]\mathbf{u}(t) =$$

$$= r^{-1}\left[\mathbf{u}(t) - \frac{3r_N}{r + 3r_N} \times \frac{\mathbf{j}^\wedge\mathbf{u}(t)}{3}\mathbf{j}\right] = r^{-1}\left[\mathbf{u}(t) - \sigma\mathbf{u}_0(t)\right]$$

where $\mathbf{u}_0(t) = \frac{\mathbf{j}^\wedge\mathbf{u}(t)}{3}\mathbf{j} = \frac{\mathbf{j}^\wedge\mathbf{u}(t)}{\sqrt{3}} \times \frac{\mathbf{j}}{\sqrt{3}} = u_0(t)\mathbf{j}$– the voltage zero sequence component; $\sigma = 3r_N/(r + 3r_N)$– the optimum value of the attenuation coefficient of this component that provides minimal power losses [18]. Substituting these values in (9), we obtain an expression for the active current of a three-phase four-wire system, which is used for practice

(11) $$\mathbf{i}_A(t) = \frac{P}{\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{u}_\sigma(t)dt}\mathbf{u}_\sigma(t),$$

where $\mathbf{u}_\sigma(t) = \mathbf{u}(t) - \sigma\mathbf{u}_0(t)$.

Neutral current caused by active current vector is given by

(12) $$i_{NA}(t) = \mathbf{j}^\wedge\mathbf{i}_A(t) = \frac{P}{\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{u}_\sigma(t)dt}\mathbf{j}^\wedge\mathbf{u}_\sigma(t) =$$

$$= \frac{P[\mathbf{j}^\wedge\mathbf{u}(t) - \sigma\mathbf{j}^\wedge\mathbf{j}u_0(t)]}{U^2 - \sigma U_0^2} = \frac{3P(1-\sigma)}{U^2 - \sigma U_0^2}u_0(t),$$

where $U_0^2 = \mathbf{u}_0 \circ \mathbf{u}_0 = \frac{3}{T}\int_0^T u_0^2(t)dt.$

Expression (12) completely coincides with the optimal value of the neutral current in the paper [13] that minimizes instantaneous power losses, after the replacement of the instantaneous quantities instead of the integral ones.

The significance of the proposed formulas for practice

Let us compare the apparent powers that are determined by the expressions (4) and (6), finding their values as the maximum active powers caused by the corresponding active currents at the same value of power losses ΔP. Power losses during the flow of Fryze's active current (1) proportional to the voltage vector, would be

$$\Delta P = \frac{G_F^2}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}\mathbf{u}(t)dt,$$

where the magnitude of the scalar factor is given by

$$G_F = \sqrt{\Delta PT\left[\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}\mathbf{u}(t)dt\right]^{-1}}.$$

Similarly, we find the power losses from the flow of an active current (9) proportional to the short-circuit current, as follows:

$$\Delta P = \frac{G_A^2}{T}\int_0^T [\mathbf{R}^{-1}\mathbf{u}(t)]^\wedge\mathbf{R}[\mathbf{R}^{-1}\mathbf{u}(t)]dt = \frac{G_A^2}{T}\int_0^T \mathbf{u}(t)^\wedge\mathbf{R}^{-1}\mathbf{u}(t)dt$$

and the magnitude of the scalar factor is

$$G_A = \sqrt{\Delta PT\left[\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}^{-1}\mathbf{u}(t)dt\right]^{-1}}.$$

The ratio of apparent powers founded as maximum active ones is specified by

(13) $$\frac{S}{S_B} = \frac{G_A}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}^{-1}\mathbf{u}(t)]dt \div \frac{G_F}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{u}(t)dt =$$

$$= \sqrt{\frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}^{-1}\mathbf{u}(t)dt \times \frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}\mathbf{u}(t)dt} / U^2.$$

and is determined only by the voltage vector and the parameters of the resistance matrix. Let us single out the orthogonal components of the phase voltage vector

$$\mathbf{u}(t) = \mathbf{u}_{\alpha\beta}(t) + \mathbf{u}_0(t),$$

where $\mathbf{u}_{\alpha\beta}(t)$ - orthogonal component localized in the *αβ* – plane [5], then

$$\mathbf{R}^{-1}\mathbf{u}(t) = r^{-1}[\mathbf{u}_{\alpha\beta}(t) + (1-\sigma)\mathbf{u}_0(t)];$$

$$\mathbf{R}\mathbf{u}(t) = r\left[\mathbf{u}(t) + \frac{3r_N}{r} \times \frac{\mathbf{j}^T\mathbf{u}(t)}{3}\mathbf{j}\right] = r\left[\mathbf{u}_{\alpha\beta}(t) + \frac{\mathbf{u}_0(t)}{1-\sigma}\right].$$

With this in mind the expression (13) is transformed to the form

(14) $$\frac{S}{S_B} = \frac{\sqrt{[U_{\alpha\beta}^2 + U_0^2(1-\sigma)][U_{\alpha\beta}^2 + (1-\sigma)^{-1}U_0^2]}}{U^2},$$

where $U_{\alpha\beta}^2 = \mathbf{u}_{\alpha\beta} \circ \mathbf{u}_{\alpha\beta} = U^2 - U_0^2$- square of the effective value of the corresponding orthogonal component.

Fig. 2 shows the graphs of apparent power relative difference

$$\delta_S = \frac{S - S_B}{S_B}\times 100\% = \left[\sqrt{1+(1-\sigma)^{-1}\sigma^2\Delta_0^2(1+\Delta_0^2)} - 1\right]\times 100\%$$

as function of voltage zero-sequence factor $\Delta_0^2 = U_0^2 / U^2$ and the relationship between the resistances r and r_N.

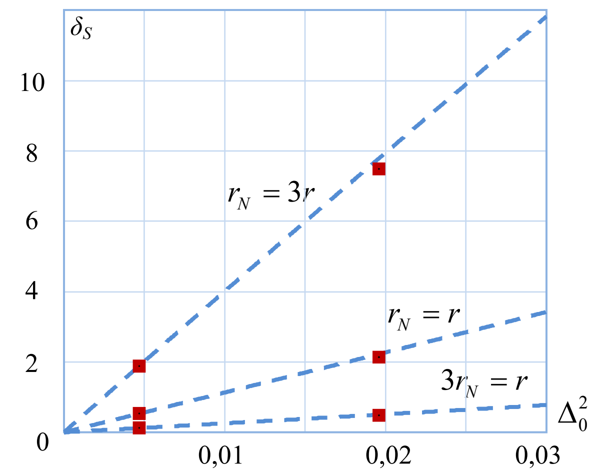

Fig. 2. Apparent power relative difference versus voltage zero-sequence factor under different values of transmission line resistances (dotted lines) and computer simulation results (□ marked pots).

The discrepancy between the values of the apparent powers may exceed 10% for large relative values of the neutral resistance and zero-sequence factor. This leads to overestimated value of the power factor, the denominator of which is the Buchholz's apparent power (4), and insufficient power losses minimization under the SAF's control strategy that forms Fryze's active current (1).

The minimum possible power losses in the transmission line are realized with the SAF's optimal control strategy that forms the active current (9):

(15) $$\Delta P_{MIN} = \frac{k^2}{T}\int_0^T [\mathbf{R}^{-1}\mathbf{u}(t)]^\wedge \mathbf{R}\mathbf{i}_S(t)dt = \frac{P^2}{P_S^2}\times P_S = \frac{P^2}{P_S}.$$

We find the power losses gain in the application of SAF with optimal control strategy using (8), (15) and neglecting its own losses:

(16) $$W = \Delta P / \Delta P_{MIN} = (S^2 / P_S) \div (P^2 / P_S) = S^2 / P^2 = \lambda^{-2},$$

where $\lambda = P/S -$ power factor, in which the value of apparent power is represented by the formula (8).

The last ratio leads to calculation formula for the experimental determination of the power factor:

(17) $$\lambda = W^{-0.5} = \sqrt{\Delta P_{MIN} / \Delta P}.$$

We find the power losses gain in the formation by SAF of active currents (1) and (9):

(18) $$W_F = \frac{1}{\lambda_F^2} = \frac{\frac{1}{T}\int_0^T \mathbf{i}_F^\wedge(t)\mathbf{R}\mathbf{i}_F(t)dt}{\Delta P_{MIN}} = \frac{\frac{(P/U^2)^2}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}\mathbf{u}(t)dt}{P^2 / P_S} =$$

$$= \frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}^{-1}\mathbf{u}(t)dt \times \frac{1}{T}\int_0^T \mathbf{u}^\wedge(t)\mathbf{R}\mathbf{u}(t)dt / U^2.$$

By comparing formulas (14), (17), (18), we establish that

(19) $$S / S_B = \sqrt{W_F} = 1/\lambda_F = \sqrt{\Delta P_F / \Delta P_{MIN}}.$$

In particular, the ratio of apparent powers $S/S_B = 1.1$ corresponds to the power factor $\lambda_F = 0.909$ and the power losses gain $W_F = \Delta P_F / \Delta P_{MIN} = 1.21$. Thus, in the presence of voltage zero-sequence component SAF control strategy, which implements the proposed active current, provides up to 21% power losses gain in the transmission line compared to the original Fryze's definition.

Experimental verification of the proposed formulas

For the experimental verification of the proposed formulas, it is sufficient to measure and compare the power losses in the transmission line with different SAF control strategies that form the currents of the three-phase source by formulas (1) and (11). Since the difference between the proposed formulas and definitions is manifested for the non-zero value of the zero-sequence factor, let's set a non-symmetric vector of phase voltages

$$\mathbf{u}(t) = \begin{Vmatrix} u_A(t) \\ u_B(t) \\ u_C(t) \end{Vmatrix} = \begin{Vmatrix} (U_m + \Delta U)\sin(\omega t) \\ (U_m - \Delta U)\sin(\omega t - 2\pi/3) \\ (U_m - \Delta U)\sin(\omega t + 2\pi/3) \end{Vmatrix}.$$

In [18] it is shown that for such a voltage vector, the parameter $\Delta_\perp^2 = U_0^2 / U_{\alpha\beta}^2$ is related to the amplitude relative instability $\delta = \Delta U / U_m$ by the relation

(20) $$\Delta_\perp^2 = \frac{4\delta^2}{9 - 6\delta + 5\delta^2}.$$

Given that $\Delta_0^2 = U_0^2 / U^2 = \Delta_\perp^2 / (1 + \Delta_\perp^2)$,

$$\Delta_0^2(\delta = 0.1) = 0.00471; \Delta_0^2(\delta = 0.2) = 0.0196.$$

Computer simulation of a three-phase four-wire system was carried out with the specified supply voltages and nonlinear load as three-phase diode rectifier scheme (Fig.3) in the PLECS environment with parameters $U_m = 220\sqrt{2}V; R_L = 1\Omega; r = 2\times 10^{-4}\Omega$. Active currents in the transmission line according to (1), (11) were formed by active compensator with a control system implemented on dependent current sources.

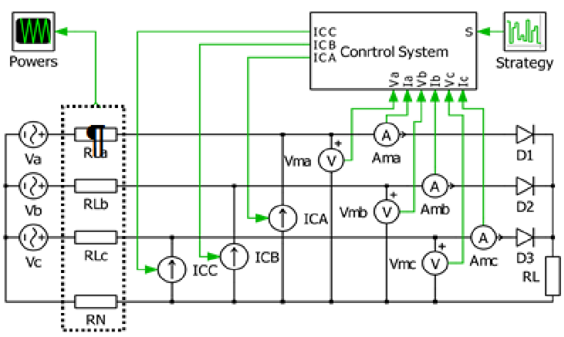

Fig. 3. Computer model of three-phase four-wire power system

Instantaneous power losses were measured on resistors of transmission line and their waveforms are presented on diagram (Fig. 4) for parameter set r_N = 3r; $\delta = 0.2$. The average values of power losses are represented by dashed lines and for this case are $\Delta P_F = 6.946W$ and $\Delta P_{MIN} = 6.012W$ respectively. According to this diagram, the power losses gain in comparison with Fryze's strategy is $W_F = 6.946/6.012 = 1.155$, and apparent power relative difference is $\delta_S = \left(\sqrt{\Delta P_F / \Delta P_{MIN}} - 1\right)\times 100\% = 7.487\%.$.

Both these values are entered in the corresponding cells of Table 1 and δ_S is plotted on the theoretical dependency graph (Fig. 2) with $\Delta_0^2 = 0.0196(\delta = 0.2)$. The results of similar experiments with other values of the transmission line parameters and asymmetry are reflected by other 5 experimental points in Fig. 2 and summarized in the Table 1.

Table 1. The results of similar experiments

		$r_N = 3r$	$r_N = r$	$3r_N = r$
δ = 0.1 Δ^2*=0.00471*	ΔP_F ,W	6,351	6,179	6,122
	ΔP_{MIN} ,W	6,119	6,115	6,108
	W_F	1,038	1,010	1,002
	δ_S ,%	1.877	0.525	0.116
δ = 0.2 Δ^2*=0.0196*	ΔP_F ,W	6,946	6,253	6,022
	ΔP_{MIN} ,W	6,012	5,994	5,965
	W_F	1,155	1,043	1,009
	δ_S ,%	7.487	2.137	0.478

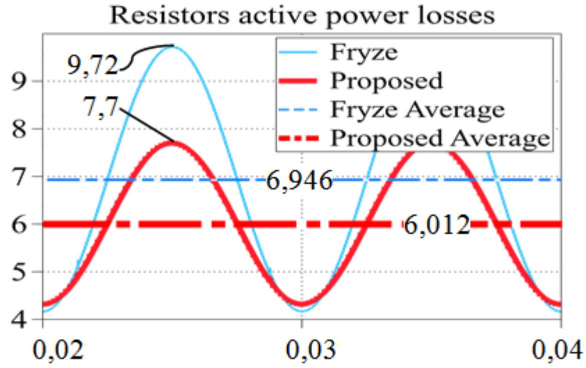

Fig. 4. Waveforms of instantaneous power losses and their average values

In general, the results of the experiment confirm the adequacy of the proposed formula of apparent power and illustrate the possibility of power losses reducing up to 15% by implementing the proposed active current compared to the original Fryze's one in the presence of voltage zero-sequence component.

Conclusions

The Fryze's active current with the given load active power and source voltages minimizes the rms value of the line currents that corresponds to current multiplier of the Buchholz's formula of apparent power.

A new formula for apparent power of the three-phase four-wire system was justificated by theoretical and experimental researches. It is equal to the average geometric value of the currents power losses and short circuit power of the voltage source loaded by transmission line resistances. It definition is fully consistent with modern standards, allows us to correctly calculate both the power factor and power losses gain as well as formulate SAF's control strategy with unit power factor.

The generalized formula for introduced by Professor Fryze concept of power system's active current is grounded. It is part of the short-circuit current that is equal to ratio between load power and short-circuit power of the voltage source. This active current transfers to a load of the three-phase power supply system the given energy with minimal power losses in transmission line. Computer simulation showed that SAF control strategy, which implements the proposed active current, provides up to 15% power losses gain in the transmission line compared to the original Fryze's definition in the presence of voltage zero-sequence component.

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