PRZEGLĄD ELEKTROTECHNICZNY

Ukazuje się od 1919 roku 4'20

Organ Stowarzyszenia Elektryków Polskich • Wydawnictwo SIGMA-NOT Sp. z o.o.

Invited paper Leszek S. CZARNECKI

School of Electrical Engineering and Computer Science, Louisiana State University, Baton Rouge, USA

doi:10.15199/48.2020.04.01

Currents' Physical Components (CPC)–based Power Theory A Review Part II: Filters and reactive, switching and hybrid compensators

Abstract. *The CPC-based power theory (PT) provides fundamentals for the development of the reactive compensators needed for the improvement of the effectiveness of the energy transfer. It also provides fundamentals for the development of algorithms for control of switching compensators and hybrid compensators that serve the same purpose. These fundamentals have been developed, along with the CPC, step-by-step, with partial results published in Polish, German, English and American journals and in conference proceedings, often not reported on the main databases and consequently, difficult to be found. This Review provides a draft of these CPC-based fundamentals of compensation with references to more detailed results.*

Streszczenie. *Teoria mocy obwodów i systemów elektrycznych, oparta na koncepcji Składowych Fizycznych Prądów, tworzy podstawę teoretyczną syntezy kompensatorów potrzebnych do poprawy skuteczności przenoszenia energii. Dotyczy to także podstaw konstrukcji algorytmów sterowania kompensatorów kluczujących oraz hybrydowych. Metody kompensacji, oparte na teorii mocy CPC, były rozwijane stopniowo, z częściowymi wynikami publikowanymi w polskich, niemieckich, angielskich oraz amerykańskich czasopismach i materiałach konferencyjnych, często nie notowanych w głównych indeksach i trudnych do odnalezienia. Przegląd ten zestawia wynikające z teorii mocy CPC metody syntezy kompensatorów reaktancyjnych i algorytmy sterowania kompensatorów kluczujących, ze wskazaniem źródeł bardziej szczegółowych wyników.* **(Teoria mocy oparta na koncepcji Składowych Fizycznych Prądów (CPC). Część II: Filtry oraz compensatory reaktancyjne, kluczujące i hybrydowe. Artykuł przeglądowy)**

Keywords: Active filters, instantaneous reactive power, unbalanced loads, asymmetrical systems, nonsinusoidal systems.
Słowa kluczowe: Filtry aktywne, chwilowa moc bierna, odbiorniki niezrównoważone, systemy niesymetryczne, niesinusoidalne.

Introduction

A search for theoretical fundamentals for design or control of compensators that are capable of improving the effective-ness of the electric energy transfer is one of the main practical objectives of the power theory development.

A reduction of the reactive power Q was traditionally the only goal of compensation. In systems with sinusoidal voltages and currents, a shunt capacitor of the capacitance

$$C = \frac{Q}{\omega U^2} \tag{1}$$

compensates, at the voltage of the rms value U, the reactive power Q entirely.

Steinmetz [1] included, moreover, symmetrization of the load current in compensation goals. Also reduction of the cur-rent distortion, transients, or effects of the load power variabi-lity could be now the compensation objectives [23, 35].

This search for theoretical fundamentals of compensation in the presence of the voltage distortion has occurred to be one of the toughest challenges of electrical engineering in the XX century. It was the main practical goal of the power theory (PT) development, drafted in Part I [46] of this Review. Unfortunately, most of the "schools" of the PT have failed to provide any fundamentals for a compensator design or have provided some of them, but at restricted conditions.

Theoretical fundamentals for reactive compensators syn-thesis and control were provided eventually in a frame of the CPC – based PT of electrical circuits and systems.

The first solution of reactive compensation of a single-phase linear load in the presence of the supply voltage harmonics was published [7] in 1981. Fundamentals of the synthesis of compensators for more complex loads were developed step-by-step and published in various journals and con-ference proceedings. This Review provides a draft of these, CPC-based, compensation fundamentals, as well as funda-mentals of switching and hybrid compensators control; com-piles partial results and provides information where more detailed ones can be found.

A concept of a load compensation

A reduction of the inequality between the apparent power S of the energy provider and the active power P of the energy consumer

$$S \geq P \tag{2}$$

is the main objective of the electrical loads compensation.

Since the supply voltage affects both sides of this inequa-lity, a reduction of the inequality between the supply current $\boldsymbol{i}$ and the load active current $\boldsymbol{i}_{\mathrm{Ca}}$ three-phase rms values $||.||$,

$$||\boldsymbol{i}|| \geq ||\boldsymbol{i}_{\mathrm{Ca}}|| . \tag{3}$$

is the more specific goal of the compensation.

Compensation was confined traditionally to a reduction of the reactive power Q. The CPC-based PT reveals now [44, 46] several physical phenomena, associated with various cur-rents' components, responsible for the inequality (3). Namely, the difference, in squares, between both sides of the inequa-lity (3), expressed in the CPC terms, is equal to

(4) $$||\boldsymbol{i}\,||^2 - ||\boldsymbol{i}_{\mathrm{Ca}}||^2 = ||\boldsymbol{i}_{\mathrm{Cs}}||^2 + ||\boldsymbol{i}_{\mathrm{Cr}}||^2 + ||\boldsymbol{i}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}_{\mathrm{G}}||^2$$

with

(5) $$||\boldsymbol{i}_{\mathrm{Cu}}||^2 = ||\boldsymbol{i}^{\,\mathrm{z}}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}^{\,\mathrm{p}}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}^{\,\mathrm{n}}_{\mathrm{Cu}}||^2$$

thus, six different components contribute to this difference. Their meanings and definitions are discussed in Part I of the Review [46]. Reduction three-phase rms value of any of these components, without increasing other ones, contributes to a reduction of the supply current, i.e., to the load compensation. Each of these currents occurs due to a different physical phe-nomenon in the load. Different compensator with specific pro-perties can be needed for their reduction.

History of the compensation methods development

The first concepts of the power theory, as suggested by Budeanu and Fryze, did not suggest any method of the loads compensation. The reduction of the reactive power Q, as defined by these theories, does not have to reduce [28] the apparent power and improve the power factor.

The first solution as to the reactive power compensation was obtained [3] by Shepherd and Zakikhani in 1972. Accord-ing to [3], if the supply voltage has the waveform

(6) $$u = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n)$$

and the load current is

(7) $$i = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \alpha_n - \varphi_n)$$

then, a capacitor connected as shown in Fig. 1,

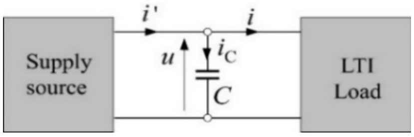

Fig. 1. An LTI load with a compensating capacitor.

reduces the supply current $i'(t)$ rms value to its minimum, if the capacitance, called ***the optimal***, C_{opt}, has the value

(8) $$C = C_{\mathrm{opt}} = \frac{\sum_{n=1}^{\infty} nU_n I_n \sin\varphi_n}{\omega_1 \sum_{n=1}^{\infty} n^2 U_n^2}\,.$$

Its calculation requires that the phase-shift φ_n if the voltage and current harmonics are known. Unfortunately, at the time when this result was obtained, instrumentation for the Digital Signal Processing was not jet developed and this phase-shift practically was not available by a measurement. This obstacle was removed by Kusters and Moore [6], who found an equi-valent formula for the optimal capacitance in the time-domain, i.e., without the use of harmonics, namely

(9) $$C_{\mathrm{opt}} = -\frac{Q_{\mathrm{C}}}{||\,u\,||\,||\,\dot{u}\,||}$$

where

(10) $$\dot{u}(t) = \frac{d}{dt}u(t), \qquad Q_{\mathrm{C}} = ||u||\,||i_{\mathrm{qC}}||\ \mathrm{sgn}(\dot{u}, i)$$

with

(11) $$i_{\mathrm{qC}}(t) = \frac{(\dot{u}, i)}{||\dot{u}||^2}\,\dot{u}(t)\,.$$

This method of the capacitance calculation was recom-mended [5] by the International Electrotechnical Commis-sion. Unfortunately, as it was shown in [8, 11], in real power systems, where the supply sources have an inductive impe-dance, the formula (9) does not provide the right value of the optimal capacitance.

Some progress in methods of compensation was achie-ved with the development of the Instantaneous Reactive Power (IRP) p-q Theory [9] by Nabae, Akagi and Kanazawa. They introduced the concept of the instantaneous active and reactive powers p and q, defined as

(12) $$p = u_\alpha\, i_\alpha + u_\beta\, i_\beta\,, \qquad q = u_\alpha\, i_\beta - u_\beta\, i_\alpha$$

where the quantities with indices α and β are the results of Clarke's Transform of three-phase voltages and currents at the load terminals to α and β orthogonal coordinates, namely

(13) $$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} x_{\mathrm{R}} \\ x_{\mathrm{S}} \end{bmatrix}.$$

The power properties of the load are described in the IRP p-q Theory in terms of two powers, the instantaneous reactive power q and the instantaneous active power p. The last power has a constant and an oscillating component

(14) $$p = \bar{p} + \tilde{p}\,.$$

According to IRP p-q Theory, a compensator, connected as shown in Fig. 2, should be controlled in such a way, that it burdens the supply source with the negative values of the load instantaneous reactive power and the oscillating com-ponent of the load instantaneous active power. Thus, in effect of such control, the instantaneous active power of the supply source after compensation should be constant.

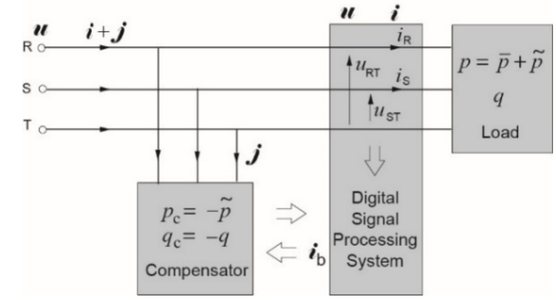

Fig. 2. Control idea of a switching compensator.

Such compensators are commonly known as "active power filters" or "active harmonic filter". Usually, a voltage source inverter is the main component of such a filter. The name an "active power filter" does not describe correctly the properties of such a device, however. It is not active, but a passive device. It does not operate, moreover, as a filter. It burdens the supply lines with a current $\boldsymbol{j}$ which compensates undesirable components of the supply current. Thus it is a compensator, but not a filter. It can compensate not only harmonics but also the reactive and unbalanced currents and even some transient components of the load current. Thus the adjective "harmonic" is too narrow for such a compen-sator. Because the main feature of such a compensator is shaping its current by fast switching of transistors, it is refer-red to [33, 35, 36] as a "***switching compensator***".

The control strategy suggested by the IRP p-q theory has become very popular. This strategy is right on the condition that the supply voltage is sinusoidal and symmetrical, how-ever. When this condition is not satisfied, then the IRP p-q Theory-based algorithm generates erroneous control signals for the compensator. This is demonstrated on a circuit with a resistive load, shown in Fig. 3, the load that does not require any compensation.

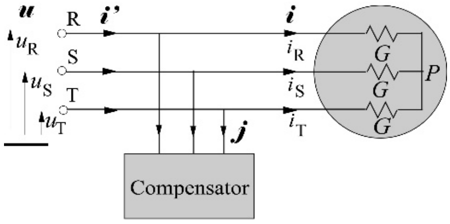

Fig. 3. A balanced purely resistive load with a compensator.

If the supply voltage in such a circuit is distorted by the 5th order harmonic, so that

(15) $$\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_5$$

and assuming for the sake of simplicity that

(16) $$u_{R1} \triangleq \sqrt{2}\, U_1 \cos\omega_1 t, \qquad u_{R5} \triangleq \sqrt{2}\, U_5 \cos 5\omega_1 t$$

then, as it was proven in [34], the instantaneous power is

(17) $$p = P_1 + P_5 + 6GU_1U_5 \cos 6\omega_1 t = \bar{p} + \tilde{p}$$

where P_1 and P_5 denote the active power of 1st and 5th order harmonics. It means that the instantaneous active power of an ideal load, which does not require any compensation, can contain an oscillating component. A compensator that com-pensates this component loads the source [34] with an addi-tional current which increases the apparent power S of the supply, thus instead of improving, it reduces the power factor. A similar situation occurs when the supply voltage is asym-metrical even if this voltage is sinusoidal. The instantaneous active power $p(t)$ of the balanced resistive load at such a voltage contains an oscillating component which does not affect the load power factor. The power factor is reduced [37], however, when a compensator, controlled according to the IRP p-q Theory, attempts to compensate this oscillating com-ponent. It means that in the presence of the supply voltage distortion or asymmetry, the IRP p-q Theory-based compen-sators control algorithms are erroneous.

Some attempts aimed at the development of a compensa-tion method were based on the Conservative Power Theory (CPT), suggested by Tenti and others [30]. The CPT is closely related to the Budeanu's power theory, however, only formu-lated in the time-domain. Therefore, as demonstrated in [41,43], the CPT-based methods of the compensator synthe-sis do not provide the right values of its parameters.

Parameters of a compensator can be found, of course, using optimization procedures, which minimizes the supply current rms value. The power theory of electrical circuits is even not needed for that. A lot of computation is needed for such procedures, however, so that at adaptive compensation of time-varying loads, optimization procedures could be too slow. In such a situation, the parameters of the compensator should be calculated from algebraic formulas.

Loading and supply qualities

Reduction of the energy transfer effectiveness in balan-ced systems with sinusoidal voltages and currents is caused only by a phase-shift between the load current and the supply voltage. In unbalanced systems with nonsinusoidal voltages and currents, there are several different agents and mecha-nisms of this effectiveness degradation. This effectiveness depends, first of all, on the ***loading quality*** (LQ) of electrical loads. It is illustrated in the diagram shown in Fig. 4.

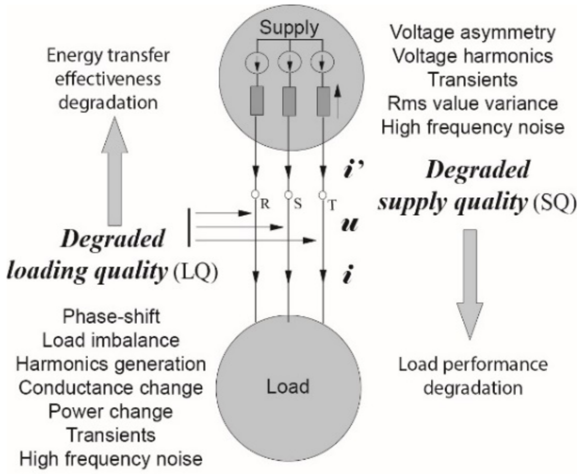

Fig. 4. A diagram that illustrates the effects of the physical phenomena in the load upon the supply quality.

The loading quality can be regarded as an ideal when the load is purely resistive, balanced and time-invariant. Any deviation from such properties can be regarded as a degradation of the loading quality. A phase-shift, the load imbalance, generation of the current harmonics, the load power variation, transients, and the high-frequency noise, contribute to the loading quality degradation. These phenomena contribute to the degradation of the effectiveness of the energy transfer from the supply source to electrical loads.

Synchronous generators in power systems produce a voltage which can be regarded, with high accuracy, as sym-metrical and sinusoidal. Loads with a degraded LQ can cause, due to a voltage drop on the power system impedan-ces, distortion, asymmetry, rms value variation, transients and the high-frequency noise in the distribution voltage. All these agents contribute to a degradation of the ***supply quality*** (SQ) which can degrade the performance of electrical loads. This is illustrated in the diagram shown in Fig. 4.

Observe that some phenomena that degrade the LQ, such a phase-shift, load imbalance, harmonics generation or the load conductance change, can be specified and observed in a frequency-domain. Remaining ones are random, obser-ved in a time-domain.

Filters and compensators

A shunt device connected in the power system as shown in Fig. 4, or sometimes a series-shunt [15, 23, 29] device, that modifies the supply current ***i'*** in such a way that some of its components are reduced, is referred to, depending on the operation principle, as a compensator or as a filter.

A device that loads the system with a required specific current is referred to as ***a compensator***. A device that for some frequencies has lower impedance than for other ones is referred to as ***a filter***. Some devices, such as ***resonant harmonic filters*** (RHF) are at the same time compensators and filters. They compensate the reactive current of the load by a capacitive current, and they provide a low impedance path for some current harmonics originated in the load.

Compensators can be built as reactive ones (RC), which change the susceptance as seen from the supply source, or as switching compensators (SC), which load the system with currents shaped by fast switched transistors.

The SCs can reduce all effects of the degraded loading quality, while the RCs can reduce only the effects of the voltage and current phase-shift and the load imbalance. Moreover, the SCs, by the principle of their operation, have the adaptive property, while RCs are essentially fixed-parameter devices. They can be, however, converted to adaptive com-pensators using thyristor switched inductors (TSI). Since thyristors have the switching power by approximately one order higher than transistors used in switching compensators, the adaptive reactive compensators can have the power much higher than the power of switching compensators. Thyristors' switching frequency is low, however, so that the adaptive RCs can reduce only the current components of the fundamental frequency, while the SCs can reduce the current components even in the frequency range of several kHz. To exploit advantages of reactive and switching compensators, they can be built as ***hybrid compensators***, i.e., composed of the reactive and switching ones.

Resonant harmonic filters

Harmonics generating loads (HGL) cause distortion of the supply current. It can be expressed in the form

$$i = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{I}_n e^{jn\omega_1 t} \tag{18}$$

where N is the set of orders n of the current harmonics and $\boldsymbol{I}_n$ denotes their complex rms (crms) value. The total harmonic distortion (THD) of the load current is defined as

$$\delta_\mathrm{i} = \frac{||i_\mathrm{d}||}{I_1}, \quad ||i_\mathrm{d}|| = \sqrt{\sum_{n\in N_\mathrm{d}} I_n^2}, \quad N_\mathrm{d} = N - \{1\}\,. \tag{19}$$

Similarly, we can define the THD of the load voltage

$$\delta_\mathrm{u} = \frac{||u_\mathrm{d}||}{U_1}, \quad ||u_\mathrm{d}|| = \sqrt{\sum_{n\in N_\mathrm{d}} U_n^2} \tag{20}$$

where U_n denotes the rms values of the voltage harmonics. Resonant harmonic filtres are the oldest devices used for reduction harmonics injected into the supply systems by HGLs. Such filters are composed of, as this is illustrated for a single line in Fig. 5, a few parallel branches with an inductor and a capacitor, tuned to a resonant frequency in a vicinity of the frequency of the load generated harmonics, which should be reduced in the supply current. For the fundamental frequ-ency, such RHFs have capacitive impedance, which is used for compensation of the reactive power. The filter parameters are calculated from formulae

$$C_k = \left[1 - \left(\frac{\omega_1}{t_k}\right)^2\right]\frac{a_k Q_1}{\omega_1 U_1^2}, \qquad L_k = \frac{1}{t_k^2 C_k}. \tag{21}$$

where t_k denotes the tuning frequency of the branch which should reduce the k^{th} order current harmonic and $a_k = Q_k/Q_1$ is the reactive power allocation to that branch coefficient.

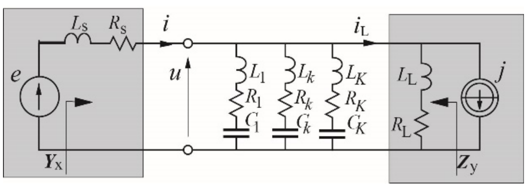

Fig. 5. A circuit with an RHF.

The filter and the supply source impedance create a cur-rent divider for the load generated harmonics. Because the filter has for selected harmonics a low impedance, the supply source is protected against the load originated harmonics

The filtering effectiveness in the reduction of the supply current and voltage harmonics reduction can be defined as

$$\varepsilon_\mathrm{i} = 1 - \frac{\delta_\mathrm{i}}{\delta_\mathrm{i0}}, \qquad \varepsilon_\mathrm{u} = 1 - \frac{\delta_\mathrm{u}}{\delta_\mathrm{u0}} \tag{22}$$

where δ_i0 and δ_u0 denote the supply current and the voltage THD in the system without a filter.

Such a method of the harmonic distortion reduction is effective as long as the internal voltage e of the distribution system is sinusoidal. When it is distorted, then the low impe-dance of the filter can cause an increase [31] of the supply current harmonics. It can be even amplified by a resonance of the filter capacitance with the supply source inductance.

There are recommendations, for example, in [25], aimed at the reduction of the effect of the supply voltage harmonics upon the filter performance, which specify the tunning frequ-ency t_k and allocation coefficients a_k. Unfortunately, such met-hods can be not effective. Optimization methods are needed instead. A weighted THD of the supply current and the voltage in a system with an RHF

$$\delta = c_\mathrm{i}\,\delta_\mathrm{i} + c_\mathrm{u}\,\delta_\mathrm{u}, \quad c_\mathrm{i} + c_\mathrm{u} = 1 \tag{23}$$

which depends on the tunning frequencies and the reactive power allocation coefficients, i.e.,

$$\delta = f\,(t_1,..,t_K, a_1,..,a_K) \tag{24}$$

should be minimized in the optimization procedure. The incor-rectness of the RHF design recommendations was validated [32] in a procedure of an optimized filter design for a four branch filter for reduction of harmonics of the order 5, 7, 11 and 13 of a load with power factor λ_1 = 0.71, at the assumption that the supply voltage is distorted by the harmonics of the same order as those generated by the load and THD equal to 2.5%, while all other harmonics contribute to THD at the level of 1%. The values of the allocation coefficients and tuning fre-quencies obtained in this procedure are compiled in Table 1.

Table 1. Allocation coefficients and tuning frequencies.

S_sc/P	20	25	30	35	40	40
a_5	0.09	0.11	0.11	0.12	0.15	0.09
a_7	0.11	0.30	0.61	0.55	0.73	0.69
a_{11}	0.60	0.46	0.21	0.14	0.08	0.10
a_{13}	0.20	0.14	0.07	0.20	0.04	0.13
t_5/ω_1	5.0	4.99	4.99	4.99	4.99	4.99
t_7/ω_1	6.99	7.00	7.02	6.95	6.88	6.86
t_{11}/ω_1	11.3	10.8	10.9	10.9	11.0	11.0
t_{13}/ω_1	13.0	13.0	13.0	12.7	13.0	13.0

This Table shows that the best allocation coefficients and tuning frequencies depend on the S_sc/P ratio, and that they do not follow any recommended pattern.

Moreover, in the presence of the distribution voltage dis-tortion, even an optimized filter might not have acceptable effectiveness. For example, at the THD of the distribution voltage at the level of 4%, the optimized filter effectiveness can have values as compiled in Table 2.

Table 2. Effectiveness of an optimized RHF at δ_e = 4%.

S_sc/P	-	20	25	30	35	40	45
ε_u	%	0.46	0.39	0.34	0.25	0.18	0.10
ε_i	%	0.32	0.30	0.25	0.20	0.16	0.10

These results demonstrate that the effectiveness of RHFs at a common level of the voltage distortion in present distribu-tion systems could not be satisfactory.

Reactive current compensation

Development of a method of compensation of the reactive power in the presence of the supply voltage distortion was one of the most important practical goals of studies upon the power theory of circuits and systems. Apart from the CPC-based PT, all approaches to its development have failed to provide fundamentals of the reactive power compensation.

According to the CPC–based PT, at the supply voltage

(25) $$u = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

the current of a linear, time-invariant (LTI) load of the admittance for the n^{th} order harmonic $\boldsymbol{Y}_n = G_n + jB_n$, is composed [7, 10, 46] of the active, scattered and reactive currents

(26) $$i = i_{\mathrm{a}} + i_{\mathrm{s}} + i_{\mathrm{r}}$$

where

(27) $$i_{\mathrm{a}} = \frac{P}{\|u\|^2}u = G_{\mathrm{e}}\, u$$

(28) $$i_{\mathrm{s}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}(G_n - G_{\mathrm{e}})\boldsymbol{U}_n\, e^{jn\omega_1 t}$$

(29) $$i_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t}\ .$$

A lossless reactive compensator of the admittance for har-monic frequencies $\boldsymbol{Y}_{Cn} = jB_{Cn}$, connected at the load terminals as shown in Fig. 6,

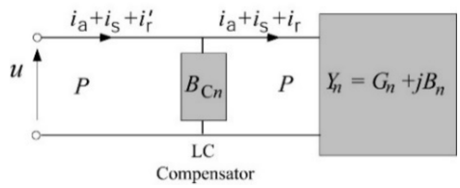

Fig. 6. A linear load with a reactive compensator.

does not affect the active power and consequently, conductances G_n, G_{e}, and the scattered current i_{s}. It modifies only the reactive current of the supply source to

(30) $$i_{\mathrm{r}}' = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j(B_{Cn} + B_n)\boldsymbol{U}_n\, e^{jn\omega_1 t}\ .$$

If for each $n \in N$

(31) $$B_{Cn} + B_n = 0$$

then the reactive current of the supply source is reduced to zero. The scattered current cannot be compensated by such a reactive compensator, however. It means, that shunt reac-tive compensators in the presence of a scattered current, can-not improve the power factor λ to unity. It was shown in [16] that it can be improved to unity with a hybrid, shunt-series reactive compensators. However, due to the complexity of such compensators, they do not have practical value.

Condition (31) enables calculation of the LC parameters of the compensator for the entire compensation of the reactive current. In fact [2], there is an infinite number of compen-sators which satisfy condition (31). Unfortunately, approxima-tely two reactive elements per harmonic are needed to build such a compensator. Thus, condition (31) has a theoretical rather than practical value. To be practical, compensator complexity has to be reduced, but it means that the reactive current cannot be compensated entirely, but only reduced.

Since the voltage sources have usually an inductive impe-dance, purely capacitive compensators can cause a resonan-ce for harmonic frequencies, thus they should be excluded from considerations. With this exclusion, the least complex compensator has the structure shown in Fig. 7.

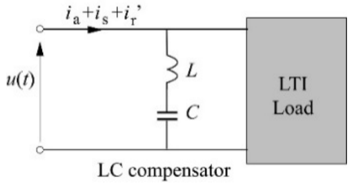

Fig. 7. A load with an LC compensator.

The system with such a compensator is prevented from a voltage resonance with the supply source inductance if the compensator impedance for all harmonics is inductive. It has to be capacitive only for the fundamental harmonic to enable compensation of the reactive power of the load.

The susceptance for harmonic frequencies of such a com-pensator is

(32) $$B_{Cn} = \frac{n\omega_1 C}{1 - n^2\omega_1^2 LC}$$

so that it changes the reactive current rms value of the supply source to

(33) $$\|i_{\mathrm{r}}'\| = \sqrt{\sum_{n\in N}\left(B_n + \frac{n\omega_1 C}{1 - n^2\omega_1^2 LC}\right)^2 U_n^2}\ .$$

It has a minimum at a capacitance when

(34) $$\frac{d}{dC}\|i_{\mathrm{r}}'\| = 0\ .$$

It was shown in paper [12], that condition (34) results in an implicit equation with respect to the capacitance C. It can be solved in an iterative procedure, with

(35) $$C_{i+1} = -\frac{\displaystyle\sum_{n\in N}\frac{nB_n U_n^2}{(1 - n^2\omega_1^2 LC_i)^2}}{\displaystyle\omega_1\sum_{n\in N}\frac{n^2 U_n^2}{(1 - n^2\omega_1^2 LC_i)^3}}$$

which starts with

(36) $$C_1 = \frac{C_0}{1 + \omega_1^2 LC_0}\ , \qquad C_0 = \frac{Q_1}{\omega_1 U_1^2}\ .$$

It results in a sequence of capacitances which converge to the capacitance that satisfies (34). The inductance L should be selected, at the designer discretion, such that

(37) $$\frac{1}{\sqrt{LC}} < 2\omega_1\ .$$

Unfortunately, a cut-and-trial approach may be needed for selection of this inductance, but this fits the iterative procedure of the compensator capacitance calculation which cannot be avoided.

The capacitance of an LC compensator, as shown in [14], can be found also in a time-domain, i.e., without using the CPC decomposition and the concept of harmonics.

As long a three-phase load is balanced, then the results obtained for compensation of the reactive current in single-phase systems can be applied, phase-by-phase, to three-phase systems. When the load is unbalanced, compensation of the reactive current can be combined with compensation of the unbalanced current.

Reactive compensation of three-phase loads supplied from three-wire lines

The load balancing was solved, for the first time, by Steinmetz in 1917 [1]. He observed that the load imbalance and consequently, the supply current asymmetry, creates rip-ples in the instantaneous power. According to [1], the supply current symmetry can be restored by a reactive compensa-tor known as a Steinmetz circuit. This solution applies only to circuits and systems with sinusoidal voltage, however.

If the supply voltage of a three-phase LTI load is symmetrical but nonsinusoidal, then the load current $\boldsymbol{i}$, apart from the active, scattered and the reactive currents $\boldsymbol{i}_a$, $\boldsymbol{i}_s$ and $\boldsymbol{i}_r$, contains [16, 46] also un unbalanced current

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{Y}_{\mathrm{u}n}\,\boldsymbol{1}_n^{*}\boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \,. \tag{38}$$

In this formula

$$\boldsymbol{1}_n = \begin{bmatrix} 1 \\ 1e^{-jn\frac{2\pi}{3}} \\ 1e^{jn\frac{2\pi}{3}} \end{bmatrix} = \begin{bmatrix} 1 \\ \beta_n^{*} \\ \beta_n \end{bmatrix} \tag{39}$$

and

$$\boldsymbol{Y}_{\mathrm{u}n} = Y_{\mathrm{u}n}\, e^{j\psi_n} = -(\boldsymbol{Y}_{\mathrm{ST}n} + \beta_n \boldsymbol{Y}_{\mathrm{TR}n} + \beta_n^{*}\boldsymbol{Y}_{\mathrm{RS}n}) \tag{40}$$

is the load unbalanced admittance. Symbols $\boldsymbol{Y}_{\mathrm{ST}n}$, $\boldsymbol{Y}_{\mathrm{TR}n}$, and $\boldsymbol{Y}_{\mathrm{RS}n}$, stand for the line-to-line equivalent admittances of the load for harmonics of the n^{th} order, as shown in Fig. 8. The reactive current of such a load is

$$\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_{\mathrm{e}n}\,\boldsymbol{1}_n \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{41}$$

where

$$B_{\mathrm{e}n} = \mathrm{Im}\{\boldsymbol{Y}_{\mathrm{RS}n} + \boldsymbol{Y}_{\mathrm{ST}n} + \boldsymbol{Y}_{\mathrm{TR}n}\} \,. \tag{42}$$

The unbalanced current, along with the reactive one, can be compensated entirely [16, 19] by a reactive compensator of the Δ structure shown in Fig. 8.

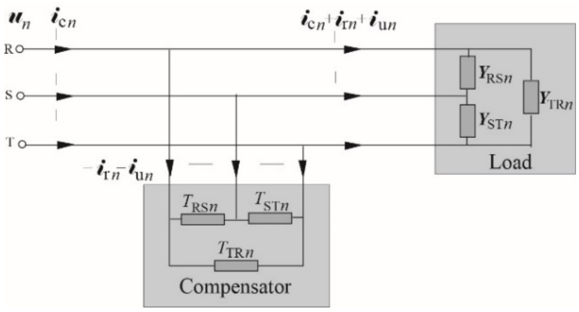

Fig. 8. Equivalent parameters of a three-phase load and a compensator for the n^{th} order harmonic.

Equivalent parameters of the circuit and the current com-ponents for the n^{th} order harmonic are shown in this figure. Symbol $\boldsymbol{i}_{\mathrm{c}n}$ denotes the current component not affected by the compensator.

The compensator reduces the n^{th} order harmonic of the reactive and unbalanced currents of the supply entirely if

$$B_{\mathrm{e}n} + (T_{\mathrm{RS}n} + T_{\mathrm{ST}n} + T_{\mathrm{TR}n}) = 0 \tag{43}$$

$$\boldsymbol{Y}_{\mathrm{u}n} - (T_{\mathrm{ST}n} + \beta_n T_{\mathrm{TR}n} + \beta_n^{*} T_{\mathrm{RS}n}) = 0 \,. \tag{44}$$

These two conditions result in the compensator susceptan-ces for the n^{th} order harmonic

$$\begin{aligned} T_{\mathrm{RS}n} &= \frac{1}{3}(s\sqrt{3}\,\mathrm{Re}\,\boldsymbol{Y}_{\mathrm{u}n} - \mathrm{Im}\boldsymbol{Y}_{\mathrm{u}n} - B_{\mathrm{e}n}) \\ T_{\mathrm{ST}n} &= \frac{1}{3}(2\,\mathrm{Im}\,\boldsymbol{Y}_{\mathrm{u}n} - B_{\mathrm{e}n}) \\ T_{\mathrm{TR}n} &= -\frac{1}{3}(s\sqrt{3}\,\mathrm{Re}\,\boldsymbol{Y}_{\mathrm{u}n} + \mathrm{Im}\,\boldsymbol{Y}_{\mathrm{u}n} + B_{\mathrm{e}n}) \end{aligned} \tag{45}$$

where s denotes a sequence index, equal to s = 1 for harmonics of the positive sequence and $s = -1$ for harmonics of the negative sequence. Numerical illustrations of such com-pensation can be found in [13, 42, 45]. A compensator with such susceptances could be very complex, however. A met-hod of these currents minimization by a compensator of a reduced complexity can be found in [18, 19]. These results can be extended also to systems with asymmetrical supply voltage [20, 48].

Reactive compensation of three-phase loads supplied from four-wire lines

Loads in industrial environments are usually supplied from three-phase lines with a neutral conductor, i.e., from four-wire lines. A presence of the neutral current changes, of course, compensation conditions. First of all, a zero sequence com-ponent can occur in the supply current. Its compensation requires that branches of the compensator are connected to the neutral conductor, creating a Y structure sub-compensa-tor, shown in Fig. 9. The current components of the positive and the negative sequence can be compensated [40] by a sub-compensator of a Δ structure.

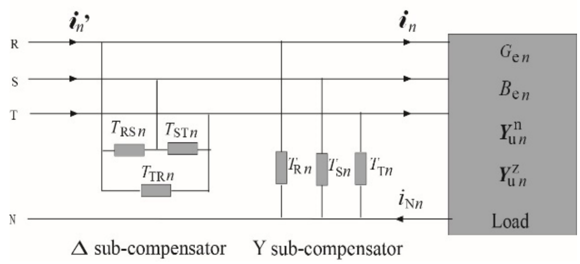

Fig. 9. A reactive compensator in a four-wire system with the load parameters for the n^{th} order harmonic.

The load current in such a system can be decomposed [39] into six Physical Components, namely

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}} \,. \tag{46}$$

Apart from the scattered current $\boldsymbol{i}_s$, which is not affected by any shunt reactive compensator, four currents contribute to the power factor degradation in such a system. These are the reactive current and three symmetrical components of the positive, negative and zero sequences of the unbalanced current $\boldsymbol{i}_{\mathrm{r}}, \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}, \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}, \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$. Their harmonics are specified by the load equivalent susceptance and three unbalanced admittan-ces of the positive, negative and zero sequences for harmonic frequencies

$$B_{\mathrm{e}n},\quad \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}},\quad \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}},\quad \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} \,.$$

Formulae for their calculation can be found in [45].

These four components of the supply current can be modi-fied by a reactive compensator. In particular, they can be reduced to zero, if the equivalent susceptance and three unbalanced admittances of the positive, negative and zero sequences for harmonic frequencies of the load with the com-pensator are equal to zero. Values of six susceptances $T_{\mathrm{XY}n}$ of the compensator have to be calculated for that.

A harmonic of the reactive current is compensated enti-rely if the equivalent susceptance of the compensated load is equal to zero. This condition results in one equation which has to be satisfied by the compensator susceptances $T_{\mathrm{XY}n}$. A harmonic of one of the unbalanced currents of a particular sequence is compensated entirely if the unbalanced admit-tance of the compensated load for that sequence is equal to zero. Such an admittance is a complex number, thus, its real and imaginary parts have to be equal to zero, and consequ-ently, the susceptances of the compensator $T_{\mathrm{XY}n}$ have to satisfy two equations. All four currents are compensated entirely on the condition that the compensator susceptances $T_{\mathrm{XY}n}$ satisfy seven equations.

Six unknown susceptances $T_{\mathrm{XY}n}$ of a compensator cannot satisfy seven equations, however. The set of these equations is contradictory. Thus, one of these four currents cannot be compensated. The selection of the current which will remain uncompensated has to be left for the compensator designer. This should be the current with the lowest three-phase rms value. It seems that in the majority of industrial situations, the lowest value has the unbalanced current of the positive sequ-ence $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$. Since also the scattered current cannot be compen-sated by a shunt reactive device, thus such a compensator cannot improve the power factor to unity.

The branches of the compensator in the presence of har-monics could be [2] very complex, however. Similarly, as in single-phase systems, approximately two reactive devices per a compensator branch are needed for each harmonic of the supply voltage. This complexity has to be substantially reduced to make reactive compensation technically accept-able. The compensator cannot have more than two reactive devices per branch.

A purely capacitive branch could cause a resonance for harmonic frequencies with the commonly inductive impe-dance of the supply source. Therefore, such branches are not acceptable. The compensator can be built [18, 47] exclusi-vely of L or LC branches shown in Fig. 10.

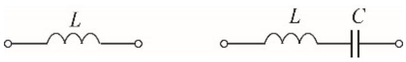

Fig. 10. Acceptable branches of a reduced complexity compensator.

An example of a synthesis procedure of such a reduced complexity compensator can be found in [47]. The results of that procedure are shown in Fig. 11. This compensator was designed at the assumption that the supply voltage funda-mental harmonic rms value was U_1 = 240 V, and the voltage was distorted by the 3rd, 5th, and 7th order harmonics of the rms value U_3 = 2%U_1, U_5 =3%U_1, and U_7 = 1.5%U_1.

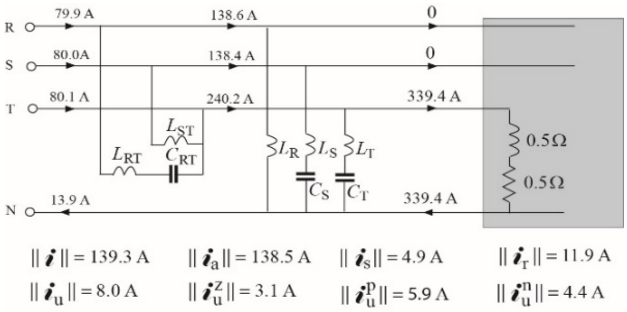

Fig. 11. An example of a reactive balancing compensator.

The compensation method drafted above results in a reactive compensator with fixed parameters. It improves the power factor λ of the supply source when its load has also fixed parameters, or they change insignificantly. Otherwise, such compensators are losing their effectiveness. Adaptive compensators are needed instead.

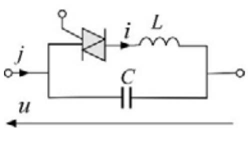

Fig. 12. A thyristor switched inductor (TSI) with a shunt capacitor.

One-ports with thyristor switched inductors (TSI) [4], and a capacitor connected as shown in Fig. 12, are commonly used in adaptive compensators of the reactive power.

This is an example of a branch that provides a thyristor controlled susceptance (TCS). Several other structures of TCS were investigated in [21].

The current of a thyristor switched inductor depends on the firing angle α of thyristors. It changes for some angle as shown in Fig. 13. Symbol i_0 denotes the current at α = 0.

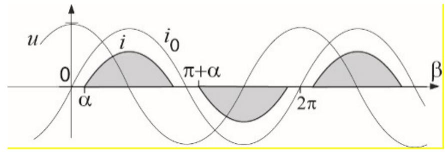

Fig. 13. The currents waveform of a thyristor switched inductor.

Assuming that the voltage $u(t)$ on the TSI branch is sinu-soidal, the ratio of the crms value of the current $i(t)$ fundamen-tal harmonic $i_1(t)$ and the branch voltage crms value

(47) $$\boldsymbol{Y}_1 = \frac{\boldsymbol{I}_1}{\boldsymbol{U}} = jT = -j(1-\frac{2\alpha+\sin 2\alpha}{\pi})\frac{1}{\omega_1 L}$$

specifies the branch susceptance T for the fundamental fre-quency. It changes with the change of the firing angle α.

The TSI branch is, unfortunately, a source of current har-monics, in particular, the 3rd order. When compensation is confined, as it is common, only to the reactive power, com-pensator is built as a balanced device, meaning it is confi-gured in Δ, and thyristors are fired, with a shift of T/6, at the same firing angle. Since the 3rd order harmonics generated in compensator branches are in-phase and mutually equal, they do not leave the compensator. They are confined to its Δ loop.

When the TSIs are used in a balancing compensator, it operates as an unbalanced device, thyristors are switched at different angles and the 3rd order harmonic generated in parti-cular, branches have different values. Consequently, the 3rd order harmonic currents are injected into the supply lines. This causes the waveform distortion and resonance of the compensator capacitance with the supply system inductance can occur. To avoid it, as suggested in [22], the capacitor should be replaced by a filter, as shown in Fig. 14, tuned to the frequency of the 3rd order harmonic.

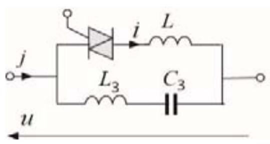

Fig. 14. A TSI with a filter of the 3rd order harmonic.

With the change of the firing angle from zero to π/2, the susceptance of the whole branch changes from its minimum value

(48) $$T_{\min} = \frac{9}{8}\omega_1 C_3 - \frac{1}{\omega_1 L}$$

to its maximum value

(49) $$T_{\max} = \frac{9}{8}\omega_1 C_3 .$$

Thus, if the required range of change of the branch suscep-tane T is known, the parameters of the branch can be calcu-lated, namely

(50) $$L = \frac{8}{9\omega_1} T_{\max}, \qquad C_3 = \frac{1}{\omega_1 (T_{\max} - T_{\min})} .$$

In a substantial presence of the 3rd order harmonic in the supply voltage, the filter L_3C_3 can cause unacceptable incre-ase of the 3rd order harmonic in the supply current. A series inductor should be added [23, 24] to the TCS branch, as it is shown in Fig. 16, to reduce this harmonic

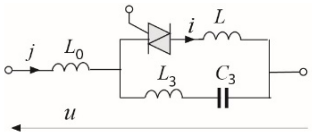

Fig. 16. A TSI with a series inductor and 3rd order harmonic filter.

Compensation with switched compensators

Adaptive reactive compensators, due to a high switching capability of thyristors, currently of the order of 50kA, can be built as devices of very high power. Due to low switching frequency, they handle mainly the current harmful compon-ents only of the fundamental frequency, however. Switching compensators (SCs) are needed [22, 30, 36, 38] for compen-sation of high-frequency components of the supply current.

Switching compensators are built as voltage source inver-ters interfaced with the distribution system by inductors L which create low-pass filters for reducing the noise created by switched transistors. The power transistors-built inverter is supplied with a DC voltage U_C on the capacitance C, created by the inverter, which has also a property of a controlled three-phase rectifier. A structure of an SC is shown in Fig. 16.

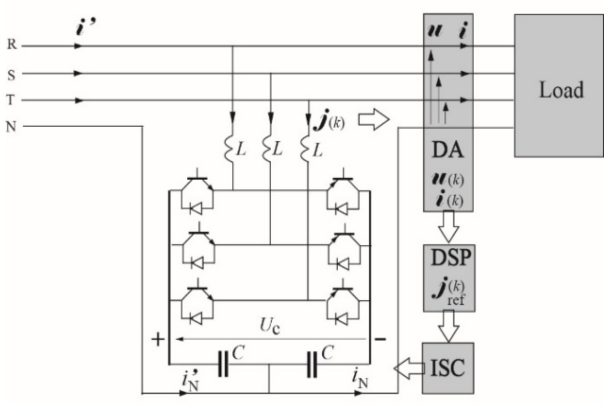

Fig. 16. A structure of a switching compensator.

It contains a Data Acquisition (DA) system which pro-vides, at the sampling frequency $f_s = 1/T_s$, samples of voltages and currents $\boldsymbol{u}(k)$ and $\boldsymbol{i}(k)$, measured at the load terminals at sequential time instants $t_k = kT_s$. The samples are processed by a Digital Signal Processing (DSP) system to create a refe-rence signal $\boldsymbol{j}_{\rm ref}(k)$ for the compensator control. This signal, along with the value of the DC voltage Uc on the capacitor, and the current of the compensator $\boldsymbol{j}(k)$, are used by Inverter Switches Control (ISC) system for shaping the compensating current at the next instant of time, t_{k+1}.

This is relatively well a grown-up technology. There is still a controversy, however, on how the reference signal should be generated. It is caused by controversy on the power theory of electrical circuits and systems. The consequences of gene-ration of this signal according to the IRP p-q Theory were discussed previously.

In the CPC-based decomposition of the load current of three-phase HGL loads supplied by a three-phase line with neutral:

(51) $$\boldsymbol{i} = \boldsymbol{i}_{\rm Ca} + \boldsymbol{i}_{\rm Cs} + \boldsymbol{i}_{\rm Cr} + \boldsymbol{i}^{\rm p}_{\rm Cu} + \boldsymbol{i}^{\rm n}_{\rm Cu} + \boldsymbol{i}^{\rm z}_{\rm Cu} + \boldsymbol{i}_{\rm G}$$

only the active current $\boldsymbol{i}_{\rm Ca}$ delivers the energy to the load per-manently. The remaining ones are not only useless but harm-ful. They reduce the power factor and should be compensa-ted. To do this, a switching compensator should generate and load the supply lines with the current

(52) $$\boldsymbol{j} = \boldsymbol{i}_{\rm Ca} - \boldsymbol{i}$$

Its value at the instant t_k is

(53) $$\boldsymbol{j}(k) = \boldsymbol{i}_{\rm Ca}(k) - \boldsymbol{i}(k)\,.$$

The value of sample $\boldsymbol{i}(k)$ is provided by a DA system, but the value of the active current $\boldsymbol{i}_{\rm Ca}(k)$ has to be calculated by the DSP system. The active current $\boldsymbol{i}_{\rm Ca}$ is defined as

(54) $$\boldsymbol{i}_{\rm Ca} = \frac{P_{\rm C}}{||\boldsymbol{u}_{\rm C}||^2}\boldsymbol{u}_{\rm C} = G_{\rm Ce}\,\boldsymbol{u}_{\rm C}$$

where the index C denotes the set of harmonic orders n for which harmonic active power of the load

(55) $$P_n = {\rm Re}\{\, \boldsymbol{U}_n^{\rm T} \boldsymbol{I}_n^* \}$$

is positive, and

(56) $$P_{\rm C} = \sum_{n\in N_{\rm C}} P_n, \qquad \boldsymbol{u}_{\rm C} = \sum_{n\in N_{\rm C}} \boldsymbol{u}_n\,.$$

Calculation of the active current requires that crms values of voltages and currents harmonics are calculated. Let us denote them by a common symbol

(57) $$\boldsymbol{X}_n = X_n e^{j\alpha_n}\,.$$

On the condition that the sampling frequency satisfies the Nyquist criterion, these crms values of the voltages and currents harmonics can be calculated at instant t_k using the Discrete Fourier Transform (DFT), having K uni-distance samples x_s, acquired in a single period T, from the formula

(58) $$\boldsymbol{X}_n(k) = \frac{\sqrt{2}}{K}\sum_{s=k-K+1}^{s=k} x_s e^{-j\frac{2\pi}{K}ns} = \frac{\sqrt{2}}{K}\sum_{s=k-K+1}^{s=k} x_s \boldsymbol{z}_{ns}, \quad s = 1...K$$

.

The fundamental harmonic u_1 of the supply voltage, usually less distorted than the current, can be used for detecting the period T and the length of the sampling window, as it is shown in Fig. 17. The complex numbers

(59) $$\boldsymbol{z}_{ns} = \frac{\sqrt{2}}{K}\, e^{-j\frac{2\pi}{K}ns}$$

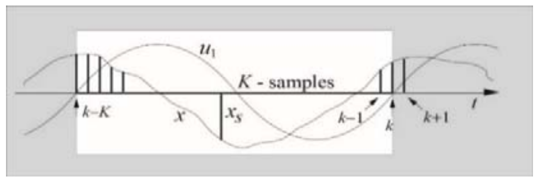

Fig. 17. A sampling window and samples of the $x(t)$ quantity.

to avoid repetition of their calculation can be tabulated for each harmonic order n from the set N. The number of $2K$ multiplications is needed for calculating the crms value $\boldsymbol{X}_n(k)$. This number can be reduced [17, 26, 27, 33] to only two multi-plications, by calculating $\boldsymbol{X}_n(k)$ using the iterative formula

(60) $$\boldsymbol{X}_n(k) = \boldsymbol{X}_n(k-1) + (x_k - x_{k-K})\,\boldsymbol{z}_{nk}\,.$$

Observe that for periodic quantities $x_k = x_{k-K}$, so that the crms value $\boldsymbol{X}_n(k)$ remains constant.

Having the crms values of the supply voltage and current harmonics at the load terminals R, S, and T, the sign of the harmonic active power

(61) $$P_n(k) = {\rm Re}\{\, \boldsymbol{U}_n^{\rm T}(k)\, \boldsymbol{I}_n^*(k)\}$$

can be examined, and the set N_C of harmonic orders n can be determined. So that

(62) $$P_C(k) = \sum_{n\in N_C} P_n(k)$$

(63) $$\|\boldsymbol{u}_C(k)\|^2 = \sum_{n\in N_C} \|\boldsymbol{u}_n(k)\|^2 = \sum_{n\in N_C} [X^2_{Rn}(k) + X^2_{Sn}(k) + X^2_{Tn}(k)]$$

as well as, the conductance $G_{Ce}(k)$ can be calculated.

The set N_C and the conductance $G_{Ce}(k)$ usually do not change very fast, so that approximately $G_{Ce}(k) = G_{Ce}$, and it can be calculated every few periods T or even every few sec-onds to be next used in formulae (54) and (53) for the com-pensator current $\boldsymbol{j}$ at the instant t_k calculation, namely

(64) $$\boldsymbol{j}(k) = G_{Ca}\sqrt{2}\,\mathrm{Re}\sum_{n\in N_C} U_n(k)e^{j\frac{2\pi}{K}nk} - \boldsymbol{i}(k)\ .$$

This value of the compensating current is not the one that should load the compensated system. It is calculated based on the state of the system in the instant t_k and some time is needed for completing all calculations. Assuming that they can be completed in time no longer than the sampling interval T_s, and currently built microprocessors are fast enough for that, the value of the compensating current in the instant t_{k+1}, i.e., $\boldsymbol{j}(k+1)$ is needed. It can be predicted by a linear extra-polation, namely

(65) $$\boldsymbol{j}(k+1) = \boldsymbol{j}(k) + \Delta\boldsymbol{j},\quad \Delta\boldsymbol{j} = \boldsymbol{j}(k) - \boldsymbol{j}(k-1)\ .$$

At the same time, at the instant t_{k+1}, the next set of samples of the voltages and currents is acquired, so that the value of the compensating current can be updated. In such a way, this CPC-based algorithm provides a semi-instantaneous control of a switching compensator. Only at major changes of the load, sort of ON/OFF switching, this semi-instantaneous control is delated by one period T. In effect of such a control of the SC, all harmful components of the supply current $\boldsymbol{i}'(t)$ are compensated and it is in-phase with the voltage $\boldsymbol{u}_C(t)$.

When the internal voltage $\boldsymbol{e}(t)$ of the distribution system is sinusoidal, then the set N_C is composed of only the order of the fundamental harmonic, n=1, and the calculation of the compensator current can be substantially simplified. There is no need for calculation of the crms values of the higher-order harmonics of voltages and currents.

Hybrid compensators

Switching compensators have a capability of reduction of all undesirable and harmful components of the supply current and they have a natural adaptive property. Relatively low power, confined by a switching capability of transistors, is the main disadvantage of these devices, however. Their power might not be sufficient for some applications. This obstacle can be reduced using a ***hybrid compensator***.

Transistors' switching power declines with the switching frequency increase, while the power factor λ is degraded mainly by the current components of the fundamental frequ-ency. Therefore, there are two ways for increasing the com-pensator power.

1. The current harmful components of the fundamental fre-quency are compensated by an adaptive reactive com-pensator (ARC) [22,] while the goal of an SC is confined to compensation of the higher-order current harmonics. It results in a hybrid compensator shown in Fig. 18.
2. The current harmful components of the fundamental fre-quency are compensated by a switching compensator with a low switching frequency (LfSC), while an SC with high switching frequency (HfSC), connected in parallel to the first one, as shown in Fig. 19, compensates the higher order harmonics.

Since the switching power of thyristors, needed for adaptive compensators construction, is approximately by one order higher than the power transistors, the first approach enables the construction of compensators of much higher power than the second approach.

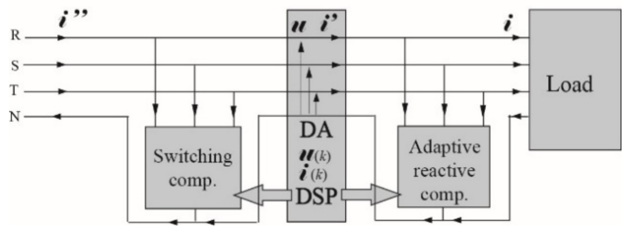

Fig. 18. A hybrid compensator build of ARC and SC.

These two compensators can have separate DA and DSP sytems or common ones. In the situation, as shown in Fig. 18, the reactive compensator is controlled in a closed-loop, so that the stability of the control should be taken into account. The SC is controlled in an open loop, so that stability issues do not occur. Observe, that the SC can compensate not only the current harmonics generated in the load but also produced in the ARC by thyristors switched inductors.

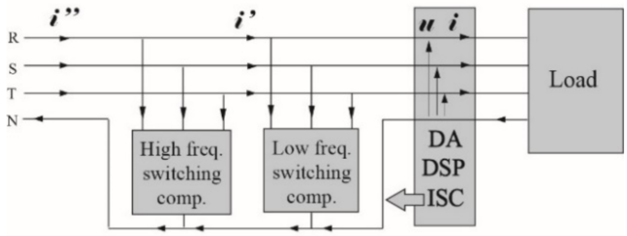

Fig. 19. A hybrid compensator build of LfSC and HfSC.

An illustration of an unbalanced and harmonics genera-ting load compensated by a hybrid compensator composed of low and high frequency switched compensator is shown in Fig. 20.

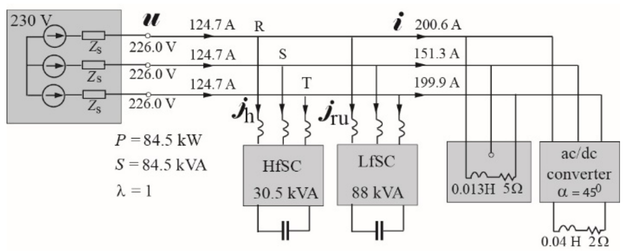

Fig. 20. An example of a load and a hybrid compensator.

The load is composed of an unbalanced load and ac/dc three-phase converter operated firing angle of α = 45°. The waveform of the voltage of line R and currents in lines R and S are shown in Fig. 21.

The control of such a hybrid compensator requires that the reference signal $\boldsymbol{j}(t)$ is decomposed into a fundamental harmonic component $\boldsymbol{j}_1(t)$ and remaining harmonics compo-nent $\boldsymbol{j}_h(t)$, namely

(66) $$\boldsymbol{j}(t) = \boldsymbol{j}_1(t) - \boldsymbol{j}_h(t)\ .$$

Since the load current fundamental harmonic is

(67) $$\boldsymbol{i}_1 = \boldsymbol{i}_{1Ca} + \boldsymbol{i}_{1Cs} + \boldsymbol{i}_{1Cr} + \boldsymbol{i}_{1Cu} = \boldsymbol{i}_{1Ca} - \boldsymbol{j}_1$$

thus

(68) $$\boldsymbol{j}_1 = \boldsymbol{i}_{1\mathrm{Ca}} - \boldsymbol{i}_1 \ .$$

The compensating current of the LfSC at the instant t_k is

(69) $$\boldsymbol{j}_1(k) = \sqrt{2}\,\mathrm{Re}\{[G_{\mathrm{Ca}}\boldsymbol{U}_1(k) - \boldsymbol{I}_1(k)]e^{j\frac{2\pi}{K}k}\}$$

while this current of the HfSC

(70) $$\boldsymbol{j}_\mathrm{h}(k) = \boldsymbol{j}(k) - \boldsymbol{j}_1(k) \ .$$

It means that these currents can be separated with the Fourier analysis confined only to the fundamental harmonics of the load voltages and currents. The effects of such compensation are shown in Fig. 22.

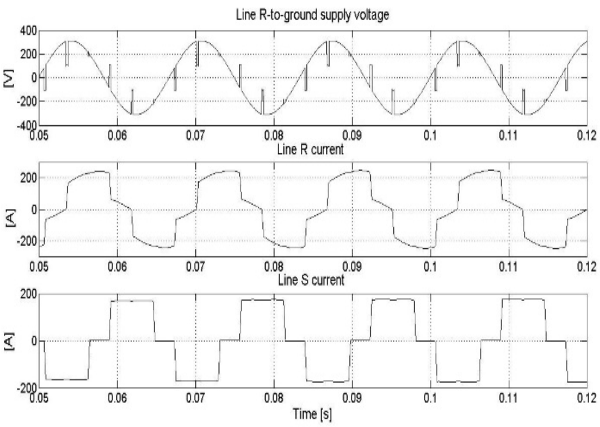

Fig. 21. Voltage $u_\mathrm{R}(t)$ and currents $i_\mathrm{R}(t)$, and $i_\mathrm{S}(t)$ waveforms.

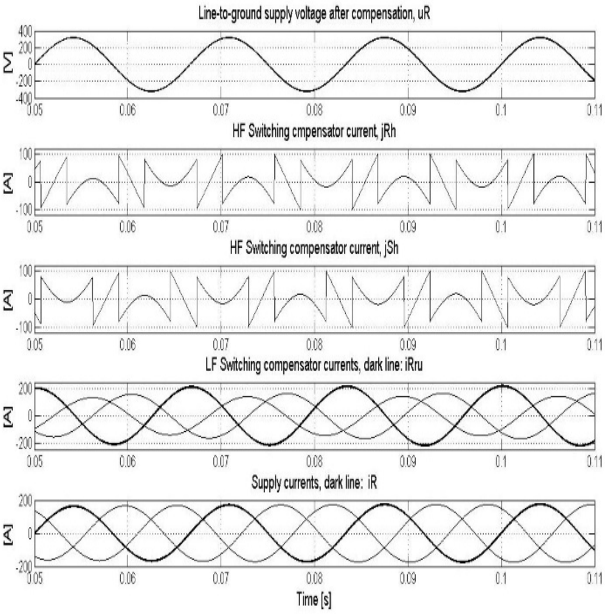

Fig. 22. Compensator currents and effects of compensation.

Summary

With major progress in our comprehension of the energy transfer - related physical phenomena in electrical circuits and systems, mainly due to the development of the Currents' Phy-sical Components (CPC) – based power theory, the main questions related to compensation in such circuits and sys-tems have been answered. This is the very practical merit of this theory development. It also provided a tool for verification and a reference for methods of compensation suggested by other power theories, in particular, it revealed limitations of the Instantaneous Reactive Power p-q Theory as applied to switching compensators control.

The compensation methods have been developed with some natural delay with respect to the development of the CPC-based PT. Therefore, only the main scaffolder, meaning ideas and fundamentals for the compensation methods are ready now. This scaffolder can be filled with more detailed results. In particular, hybrid compensators stand currently for not a well-developed technology jet. This is because some results, in particular, those related to the CPC- based reactive balancing in four-wire systems with nonsinusoidal voltages, were obtained quite recently. Thus, hybrid compensation is still an "uncharted area" and new developments could be
expected. This paper was focused on the CPC-related methods of compensation, but other approaches should be carefully observed as well.

Author: *Prof. dr hab. inż. Leszek S. Czarnecki, IEEE Life Fellow, Distinguished Professor at the School of Electrical Eng. and Comp. Science, Louisiana State Univ., 824 Louray Dr., Baton Rouge, USA, LA 70808, lsczar@cox.net, lczarn1@lsu.edu, Intern.: czarnecki.study*

REFERENCES

[1] Steinmetz, Ch.P., Theory and calculation of electrical apparatur, McGraw-Hill Book Comp., New York, 1917.
[2] Balabanian, N., Network Synthesis, Prentice-Hall, Englewood Cliffs, New York, 1958.
[3] Shepherd, W. Zakikhani, P., Suggested definition of reactive power for nonsinusoidal systems, *Proc. IEE,* Vol. 119, No. 9, pp. 1361-1362, 1972.
[4] Steeper, D.E., Stratford, R.P. Reactive compensation and har-monic suppression for industrial power systems using thyristor converters", *IEEE Trans. on IA*, Vol. 12, No. 3, pp. 232-254, 1976.
[5] International Electrotechnical Commission, (1979), Techn. Com-mittee No. 25, Working Group 7, Report: *Reactive power and distortion power*, Doc. No. 25, 113 (Secr.).
[6] Kusters, N.L., Moore, W.J.M., On the definition of reactive power under nonsinusoidal conditions, *IEEE Trans. Power Appl. Syst.,* Vol. PAS-99, No. 3, pp. 1845-1854, 1980.
[7] Czarnecki, L.S., Minimization of distortion power of nonsinusoi-dal source applied to linear loads, *Proc. IEE,* Vol. 128, Pt. C, No. 4, pp. 208-210, 1981.
[8] Czarnecki, L.S., Additional discussion to "Reactive power under nonsinusoidal conditions", *IEEE Trans. on Power and Systems*, Vol. PAS-102, No. 4, pp. 1023-1024, 1983.
[9] Akagi, at all., Instantaneous reactive power compensator compri-sing switching devices without energy storage components, *IEEE Trans. on Ind. Appl.*, IA-20, No. 3, pp. 625-630, 1984.
[10] Czarnecki, L.S., Considerations on the reactive power in non-sinusoidal situations, *IEEE on Instr. Meas.,* Vol. IM-34, No. 3, pp. 399-404, 1984.
[11] Czarnecki, L.S., Comments on the Kusters and Moore's power definitions in circuits with nonsinusoidal waveforms (in Polish), *Rozprawy Elektrotechn.*, Tom XXX, Z.3-4, pp. 1089-1099, 1984.
[12] Czarnecki, L.S., Minimization of reactive power in nonsinusoidal situation, *IEEE Trans. on IM.,* Vol. IM-36, No. 1, pp. 18-22, 1987.
[13] Czarnecki, L.S., Reactive and unbalanced currents compensa-tion in three-phase circuits under nonsinusoidal conditions, *IEEE Trans. IM,* IM-38, No. 3, pp. 754-459, 1989.
[14] Czarnecki, L.S., Time-domain approach to the reactive current minimization in nonsinusoidal situations, *IEEE Trans. on Instr. Meas.,* Vol. IM-39, No. 5, pp. 698-703, 1990.
[15] Czarnecki, L.S., Scattered and reactive current, voltage, and power in circuits with nonsinusoidal waveforms and their com-pensation, *IEEE Trans.on IM,* Vol. 40, No. 3, pp. 563-567, 1991.
[16] Czarnecki, L.S., Reactive minimization of unbalanced currents in three-phase asymmetrical circuits with nonsinusoidal voltage, *IEE Proc. Pt. B*, Vol. 139, No. 4, pp. 347-354, 1992.
[17] Czarnecki, L.S., Two algorithms of the fundamental harmonic complex RMS value calculation, *Archiv fur Elektrotechnik,* (75), pp. 163-168, 1992.

[18] Czarnecki, L.S., Minimization of unbalanced and reactive currents in three-phase asymmetrical circuits with nonsinusoidal voltage, *Proc. IEE,* Vol. 139, Pt. B, No. 4, pp. 347-354, 1992.
[19] Czarnecki, L.S., Power factor improvement of three-phase unba-lanced loads with nonsinusoidal voltage, *European Trans. on Elect. Power Syst., ETEP,* Vol. 3, No. 1, pp. 67-74, 1993.
[20] Czarnecki, L.S., Supply and loading quality improvement in sinu-soidal power systems with unbalanced loads supplied with asym-metrical voltage, *Arch. fur Elektrotechnik,* 77, pp. 69-177, 1994.
[21] Czarnecki, L.S. Hsu, M.S., Thyristor controlled susceptances for balancing compensators operated under nonsinusoidal condi-tions, *Proc. IEE, B, EPA*., Vol. 141, No. 4, pp. 177-185, 1994.
[22] Czarnecki, L.S., A combined time-domain and frequency-domain approach to hybrid compensation in unbalanced nonsinusoidal systems, *European Trans. on Electric Power, ETEP,* Vol. 4, No. 6, pp. 477-484, 1994.
[23] Czarnecki, L.S., Dynamic, power quality oriented approach to theory and compensation of asymmetrical systems under nonsi-nusoidal conditions, *European Trans. on Electric Power, ETEP,* Vol. 5, pp. 347-358, 1994.
[24] Czarnecki, L.S., Hsu, S.M., Chen, G., Adaptive balancing compensator, *IEEE Trans. on Pow. Del.,* Vol. 10, No. 3, pp. 1663 - 1669, 1995.
[25] Bonner, J.A. at all., Selecting ratings for capacitors and reactors in applications involving multiple single-tuned filters, *IEEE Trans. on Pow. Del.,* Vol.10, pp. 547-555, 1995.
[26] Czarnecki, L.S., Power theory of electrical circuits with quasi-periodic waveforms of voltages and currents, *Europ. Trans. on Elect. Power, ETEP,* Vol. 6, No. 5, pp. 321-328, 1996.
[27] Czarnecki, L.S., Chen, G., Staroszczyk, Z., Application of running quantities for control of adaptive hybrid compensator, *Europ. Trans. on Elect. Power, ETEP,* Vol. 6, No. 5, pp. 337-344, 1996.
[28] Czarnecki, L.S., Budeanu and Fryze: two frameworks for interpreting power properties of circuits with nonsinusoidal voltages and currents, *Archiv fur Elektrotechn.*, 81, No. 2, pp. 5-15, 1997.
[29] Hsu, S.M. Czarnecki, L.S., Adaptive blocking compensator, *IEEE Trans. on Pow. Del., Vol. 18, No. 3,* pp. 895-902, 2003.
[30] Tenti, P., Mattavelli, P., A time-domain approach to power terms definitions under nonsinusoidal conditions, *6th Int. Workshop on Power Definitions under Nonsinusoidal Conditions*, Milano, 2003.
[31] Czarnecki, L.S., Ginn, H.L., Effects of damping on the performance of resonant harmonic filters*, IEEE Trans. on Power Deliv.*, Vol. 19, pp. 846-853, 2004.
[32] Czarnecki, L.S., Ginn, H.L., The effect of the design method on the efficiency of resonant harmonic filters, *IEEE Trans. on Power Delivery*, Vol. 20, No. 1, pp. 286-291, 2005.
[33] Ginn, H.L., Chen, G., Switching compensator control strategy based on CPC power theory, *Przegląd Elektrotechniczny*, Vol. 84, No. 6, pp. 23-27, 2008.
[34] Czarnecki, L.S., Effect of supply voltage harmonics on IRP-based switching compensator control, *IEEE Trans. on Power Electro-nics*, Vol. 24, No. 2, pp. 483-488, 2009.
[35] Czarnecki, L.S, Pearce, S.E., Compensation objectives and CPC – based generation of reference signals for shunt switching com-pensator control, *IET, Pow. Elect.*, Vol. 2, No. 1, pp. 33-41, 2009.
[36] Akagi, H., at all., Compensation algorithms based on the p-q and CPC theories for switching compensators in micro-grids, *Proc. of COBEP Power Electronics Conf.,* Brazilian DOI: 10.1109/ COCEP.2009.5347593, pp. 32-40, 2009.
[37] Czarnecki, L.S., Effect of supply voltage asymmetry on IRP p-q - based switching compensator control, *IET Proc. on Power Elec-tronics*, Vol. 3, No. 1, pp. 11-17, 2010.
[38] Popescu, M., Bitoleanu, A., Suru, V., Currents' Physical Compo-nents theory implementation in shunt active power filtering for unbalanced loads, *International School on Nonsinusoidal Cur-rents and Compensation, ISNCC,* Dig. Obj. Ident.: 10.1109 /ISNCC.2013. 6604447, 2013.
[39] Czarnecki, L.S., Haley, P.H., Currents' Physical Components - CPC - in four-wire systems with nonsinusoidal symmetrical voltage, *Przegląd Elektrotechn.*, R.91, No. 6, pp. 48-53, 2015.
[40] Czarnecki, L.S., Haley, P.H., Unbalanced power in four-wire sys-tems and its reactive compensation, *IEEE Trans. on Pow. Del.,* Vol. 30, No. 1, pp. 53-63, 2015.
[41] Czarnecki, L.S., Comparison of the Conservative Power Theory (CPT) with Budeanu's power theory, *Annals of the University of Craiova,* 2016.
[42] Czarnecki, L.S., Bhattarai, P.D. A method of calculation of LC parameters of balancing compensators for AC arc furnaces, *IEEE Trans. on Pow. Del.*, Vol. 32, No. 2, pp. 688-695, 2017.
[43] Czarnecki, L.S., Critical comments on the Conservative Power Theory (CPT), *Przegląd Elektrotechniczny*, R3, No. 1, pp. 268□274, 2017.
[44] Czarnecki, L.S., A Key Note: Degradation of the energy transfer effectiveness described in terms of CPC-based PT, Intern. Conf. on Energy Science and Electr. Eng., (ICESEE), Budapest, 2017.
[45] Czarnecki, L.S., Ezeonwumelu, I.L., Considerations on direct bal-ancing of ultra-high power AC arc furnaces in uneasy state", *IEEE Int. Conf. on Environment and Electr. Eng. (EEEIC),* 2018.
[46] Czarnecki, L.S., Currents' Physical Components (CPC) – based Power Theory. A Review, Part I: Power properties of electrical circuits and systems, *Przegląd Elektrotechniczny,* ISSN 0033-2097, R. 95, Nr. 95, pp. 1-11, 2019.
[47] Czarnecki, L.S., CPC – based reactive balancing of linear loads in four-wire supply systems with nonsinusoidal voltage, *Przegląd Elektrotechniczny*, R. 95, Nr. 95, pp. 1-8, 2019.
[48] Czarnecki, L.S., Bhattarai, P.D., CPC□based reactive compensa-tion of linear loads supplied with asymmetrical nonsinusoidal vol-tage, *Annals Univ. of Craiova*, Vol. 42, No. 10, pp. 1-9, 2019.