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Multiobjective Design of Permanent Magnet Generator with Dimensionality Reduction in Criteria Space

Streszczenie. *W artykule przedstawiono rozwiązanie problemu optymalizacji wielokryterialnej poprzez redukcję wymiarów w przestrzeni kryteriów. Rozważono właściwości zbioru Pareto w zadaniu projektowania generatora z magnesami trwałymi. Zaprezentowano i porównano aproksymacje frontu Pareto przy optymalizacji z ograniczeniami przy redukcji z pięciu dwóch kryteriów.*

Abstract. *This paper addresses the problem of dimensionality reduction while preserving the characteristics of the Pareto set approximation in multiobjective optimization. The real-life engineering design problem for permanent magnet generator is considered. The Pareto front approximations with constraints, ranging from the five objectives to the set of two, are presented and compared. (**Redukcja wymiarów przestrzeni kryteriów w polioptymalizacji generatora z magnesami trwałymi**).*

Słowa kluczowe: optymalizacja wielokryterialna z ograniczeniami, maszyna z magnesami trwałymi, front Pareto, redukcja wymiarów.
Keywords: multiobjective optimization, permanent magnet electric machine, Pareto front, dimensionality reduction .

Introduction.

The field of multiobjective evolutionary algorithms has been rapidly growing over the last decade [3]. Most publications deal with two- or three-objectives problems, except "test beds" that are capable of controlled simulation of the algorithms' features. The main reason is that a high number of objectives causes additional challenges compared to low-dimensional problems.

In single-objectives optimization designer obtains one optimal solution. For a nontrivial multi-objectives optimization problem, no single solution exists that simultaneously optimizes each objective. In that case, the objective functions are said to be conflicting, and there exists a possibly infinite number of Pareto optimal solutions [3]. Because such a set in multidimensional space cannot be ordered completely, one needs extra preference information coming from a designer, or generally speaking a Decision Maker (DM), to be able to select the most preferred solution

The main problem is that an increase of dimensionality of the Pareto set causes an exponential increase of points necessary for its representation [4] and a resulting difficulty in visualizing results to the DM. Works addressing the problem of dimensionality reduction has been presented recently [4], [5], [6].

The problem of electric machine design is one of the most challenging in machine design, as it has space-distributed nonlinearities and numerous constraints. These facts justify great interest in application of genetic algorithms. The exhaustive literature summary on this topic can be found in [8]. In recent review [1] it is shown Pareto-based multiobjective approach is still not used very often.

Multi-criteria dimensionality reduction

Using scalar objectives in complex multiobjective optimization problems with constraints leads to solutions relying solely on the DM's preferences This may be considered as pre-optimization. The disadvantage of this approach is the inability to assess how preferences affect the final result.

When specifying preferences in post-optimization, one has to examine the set of Pareto optimal solutions. Then the DM chooses preferred solution based on the trade-offs observed among the set. When solving the problem with too many objective functions there are difficulties with the description (preferably visualization) of the set of Pareto optimal solutions.

The main goal of this paper is to find out whether it is possible to omit some of objective functions while preserving the solution characteristics. Also under which conditions such an objective reduction is feasible, and how such a minimum set of objectives can be computed.

In the proposed procedure the first step is to find a 'non-essential' objective, whose dropping does not affect the set of efficient solutions. Principal Component Analysis, henceforth referred to as PCA, is the method for reducing the number of objectives [4]. It aims to keep those objectives that can explain most of the variance in the objective space.

The next step is to reduce the dimensionality of the problem. One possibility is to remove one of the objectives. In the case that more than one objective is identified as redundant. The proposed procedure leads to the replacement of a non-redundant objective with a new one which combines features of all the removed ones, tailored to the DM preferences (as discussed in [3], [7]).

In this paper the dimensionality reduction is presented. To prove its quality the relationships between selected design variables computed from the Pareto-optimal sets approximation before and after objective replacement is analyzed. It is shown that they possess the same structures, which justifies dimensionality reduction.

Application of reduction of criteria

Modern design of electrical machines is carried out by means of numerical approach, usually using the finite element technique. Because of the complexity of electric machines, the model derivation is included.

The key factor is to prepare the parameterization algorithms to enable analysis. Due to the fact that 3D models require vast amount of calculations this paper focuses on the novel 2D model. In this case the fundamental challenge is modelling of slot skewing, which is the main factor pushing down the torque ripples. The application of low-pass filtering (in space) provides us with the tool for fast computation of that effect. The presented model is derived using this approach.

This paper has the following outline. Section two provides basic concepts of the dimensionality reduction. Section three introduces the permanent magnet generator design problem followed by a detailed description of the novel model.

After performing a Pareto optimization in full five-dimensional objective space, the problem of dimensionality reduction is addressed in Section four.

Dimensionality Reduction in Multiobjective Optimization.

The PCA is a technique for simplifying data sets for analysis by reducing their dimensionality. In mathematical terms, M potentially highly correlated variables are transformed into a set of $L < M$ weakly correlated variables. These new variables are a linear combination of the original variables and can be used to express the data in a reduced form while retaining those characteristics of the data set that contribute most to its variance. It is achieved by keeping lower-order principal components and ignoring higher-order ones. Such low-order components often contain the "most important" aspects of the data.

Two types of methods are used for performing PCA. The first type is based directly on data analysis (e.g. neural networks with Hebbian learning). In this paper the covariance method is used. The principal components are computed using a standard eigenvalue-eigenvector approach available in scientific software MATLAB [9].

The data set of N observations (solutions) is organized in a data matrix. It consists of N vectors each containing values of M criteria functions per column.

The first step is to standardize the data by storing mean-subtracted data in the $M \times N$ matrix **X**. Next an $M \times M$ empirical covariance matrix is calculated from the outer product of a matrix **X** with itself.

$$\mathbf{C} = \mathrm{E}[\mathbf{B} \otimes \mathbf{B}] = \mathrm{E}[\mathbf{B} \cdot \mathbf{B}^{\mathrm{T}}] = \frac{1}{N-1}\mathbf{B} \cdot \mathbf{B}^{\mathrm{T}} \quad (1)$$

where **B** is the matrix whose elements are deviations from the mean of each row of the data matrix **X**, E is the expected value operator, $\otimes$ - the outer product, $(.)^{T}$ - the transpose operator.

In the next step the $M \times M$ diagonal matrix **D** consisting of the set of all eigenvalues along the principal diagonal and the $M \times M$ matrix **V** consisting of the set of all eigenvectors (one eigenvector per column) of the covariance matrix **C**; they are computed from the equation

$$\mathbf{C} \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{D} \quad (2)$$

The columns of the matrix **V** and eigenvalues from the matrix **D** are sorted in order of decreasing eigenvalue. The eigenvector corresponding to the largest eigenvalue is referred to as the first principal component, the one corresponding to the second largest eigenvalue is called the second principal component and so on.

The eigenvalues represent the distribution of the data's energy among each of the eigenvectors, where the eigenvectors form a basis for the data. The elements of the principal component denote the relative contribution of each objective function towards this vector. A positive value signifies increase in objective value when moving along this principal component and a negative value denotes a corresponding decrease. By picking the most-negative and the most-positive elements from a principal component, one can identify two most conflicting objectives.

In the objective dimensionality reduction problem, the PCA NSGA-II procedure outlined in [4] is used.

Design Issues.

The novel design algorithm of the permanent magnet machine is presented. Generally electric machine design aims to maximise their efficiency, minimise weight, power loss or any other technically justified criterion. The modern design is realised by means of numerical approach, usually through finite element technique. The key factor is to prepare the parameterisation algorithms to enable introduction of geometrical details necessary for the correct element mesh of the machine.

Description of the Design Problem.

The permanent magnet machine model is based on several assumptions; the most important are:

- the torque angle of the permanent magnet machine is close to $\pi/2$, which is the generally accepted value,
- the acceptable saturation level of the core and the temperature in the slots are calculated externally and denoted as the B_{nt} and T_{app} values,
- the flux density magnitudes in slots and yoke are equal,
- the leakage flux is neglected.

Permanent Magnet Generator Model.

The equations used in the design are listed hereafter. Each of them introduces and defines graphically new variable. The basic dimensions of the machine are presented in figures 1-3.

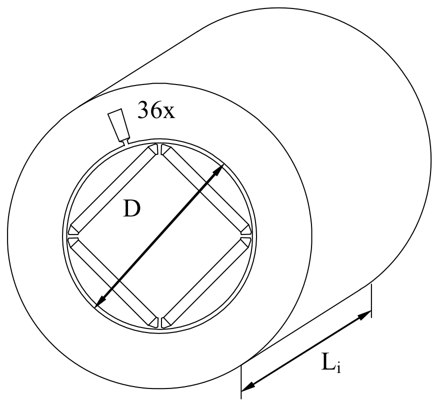


Fig.1. Machine dimensions and graphic definition of optimisation problem argument D.

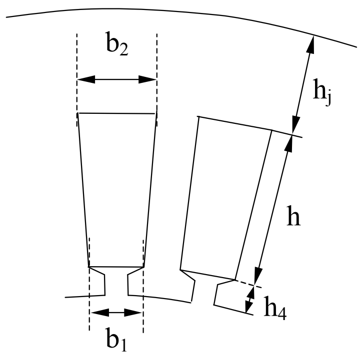


Fig. 2. Machine dimensions and graphic definition of argument h.

Geometrical details of embedded magnets in the rotor are shown in figure 3. Dimensions of magnetic isthmus preventing the magnets from magnetic short-circuiting play an important role in the overall performance of the machine.

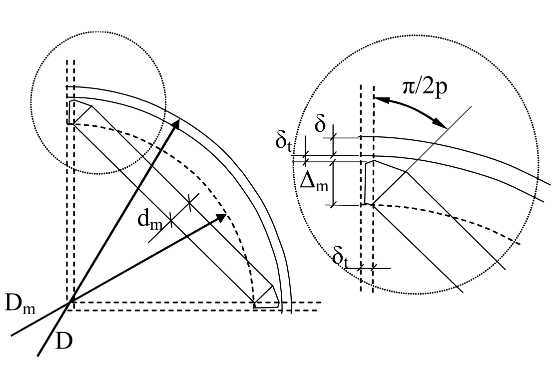


Fig.3. Machine geometry description.

Magnet width :

$$d_m = D - 2(\Delta_m + \delta + 2\delta_t)\sin\left(\frac{\pi}{2p}\right). \quad (3)$$

Effective magnetic pole pitch :

$$\tau_B = \frac{\pi D}{2p} - \Delta_m \sin\left(\frac{\pi}{2p}\right) - \delta_t \quad (4)$$

The distribution of the flux density (radial component) in the air gap is presented in figure. 4.

The value of magnitude B_δ is obtained from the magnetisation curve of permanent magnets, but without taking into account the core saturation.

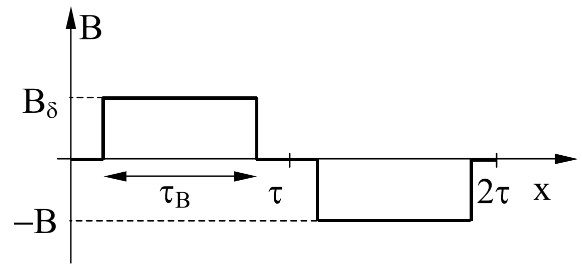


Fig.4. Radial magnetization in the slot

Flux density of the permanent magnet :

$$B_m = B_r\left(1 + \mu_{rm}\frac{d_m}{\tau_B}\frac{\delta}{\Delta_m}\right)^{-1} \quad (5)$$

Yoke height (the saturation level of the core is set to B_{nt}:

$$h_j = \frac{1}{2}\frac{B_m}{B_{nt}}d_m \quad (6)$$

Width of stator tooth and slot dimensions (see figure 2) :

$$b_z = \frac{B_m}{B_{nt}}\frac{d_m}{N}2p \quad (7)$$

$$b_1 = \frac{\pi(D + 2h_4)}{N} - b_z \,,\; b_2 = \frac{\pi(D + 2h_4 + 2h)}{N} - b_z \quad (8)$$

Length of the core (the winding temperature is set to T_{app}) :

$$L_i = \frac{P_N}{1{,}11\,p\,n\,N\,B_m\,d_m}\sqrt{\frac{\frac{b_1 + b_2}{\lambda_c 8h} + \frac{2N}{\alpha\pi(D + h_4 + h + h_j)}}{4\gamma_{Cu}\,k_v\,(b_1 + b_1)\,h\,T_{app}}} \quad (9)$$

Coil turn number :

$$z' = \frac{2m_l U_f}{4{,}44\,p\,nN\xi_1\,B_m\,d_m\,L_i}, \quad (10)$$

where

$$q = \frac{N}{2pm_l} \qquad \xi_1 = \sin\left(\frac{\pi}{2}\frac{s}{\tau}\right)\sin\left(\frac{\pi}{2m_l}\right)q\sin\left(\frac{\pi}{2m_l q}\right)^{-1}.$$

Diameter of conductor in the coil :

$$d = \sqrt{\frac{2}{\pi}k_v h\frac{b_1 + b_2}{z'}} \quad (11)$$

Mean length of the turn l_z :

$$l_z = 2L_i + \pi^2 Ds(2p\tau)^{-1}. \quad (12)$$

Weight of lamination - the first objective function :

$$G_{Fe} = \rho_{Fe}(D + h_4 + h + h_j)^2 L_i . \quad (13)$$

Weight of permanent magnets - the second objective function:

$$G_{Pm} = 2\rho_{Pm}p\Delta_m d_m L_i. \quad (14)$$

Weight of the winding - the third objective function:

$$G_{Cu} = \rho_{Cu}z' N\pi d^2 l_z / 8 . \quad (15)$$

Copper power loss - the fourth objective function:

$$\Delta P_{Cu} = N\left(\frac{P_N}{U_f m_l}\right)^2 \rho_{Cu}\left(1 + \alpha_R T_{app}\right)\frac{2z' l_z}{\pi d^2}. \quad (16)$$

Iron power loss - the fifth objective function:

$$\Delta P_{Fe} = \Delta_p B_{nt}^{\,2}\left(Nb_z h + 2h_j\left(D + h_4 + h + \frac{h_j}{2}\right)\right)L_i . \quad (17)$$

In the presented novel design of the permanent magnet machine, expressed in (3) - (12), the leakage flux is neglected i.e. it is assumed that all ampere-turns contribute to the power conversion. This simplification may lead to high value of the slot height h, therefore the constraint

$$h < 0.09 \text{ [m]}$$

is introduced.

The diameter $D \in (0.1 , 0.4)$ (m) and the slot height $h \in (0.005 , 0.09)$ (m) are the arguments of all considered objective functions (13) - (17).

The values of all constants of the analysed machine are summarised in Table 1.

Table 1. Design constants used in computations

Symbol	Constant	Value [unit]
P_N	rated power	20,0 [kW]
p	number of poles	4
n	rotation speed	25,0 [s^{-1}]
m_l	design parameter	3,0
U_f	rated voltage	230 [V]
N	number of slots	36
λ_c	thermal conductivity	0,7 [W $m^{-1}deg^{-1}$]
α	convection coefficient	20,0 [W $m^{-2}deg^{-1}$]
γ_{Cu}	copper conductivity	56 10^6 [Sm^{-1}]
k_v	slot filling factor	0,5
μ_{rm}	permeability of magnets	1,1
Δ_m	dimension (see Figure 3)	3,0 [mm]
δ_t	dimension (see Figure 1b)	1,0 [mm]
δ	dimension (see Figure 1b)	1,5 [mm]
h_4	dimension (see Figure 1b)	3,0 [mm]
B_r	magnetic remanence	1,1 [T]
B_{nt}	acceptable flux density in the core	1,6 [T]
T_{app}	acceptable temperature	120 [deg]

Multiobjective Design

In this Section the dimensionality reduction of the permanent magnet machine multiobjective design (3) - (12) is performed. The main purpose of the analysis is to compare results from the full, five-objectives problem (13) - (17), with the reduced four- and two-objectives ones.

The Non-dominated Sorting Genetic Algorithm (NSGA-II) [3] is employed in the computations. It is a very effective algorithm which incorporates elitism and does not require a sharing parameter to be chosen *a priori*. In this paper the real-coded representation of the design problem is employed.

Parents are selected from the population using the binary tournament selection method, based on the rank and crowding distance. The algorithm uses the Simulated Binary Crossover (SBX) scheme and polynomial crossover. The distribution is obtained from a uniformly sampled random number in the range (0,1). The only stopping rule considered is the number of generations *a priori* defined. From tests concerning various number of generations, $G = 100$ has been selected. It brings satisfactory Pareto front approximation for comparison of results.

Table 2. Basic parameters of the NSGA-II algorithm

Algorithm parameter		Value
Population	*N*	400
Generations	*G*	100
Pool size	*N*/2	200
Tour size		2
Crossover probability		0,9
Mutation probability		0,5

All computations were ran within the MATLAB [9] for Windows workspace. They were repeated many times, but the full statistical analysis of the results exceeds the scope of this presentation.

Objective Redundancy Analysis

The first step is to examine the full set of five objectives functions (13) - (17). The approximation of the Pareto set obtained after *G* generations is used to evaluate the fit of the population. For each i-th objective function one obtains the *N* values. They are placed in a row vector X_i so the results form the M x N matrix. After finding the covariance matrix C (1), the $M \times M$ eigenvector matrix V (2) is computed and principal components are analysed. Sample results for the full set of M=5 objective PCA are presented in form of eigenvalues, arranged in order of decreasing eigenvalue, and corresponding eigenvectors (18). The results and values presented hereafter are typical for the evolutionary computations performed by authors during this study.

$$\lambda^T = [3.2469 \quad 1.1795 \quad 0.4714 \quad 0.0745 \quad 0.0277]$$

$$V^T = \begin{bmatrix} 0.5416 & 0.5328 & -0.4503 & 0.1668 & 0.4384 \\ -0.0974 & -0.0610 & -0.0429 & -0.8682 & 0.4808 \\ 0.0564 & 0.2419 & 0.8458 & 0.1773 & 0.4376 \\ 0.5781 & -0.7744 & 0.0418 & 0.1131 & 0.2270 \\ -0.5998 & -0.2326 & -0.2798 & 0.4173 & 0.5776 \end{bmatrix} \quad (18)$$

Analysing elements of the eigenvector v_1 corresponding to the largest eigenvalue λ_1 (first principal component) one can observe that the first (equation 13) and second (equation 14) objectives are the least conflicting. On the other hand the third one (equation 15) is the most conflicting with the first (and second as well). It may be concluded that either the first or second objective may be redundant in this problem. In further research the second objective will be discarded.

After removing the second objective (14) and replacing of the first one (13) by the sum of the first (13) and second one (14) one has to repeat the calculations for reduced four-dimensional objective space now defined.

The second step of dimensionality reduction procedure concerns the four-dimensional objective vector defined as $[G_{Fe}+G_{Pm}, G_{Cu}, \Delta P_{Cu}, \Delta P_{Fe}]$. The PCA analysis gives *4* x *4* covariance matrix C (1) whose eigenstructure, after arranging in order of decreasing eigenvalue, is presented in (19).

$$\lambda^T = [2.6052 \quad 0.9914 \quad 0.3782 \quad 0.0253]$$

$$V^T = \begin{bmatrix} 0.6080 & -0.5307 & 0.2727 & 0.5237 \\ -0.0482 & -0.0888 & -0.8960 & 0.4325 \\ 0.2298 & 0.8257 & 0.1445 & 0.4946 \\ -0.7584 & -0.1697 & 0.3194 & 0.5422 \end{bmatrix} \quad (19)$$

Repeating the steps performed for five-objectives optimisation one has to analyse first principal component (eigenvector v_1). It is clear from (19) that the first and the fourth objectives are the least conflicting.

After removing the fourth objective (17) and replacing of the first one (12) by the sum of the first (13) and fourth one (17) calculations are performed for reduced three-dimensional objective space. For sake of compactness, the details of the other two steps are not reported, as the procedure is analogous to the first one.

The final design problem formulation is for two objectives only. The first criterion is the weight of the winding (15). The second objective, denoted by the F, is the sum of the rest four objectives (13), (14), (16), (17). They have no physical meaning because the sum of kilograms (e.g. G_{Fe}) and watts (e.g. ΔP_{Cu}) cannot be calculated.

$$F = \alpha_1 G_{Fe} + \alpha_2 G_{Pm} + \alpha_3 \Delta P_{Cu} + \alpha_4 \Delta P_{Fe} \,. \quad (20)$$

To simplify the analysis, in all steps, all coefficients are set equal one, i.e. $\alpha_1 = \alpha_2 = 1$ [kg^{-1}], and $\alpha_3 = \alpha_4 = 1$ [W^{-1}].. The PCA NSGA-II [4] is employed in the computations.

Results Analysis

Two approximations of Pareto sets for the five-objectives and the two-objectives optimisation respectively are presented in figure. 5, showing that with dimension reduction population tends to concentrate.

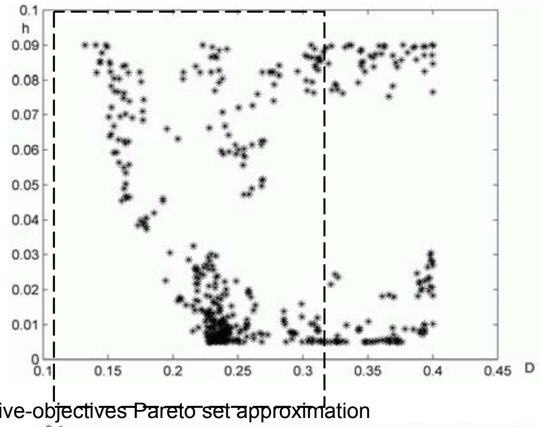
Please note the Pareto set approximation for the two-objectives optimisation in figure 5(b) is plotted on a different scale to the five-objectives result.

The result for the four-objectives optimisation is very similar to the five-objectives one and is not presented for sake of compactness.

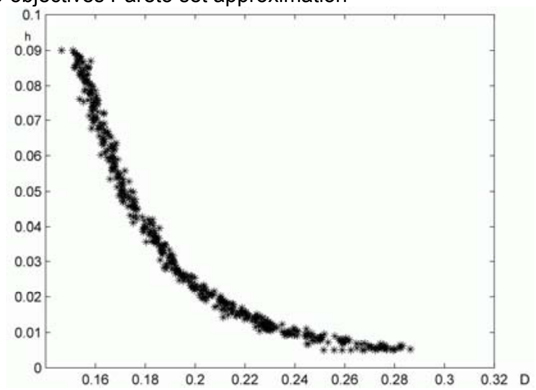
Pareto set approximation for the two-objectives optimisation may be considered a good projection of Pareto set approximation for the five-objectives problem. It is possible to explain the high concentration of solutions by comparing the 3D view of the five-objectives surfaces with surfaces for the two-objectives problem (not presented for sake of compactness).

It is evident that close to the two-dimensional Pareto-front approximation (represented by crosses) the shape of reduced objective functions almost matches the surface of the F objective (20) The cut down, loosely spaced, solutions for the five-objectives (Figure 5(a)) may be attributed to the reduced objectives.

The analysis of solutions to multidimensional problems is always challenging. In the case of five objectives, even synthetic evaluation is problematic. For this reason, and because of difficulties with multidimensional visualisation, the comparison approach is applied.



a) five-objectives Pareto set approximation



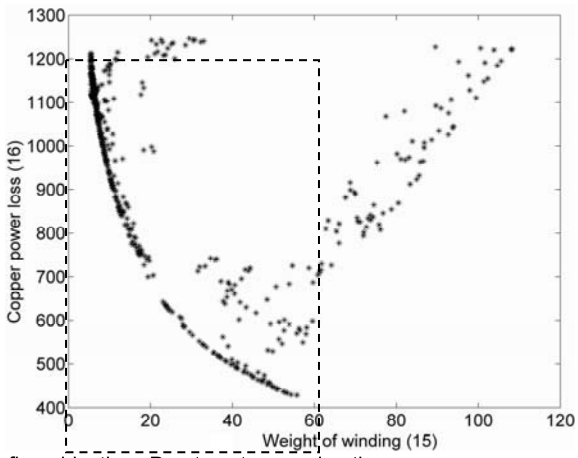
b) two-objectives Pareto set approximation

Fig.5. The Pareto sets approximations for changing number of objectives

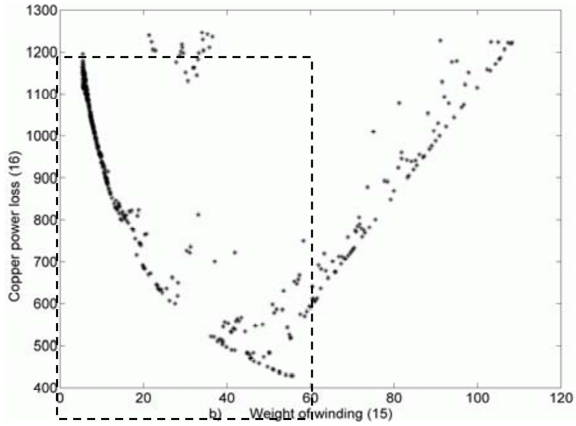
The solution sets are presented in figures 6 and 7 for selected objectives only. In figure 6 three solutions represented by characteristic $\Delta P_{Cu}(G_{Cu})$ are compared. The functions under consideration describes the generator winding ((16) and (15) respectively) which is one of the most important parts of the electric machine. The sample results in figures 6(a)-(c) are computed from the Pareto set approximations for the five-, four- and two-objectives optimisation respectively.

The characteristic $F(G_{Cu})$, (20) and (15) respectively, seems to be more synthetic for our analysis because in the two-objectives space the set $F(G_{Cu})$ is the Pareto front approximation. Applying the same comparative approach the results are presented in figures 7(a)-7(c).

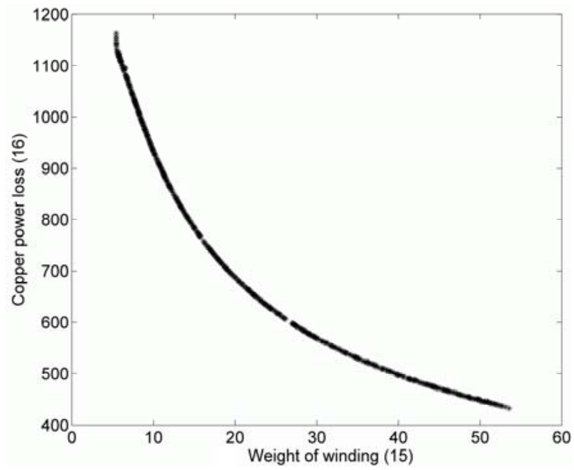
Again, the considerable parts of all three sets presented remains unchanged.



a) five-objectives Pareto set approximation



b) four-objectives Pareto set approximation

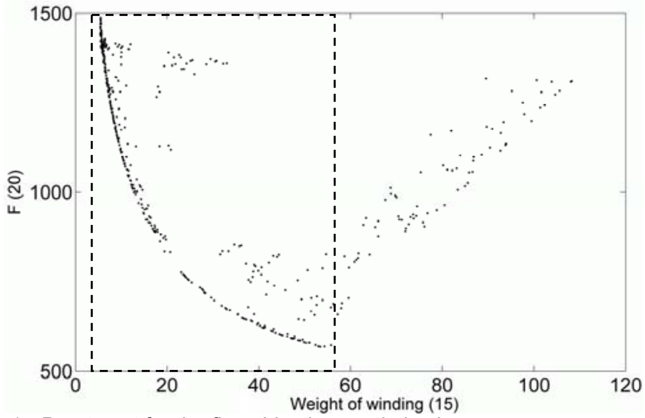


c) two-objectives Pareto set approximation

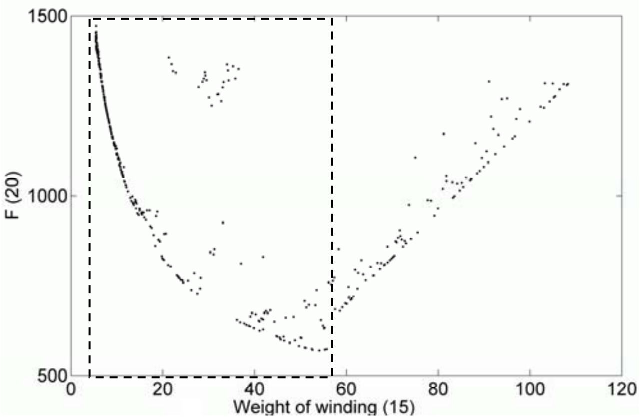
Fig.6 The weight winding (15) - copper power loss (16) for the Pareto sets approximations for a) five-, b) four-, c) two-objectives respectively. Please note the change of scale. The c) is enlargement of the dashed rectangles in a) and b). Number of points (solutions) is constant

The dimensionality reduction left this part of the $\Delta P_{Cu}(G_{Cu})$ characteristic basically unchanged. This result is projection of the Pareto set approximations concentration, as presented in figure 5.

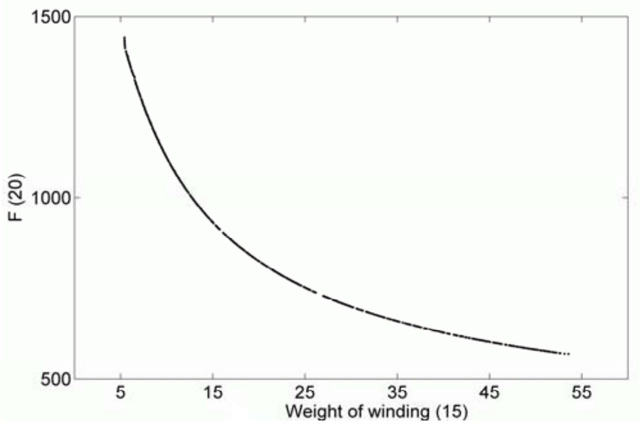
The same effect was observed over the course of this study for all other (13) - (17) design objectives not presented for the sake of compactness.



a) Pareto set for the five objectives optimisation



b) Pareto set for the four objectives optimisation



c) Pareto set for the two objectives optimisation

Fig.7. The winding weight (15) and objective defined by (20) for the Pareto sets approximation for a) five- b) four- c) two- objectives optimisation. Please note the change of scale. The 7(c) is enlargement of the dashed rectangles in a) and b). Number of points (solutions) is constant

The reduction of the five-objectives optimisation problem to the two-objectives one gives the DM a much clearer view of the Pareto front approximation. The advantage for a designer - Decision Maker is obvious.

Conclusions

The results of the Pareto-optimal design of the permanent magnet generator presented in this paper, demonstrate the Pareto-optimal set do not change much in the objective reduction process.

The comparative analysis of the selected characteristics of obtained sample results, describing the machine design, shows that applied PCA reduction procedure, implemented as the PCA NSGA-II, preserves the Pareto front approximations distribution on selected planes.

This paper provides a basis for further research with a more detailed generator models including saturation of the magnetic circuit [8]. Future research will also include more detailed statistical analysis of the results.

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