Edward KOZŁOWSKI[3], Tomasz RYMARCZYK[1,2], Grzegorz KŁOSOWSKI[3], Tomasz CIEPLAK[3]

Research & Development Centre Netrix S.A.(1), University of Economics and Innovation (2)
Lublin University of Technology (3)

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Logistic regression in image reconstruction in electrical impedance tomography

***Abstract**. The problem of image reconstruction in electrical impedance tomography (EIT) consists in both performing measurements using a set of sensors and creating of reconstruction based on these measurements. The image reconstruction requires accurate modeling of area, which presents field of view. To determine the inclusion in analyzed area the logistic regression has been applied. Additionally to select the predictors in logistic regression the elasticnet method has been used.*

Streszczenie.** Problem rekonstrukcji obrazu w elektrycznej tomografii impedancyjnej (EIT) polega zarówno na wykonywaniu pomiarów przy użyciu zestawu czujników, jak i na tworzeniu rekonstrukcji na podstawie tych pomiarów. Rekonstrukcja obrazu wymaga dokładnego modelowania obszaru, który przedstawia pole widzenia. Do określenia wtrąceń w analizowanym obszarze zastosowano regresję logistyczną. Dodatkowo do wyboru predyktorów w regresji logistycznej zastosowano metodę elasticnet. **Problem rekonstrukcji obrazu w elektrycznej tomografii impedancyjnej

Keywords: electrical impedance tomography, logistic regression, elasticnet.
Słowa kluczowe: tomografia impedancyjna, regresja logistyczna, elasticnet.

Introduction

Electrical Impedance Tomography is a nondestructive method to create an image reconstructions in different application areas (e.g. biomedical diagnosis [14], industrial imaging [4], [6], [7], [12], [17], [19], geological imaging [11] ,[31]). Many researchers have devoted our investigations to improve imaging efficacy and accuracy. As a results of exploration of researchers many methods have been created, which were directly applied into Electrical Impedance Tomography (EIT), Electrical Capacitance Tomography (ECT), Magnetic Resonance Imaging (MR), Ultrasound Tomography (UST)(see e.g. [1], [2], [6], [11], [13], [16], [22]). Main aim of tomography depends on image reconstruction, which directly connected with inverse problem solution. Some signals obtained from electrodes, transducers are mutually correlated (collinearity problem), see e.g. [3], [5], [8], [11]. In this case the application Gauss-Markov theorem to determine the unknown parameters in linear models (e.g. between conductivity and measured voltages on the electrodes) does not give satisfactory effects. To overcome this problem the researchers apply usually singular value decomposition [28], LARS [8], [28], [29], regularization methods (such as Tikhonov regularization, total variation, least angle regression, sparse regularization, see e.g. [23], [24], [25], [26], [29]) or neural network [5], [10], [20].

Our target of work does not depend on improving of image accuracy but only quickly searching the areas in imaging domain which can contain the inclusions with large probability. Obtained results can be used as a prior resolution to image reconstruction. In paper a logistic regression was proposed to solve the inverse problem in electrical impedance tomography. The areas with large probability of inclusion are determined based on measurements obtained from transducers located on the boundary of imaging domain. By setting the cut off probability level we can made the classifier, which is helpful to separate background and inclusion. Additionally by changing the required probability level we can improve imaging accuracy. To overcome the measurements collinearity problem we apply the regularization technique based on elastic net as a combination of Tikhonov regularization and Least Absolute Shrinkage and Selection Operator (LASSO) [24], [25, [28]. To test the performance of proposed method the numerical and experimental results are conducted.

Problem formulation

Main task of electrical impedance tomography consists in reconstruction of imaging domain. In many cases we must determine the areas which contain some inclusions in imaging domain. To determine the inclusions first we model a specially developed mesh which corresponds to imaging domain. From above the imaging domain is divided into k finite elements. Thus each inclusion we may designate as a set of finite elements. To answer the question does the finite element contain an inclusion it is necessary to define classifier for each finite element. The classifier is a function which has a realization in binary set. The realizations of this function correspond to inclusion existence (or not).
To create a classifier we employ logistic regression. In this way first we calculate the probability that response variable belongs to appropriate category by observing the signal $x \in R^m$ obtained from electrodes (sensors). Therefore instead of determining inclusion or not for each finite element we estimate the probabilities of inclusion. On the other words application of logistic regression allows us to determine the inclusion resolution of imaging domain. Next by using a classifier $f: R^m \to \{0,1\}$ (which would allow to classify the existence of object into categories $y = 1$ or $y = 0$) we build a reconstruction of imaging domain.

Let (Ω, F, P) be a probability space. On this space we define a random variable Y with binomial distribution, i.e. $Y: \Omega \to \{0,1\}$. Logistic regression (see e.g. [27], [28], [29]) describes probability of realization of dependent variable Y based on observation of input variables X. For convenience $P(Y = 1|X)$ denotes success probability and $P(Y = 0|X)$ defeat (unsuccess) probability. In literature **t**he ratio success probability to defeat probability

(1) $$\Theta(X) = \frac{P(Y=1|X)}{P(Y=0|X)} = \frac{P(Y=1|X)}{1-P(Y=1|X)}$$

is called the odds. The aim of logistic regression consist in determining the probability of success $P(Y = 1|X)$ based on observation X. Because the probability of success $P(Y = 1|X) \in (0,1)$, therefore from formula (1) it follows, that the odds $\Theta(X) \in (0, \infty)$ and $ln\big(\Theta(X)\big) \in (-\infty, \infty)$. The logarithm of odds is called log-odds or logit.

Application of logistic regression in EIT allows us determine the resolution of inclusion probabilities based on measurements obtained from sensors [7]. Accordingly to sequence of probabilities we may create the sequence due to classification level $s \in [0,1]$ compounded from elements from a set $\{0,1\}$ which correspond defeat and success accordingly.

Logistic regression

For any finite element we consider the training set. This data set contains both realizations of input variables (measurements from sensors) and realizations of output variable. The data set is designated as $D = \{(x_{i\cdot}, y_i)\}_{1\leqslant i\leqslant n}$, where $\{x_{i\cdot}\}_{1\leqslant i\leqslant n}$ denotes a series of input variables, $\{y_i\}_{1\leqslant i\leqslant n}$ is a series of response variable, $x_{i\cdot} \in R^m$, $y_i \in \{0,1\}$ for $1 \leqslant i \leqslant n$ and m denotes number of measurements obtained from transducers (sensors). For i –th case if the finite element contains inclusion, then we take $y_i = 1$ otherwise we put $y_i = 0$. The training set can be presented as $D = \{Y, X\}$, where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \ X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} = \begin{bmatrix} x_{1\cdot} \\ x_{2\cdot} \\ \vdots \\ x_{n\cdot} \end{bmatrix}.$$

In logistic regression (e.g. [27], [28], [29]) we analyze the linear dependencies between logit and input variables

(2) $$ln\Theta(X) = ln\left(\frac{p(\beta,X)}{1-p(\beta,X)}\right) = X\beta,$$

where $\beta = (\beta_1, \ldots, \beta_m) \in R^m$ and $p(\beta, X) = P(Y = 1|X)$. If linear equation (2) contains an intercept, then in matrix X the column that corresponds to intercept contains ones. From (2) we obtain

(3) $$p(\beta, X) = \frac{e^{X\beta}}{1+e^{X\beta}}.$$

The maximum likelihood method is applied to estimate the unknown parameters β in model (3). First we define the likelihood function as

(4) $$L(\beta, Y, X) = \prod_{i=1}^{n} p\,(\beta, x_{i\cdot})^{y_i}\left(1 - p(\beta, x_{i\cdot})\right)^{1-y_i}.$$

Solution of the task

(5) $$\max_{\beta} L(\beta, Y, X)$$

gives the estimators of unknown parameters β. Usually instead resolving the problem (5) we solve the auxiliary task

(6) $$\max_{\beta} l(\beta),$$

where the objective function is defined as logarithm of likelihood function $l(\beta) = ln\big(L(\beta, Y, X)\big)$ and equal

(7) $$l(\beta) = \sum_{i=1}^{n}\left(y_i x_{i\cdot}\beta - ln\left(1 + e^{x_{i\cdot}\beta}\right)\right).$$

First and second partial derivatives of objective function (7) are given by formulas

$$\frac{\partial l(\beta)}{\partial \beta} = X^T \begin{bmatrix} y_1 - p(\beta, x_{1\cdot}) \\ y_2 - p(\beta, x_{2\cdot}) \\ \vdots \\ y_n - p(\beta, x_{n\cdot}) \end{bmatrix}$$

and

$$\frac{\partial^2 l(\beta)}{\partial\beta\partial\beta^T} = -X^T Z(\beta) X$$

accordingly, where diagonal matrix

$$Z(\beta) = diag\big(h(\beta, x_{1\cdot}), \ldots, h(\beta, x_{n\cdot})\big)$$

and

$$h(\beta, x_{i\cdot}) = p(\beta, x_{i\cdot})\big(1 - p(\beta, x_{i\cdot})\big)$$

for $1 \leqslant i \leqslant n$. We see that the matrix of second partial derivatives is negative defined.

To determine the unknown parameters β the Newton-Raphson algorithm was applied. Application of this algorithm follows that the unknown parameters β are estimated iteratively. In the step $j+1$ the estimators are determined by formula

$$\beta^{j+1} = \beta^j + \left(\frac{\partial^2 l(\beta^j)}{\partial\beta\partial\beta^T}\right)^{-1}\frac{\partial l(\beta^j)}{\partial\beta}.$$

Elasticnet

In EIT we have a multicollinearity problem. The measurements obtained from some sensors are strongly correlated. When the independent variables (predictors) in system (2) are correlated then the direct solution of task (6) (direct application Gauss-Markov theorem) does not give the expected effect. Additionally the forecasts based on this model are unstable. Possible solution of presented problem depends on selection appropriate predictors(input variables) that should be included to regression model (2). On the one hand these predictors should influence on the value of response variable, on the other hand they should not create multicollinearity.

In literature there are many techniques (e.g. singular value decomposition, regularization, least angle regression) to solve the optimization problem (6) for the cases when the input variables are correlated, see e.g. [24],[25],[26].One of the possible ways to reduce multicollinearity dependences between predictors depends on imposing a penalty on large values of estimators and including this penalty into objective function (see e.g. [25], [27], [28]). When predictors are correlated to determine the unknown parameters of logistic regression (2) we solve the task

(8) $$\max_{\beta} \sum_{i=1}^{n}\left(y_i x_{(i)}\beta - ln\left(1 + e^{x_{(i)}\beta}\right)\right) - \lambda P_\alpha(\beta)$$

where $\lambda > 0$ and value $P_\alpha(\beta)$ denotes the penalty. Usually penalty is defined as L_2 norm and called ridge regression (Tikhonov regularization) or L_1 norm and called Least Absolute Shrinkage and Selection Operator (LASSO). The elasticnet is a connection of ridge regression and LASSO. Thus for $0 \leqslant \alpha \leqslant 1$ penalty $P_\alpha(\beta)$ is defined as linear combination of vector norm of estimators β in spaces L_1, L_2 and given by formula

$$P_\alpha(\beta) = (1-\alpha)\frac{1}{2}\|\beta\|_{L_2} + \alpha\|\beta\|_{L_1}$$

This technique implies a shrinkage of estimators of unknown parameters. Application the elasticnet method to solve the inverse problem in electrical impedance tomography allows to receive more accurate and stable reconstruction results see e.g. [5], [6], [11].

Numerical examples

The research presented in this section applies a method based on many separately trained logistic regression subsystems. Each subsystem corresponds to appropriate finite element in imaging domain. The test object was a tank filled with liquid (tap water) with 320 mm diameter. For numerical experiments a special mesh of the imaging domain has been developed that contains 2883 finite elements. The measuring system consisted of 16 electrodes, which were installed around the walls of the tank.

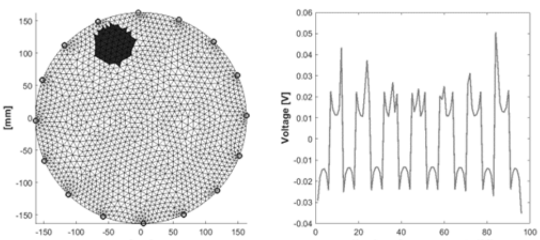

Fig.1. An example of simulation of imaging domain with a graph showing the voltage measurements.

The data was obtained through EIT. The measuring input vector (measurements obtained from electrodes) consists of 96 input variables (see Fig. 1). Each value of input vector reflects the voltage measured between appropriate pair of electrodes.

To generate an appropriate training data set, a physical model of an industrial tank was built. Using the finite element method, the tank cross-section mesh together with the 16 measuring electrodes system was designed using the MATLAB/EIDORS toolbox. Algorithms generating learning instances were also developed. Each case consists of a measurement vector and image generated on a two-dimensional mesh of pixels. It was generated 3281 cases, which next were used to estimate logistic regression models (subsystems) for each finite element from entire system (entire systems consists of 2883 subsystems). Each of the 2883 subsystems has realizations (outputs) only in binary set, which is then displayed as the pixel of the output image. Since among 96 measurements (input variables) obtained from electrodes significant number of these inputs are mutually correlated. To receive stable reconstructions it was take into account penalty $P_\alpha(\beta)$ during estimation unknown parameters in logistic regression (2). From above separately for each pixel has been solved the task (8). As a result only necessary measurements have been used into account during reconstruction. The selection of input variables has been reached by application elastic net method.

Below it will be presented the reconstruction which required application of logistic regression. Based on measures $x \in R^m$ obtained from sensors for *j*-th finite element, $1 \leqslant j \leqslant 2883$, we calculate the probability of inclusion as follows

(9) $$\hat{y}_j = \frac{e^{x\hat{\beta}_j}}{1+e^{x\hat{\beta}_j}}$$

where $\hat{\beta}_j \in R^m$ denotes the estimator of unknown parameters β for logistic regression (2). This procedure we repeat for each finite element. As a result we obtain a sequence $\{\hat{y}_j\}_{1\leq j\leq 2883}$, where $\hat{y}_j \in [0,1]$ for $1 \leqslant j \leqslant 2883$ denotes an probability inclusion for j –th finite element in imaging domain.

Thus inclusion detection in imaging domain consists in determining the finite elements, which contain inclusions, and presenting the image reconstruction. In consequence the task is to find a such classifier, which based on a sequence of probabilities shows an areas of inclusion. Accordingly for a sequence of inclusion probabilities we must define the sequence compounded from elements such as success and unsuccess due to the classification level $s \in [0,1]$. The success corresponds to existence of inclusion for finite element, otherwise finite element does not contain the inclusion. The sequence $v_{rec}(\hat{y}, l) = \{v_j(l)\}_{1\leqslant j\leqslant k}$, where

$$v_j(l) = \begin{cases} 0, & for\ \hat{y}_j < l, \\ 1, & for\ \hat{y}_j \geqslant l \end{cases}$$

we call a reconstruction of imaging domain.

Different cases of positioning objects in the imaging domain were investigated. Additionally the cut-off level was assumed to be 0.5. Fig. 2 presents the patterns and real reconstructions based on logistic regression with application an elasticnet method.

The quality (efficacy and accuracy) of reconstruction has been assessed by calculation two indices. To compare the pattern and reconstruction it has been utilized the basic property of a scalar product (Cauchy – Bunyakovsky – Schwarz inequality). Let $y = \{y_j\}_{1\leqslant j\leqslant k}$ be a pattern corresponding to measures $x \in R^m$ and $v_{rec}(\hat{y}, s)$ reconstruction which corresponds to this same measures. We define the Compatibility Ratio as follows

(10) $$CR(l) = \frac{\langle v_{rec}(\hat{y},l), y\rangle}{\sqrt{\|v_{rec}(\hat{y},l)\|\|y\|}}$$

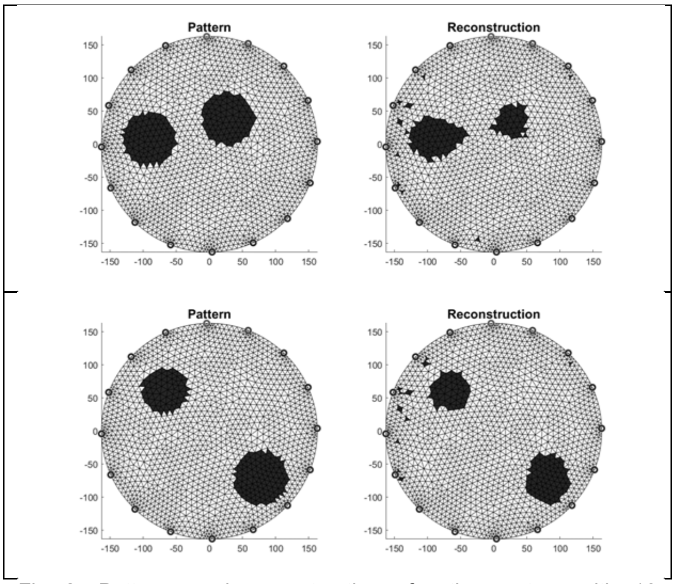

Fig 2. Patterns and reconstructions for the system with 16 measured sensors.

When vectors $v_{rec}(\hat{y}, l)$ and y are collinear ($v_{rec}(\hat{y}, l) = ry$, where $r \in R \setminus \{0\}$), then $CR(l) = 1$. Another index helpful in measuring accuracy is Relative Error of reconstruction

(11) $$RE(s) = \frac{\|v_{rec}(\hat{y},l) - y\|}{\|y\|}$$

Because the elements of sequences $v_{rec}(\hat{y}, l)$ and y come from binary set, then Relative Error shows which part of imaging domain relative to inclusion is not recognized.

Table 1 presents the values of compatibility ratio and values of relative error between patterns and reconstructions for samples presented in Fig. 2. From above we see, that areas of inclusion near boundary of imaging domain e.g. sample 1,3,4, Fig. 2 are better recognized than inclusion in centre of imaging domain. Reconstructions were made on PC, Intel Core i5 4th generation. Time of reconstruction did not exceed 10^{-3} sec.

Table 1. The parameters of the sensor

Case	Reconstruction		
	Time (sec)	*CR*	*RE*
1	0.000257	0.9844	0.1777
2	0.000263	0.9749	0.2268
3	0.000264	0.9834	0.1835
4	0.000271	0.9932	0.1163

Summary

The paper presents a method of application of logistic regression to solve the inverse problem in Electrical Impedance Tomography. The logistic regression has both advantages and disadvantages. Application of presented method allows to calculate the probability of inclusions in imaging domain. Additionally applying a classifier we can determine directly the areas with large probability of inclusion. Presented method gives a fast reconstruction and can be successfully used in industry. The study gives promising results – accuracy and sensitivity exceed 0.95, furthermore positive predicted value (called precision and characterizes a purity of performed classifier) exceed 0.95 too. Reconstruction results depend on the training set. This is only inconvenience (disadvantage) of presented methodology.

***Authors**: Edward Kozłowski, Ph.D., Lublin University of Technology, Nadbystrzycka 38A, Lublin, Poland, E-mail: e.kozlovski@pollub.pl; Tomasz Rymarczyk, Ph.D. Eng., University of Economics and Innovation, Projektowa 4, Lublin, Poland, E-mail: tomasz@rymarczyk.com ; Grzegorz Kłosowski, Ph.D. Eng., Lublin University of Technology, Nadbystrzycka 3, E-mail: g.klosowski@pollub.pl; Tomasz Cieplak, Ph.D., Lubl 8A, Lublin, Poland Lublin University of Technology, Nadbystrzycka 38A, Lublin, Poland, E-mail:* t.cieplak@pollub.pl

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