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Determination of equivalent electric parameters of heterogeneous structures

Abstract. *Equivalent values describing non-homogeneous material, calculated using the developed algorithm, can be used in modelling large-scale systems in the case when it is not possible to fully map the complex structure, which made of composite material (e.g. clinker bricks, concrete). The developed algorithm can be used to determine equivalent electric parameters of various complex structures taking into account different electric parameters and their range of single dielectric, size of holes/admixtures, frequency or the thickness of the layer through which the EM wave passes.*

Streszczenie. *Parametry zastępcze opisujące niejednorodny materiał, które są wyznaczone przez opracowany algorytm, mogą zostać zastosowane przy modelowaniu układów dużej skali, gdzie niemożliwe jest pełne odwzorowanie złożonych struktur, które są kompozytami (np. cegły klinkierowe, beton). Opracowany algorytm może być wykorzystany do obliczenia zastępczych parametrów różnych złożonych struktur przy uwzględnieniu zróżnicowania parametrów elektrycznych i ich zakresu pojedynczych dielektryków, rozmiaru wtrąceń, częstotliwości i grubości warstwy, przez którą przechodzi fala EM. (**Wyznaczanie zastępczych parametrów elektrycznych struktur niejednorodnych**).*

Keywords: heterogeneous materials, equivalent electric parameters, electromagnetic field, optimization.
Słowa kluczowe: materiały niejednorodne, elektryczne parametry zastępcze, pole elektromagnetyczne, optymalizacja.

Introduction

Issues connected with the determination of properties and formulations of substitute models of composite materials are an important and constantly developed topic [1-6, 8]. The justification of the work is the desire to create substitute models that can be used in the analysis of large-scale models. The determined equivalent electric parameters of complex materials, e.g. clinker bricks or concrete can be taken into account when calculating models on a larger scale, in which, due to the size of the created numerical models, the discrete arrangement of air areas and ceramic mass cannot be fully mapped.

In the case of time methods (e.g. FDTD, FEM), the evaluation of phenomena is carried out in a steady state [9-11], taking into account multiple wave reflections. The value of the transmission coefficient obtained in this way can be the basis for determining the equivalent electric permittivity and conductivity of the wall assuming the homogeneity of its structure. The solution to the task of homogenizing electric properties material can be obtained by forming the selection of substitute parameters as an optimization task [7, 8].

The article presents the results of a developed optimisation algorithm whose task is to determine equivalent electric parameters for a heterogeneous structure. As an example, commonly used clinker brick with holes was used. Conductivity and holes size changes were considered and equivalent electric parameters for the new model without holes were proposed. This makes it possible to analyse models on a large scale.

Influence of size holes inside the bricks on values of electric field intensity

One of the commonly used building materials is clinker brick (Fig.1) [2, 4]. The analysed model included the single-layered walls (0.12 m) consisting of clinker bricks with holes. The dimensions of the analysed brick (with the analysed frequency of f=2.4 GHz) are comparable to the length of the electromagnetic wave propagating in air λ_a=0.125 m. Figure 1 is a representation of the dimensions of the brick and their electrical reproduction, where λ_b=0.0593 m is the length of the wave in a clay material.

The variability of the width of the holes was analysed (s) along the length of the brick (l) assuming that $s\in\{0, 0.005, 0.007, 0.011, 0.013, 0.015, 0.017, 0.019$ m$\}$ [2, 4]. Typical size of holes is 0.011 m (Fig.1).

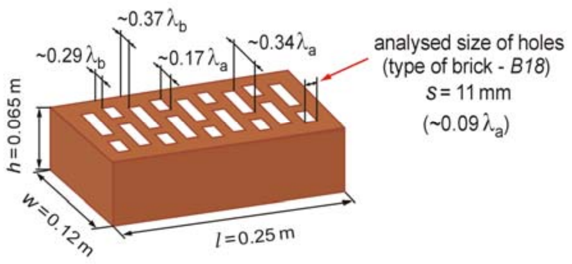

Fig.1. The electric and geometric sizes of the analysed brick

The size of the holes influences on the percentage share of the loss dielectric (clay mass) in the brick (v) was presented in Table 1. Blue colour signifies the values of the typical holes sizes and its relative volume of the clay in the brick. To analyse a relative electrical permittivity (ε_r'=4.44) was determined, whereas the conductivity was modified within the range $\sigma\in\langle 0, 0.2\rangle$ S/m [1-4, 5, 6].

Table 1. The percentage of the clay material inside the analysed brick dependent on the size of holes

Geometric size of holes inside the brick (s) [m]	Relative volume of the clay in the brick (v) [%]
0.005	90.40
0.007	86.56
0.009	82.72
0.011	78.88
0.013	75.04
0.015	71.20
0.017	67.36
0.019	63.52

The origin of the field was a sinusoidal oscillating plane wave (f=2.4 GHz). The electromagnetic field is excited in a region far away of the wall. The absorption of the incident and reflected waves are obtained using PML boundary conditions (*perfectly matched layer*) [9, 10]. These were entered in the outside areas, perpendicular to the equiphase surface. Phenomena at the edges of the wall were ignored because the Bloch's periodic boundary conditions were assumed [10].

To determine the distribution of the electromagnetic field (the relative maximum values of E_z component behind the wall), the finite difference time domain method (FDTD) was used [2, 9, 10]. The FDTD method is based on the direct numerical integration of the Maxwell's curl equations in time and space [9-11].

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \tag{1}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{2}$$

Figure 2 shows the distribution of E_z component inside the analysed area. The results of calculation are shown in the *XY* plane at the time when the steady state of the EM field distribution had been achieved.

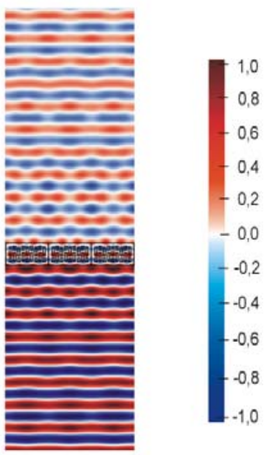

Fig.2. The 2D distribution of relative value of E_z component inside analysed area with the wall made of bricks (s =0.011 m)

A topical change of the EM wave's speed in passing through different areas of the bricks leads to the creation of momentary images of the electric field distribution. This proves the occurrence of interference. The discussed effect is particularly visible while assessing the effects occurring behind a wall made of bricks with large size of holes (s>0.011 m). In these cases the field behind the wall has higher values both minimum and maximum.

Figure 3 shows the relative maximum values of the E_z component and their dependencies on the size of holes (s) and also conductivity.

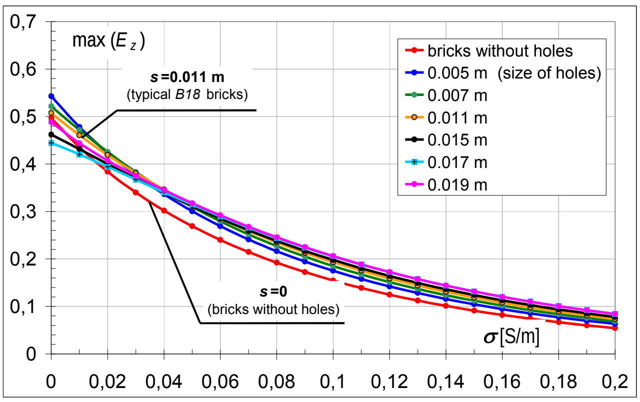

Fig.3. The relative maximum values of E_z component behind the analysed wall made of bricks with different size of holes

The higher v causes lower distortion of the wave front. At the same time, it influences negatively the ability to acquire higher field values (due to damping). In Fig. 4 the comparison of the increase of the maximum relative value of the E_z component is presented in dependence to the percentage of clay mass in the analysed wall models in the range of the most often used values of conductivity.

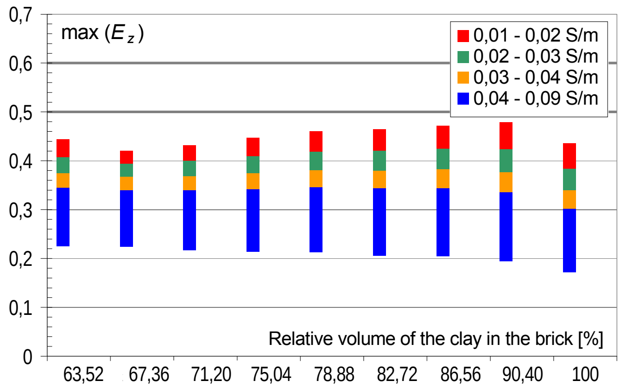

Fig.4. The dependence between max(E_z) and the relative volume of the clay inside the brick

The propagation of EM wave in the area of brick is a complex process. The number and size of the holes in the brick results in temporary changes of field image in the area close behind the wall.

Little changes in the size of the holes result in a change in the value of the electric field. For this reason, it is necessary to accurately reproduce the model and holes and also assign appropriate values for electric parameters. When analysing large-scale models, the problem is the mapping of all details of complex materials. In order to analyse large-scale models, it is necessary to apply homogeneity of the structure, e.g. bricks, with due regard to equivalent dielectric parameters.

Algorithm for determining equivalent electric parameters

Determination of equivalent electric parameters was solved by applying an optimization algorithm and solving the task of minimizing the error of approximation of substitute wall parameters (Fig. 5). As the example the wall made of clinker bricks was used.

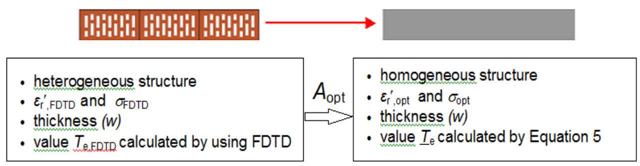

Fig.5. Task scheme and connection of input and output data in the implementation of the developed numerical algorithm (A_{opt})

The developed optimization algorithm (A_{opt}) aims to determine the equivalent values of the electric parameters of the homogenised material ($\varepsilon_r'_{,opt}$ and σ_{opt}) for the composite material under consideration. Wall thickness (internal geometry of the model) and frequency were adopted as invariants in the conducted optimization.

The analysed heterogeneous material (with holes), which was used for modelling and analysis of the walls, was replaced in the proposed algorithm by a homogeneous material. The analysed model was described with material with isotropic properties. The optimization algorithm developed and used included (Figs. 5-6):

(3) $$A_{\text{opt}} = \left\{ \varepsilon^{'}_{\text{r,min}}, \varepsilon^{'}_{\text{r,max}}, \sigma_{\min}, \sigma_{\max}, \Delta_{\varepsilon}, \Delta_{\sigma}, \delta_{A}, f_{g} \right\}$$

where: $\varepsilon_{r}'_{,\min}$ and $\varepsilon_{r}'_{,\max}$ – limitation on the domain of optimization for relative electric permittivity; $\sigma_{\min}$ and $\sigma_{\max}$ – limitation on the domain of optimization for conductivity; Δ_{ε} and Δ_{σ} – resolution of the domain optimization; δ_A – objective function; f_g – function of generating sequential solution variants.

The calculation base is the definition of the objective function δ_A, classifying the calculated variants, described by the assumed values $\{\varepsilon_r', \sigma\}$. The objective function was assumed as a relative measure of the difference between the numerical calculated value and calculated on the base of analytical dependence [11]:

(4) $$\delta_A = \frac{\left| T_{\text{e}} - T_{\text{e,FDTD}} \right|}{T_{\text{e,FDTD}}}$$

where: T_e – transmission coefficient module of the electric field value determined by the analytical method (Equation 5) for a given, iterative variable value of material parameters $\{\varepsilon_r', \sigma\}$ and a defined wall thickness (w);
$T_{e,FDTD}$ – transmission coefficient calculated by the numerical method (FDTD) for given: electric parameters of the ceramic material $\varepsilon_r'_{,FDTD}$, σ_{FDTD}, wall thickness and brick structure (Fig.1).

With orthogonal wave incident on the material boundary, the transmission coefficient of electric field in the area behind the wall is described by the formula [11]

(5) $$\underline{T}_{\text{e}} = \left| \underline{T}_{\text{e}} \right| = \left| \frac{E^{'}_{2+}}{E_{1+}} \right| = \left| \frac{\underline{T}_{\text{e1}} \cdot \underline{T}_{\text{e2}} \cdot e^{-\text{j}\underline{k}b}}{1 + \underline{\Gamma}_1 \cdot \underline{\Gamma}_2 \cdot e^{-2\text{j}\underline{k}b}} \right|$$

The search for the optimal solution was carried out taking into account (Fig.6):

1. limitation on the domain of optimization for
   $\varepsilon_r' \in \{\varepsilon_r'_{,\min}, \varepsilon_r'_{,\max}\}$, $\sigma \in \{\sigma_{\min}, \sigma_{\max}\}$, where accepted for:
   - equivalent relative electric permittivity: $\varepsilon_r' \in \langle 2, 6 \rangle$;
   - equivalent conductivity: $\sigma \in \langle 0, 0.1 \rangle$ S/m.

   The limit values were determined on the base of data in [1-4].
2. defining the limiting resolution of searching the solution Δ_ε, Δ_σ constituting the maximum differences in the values of ε_r' and σ, by which the considered variants may differ, and on this base determining the number of iterations of the tasks performed when selecting $\varepsilon_r'_{,opt}$ (N_ε) and σ_{opt} (N_σ):

(6) $$\text{N}_{\varepsilon} = \left(\varepsilon^{'}_{\text{r,max}} - \varepsilon^{'}_{\text{r,min}} \right) / \Delta_{\varepsilon}$$

(7) $$\text{N}_{\sigma} = \left(\sigma_{\max} - \sigma_{\min} \right) / \Delta_{\sigma}$$

In order to determine the replacing electric parameters, a opimalization algorithm was developed based on the analytical [10] and numerical methods [8-9]. The overall structure of the tasks carried out was divided into two main stages. The developed scheme of the error minimization algorithm includes the elements defined before the start of calculations (stage I), as well as presents the process of optimisation calculations (stage II).

Stage II relies on iteratively performing calculations for assumed, selected subsequent values of equivalent parameters $\{\varepsilon_r', \sigma\}$.

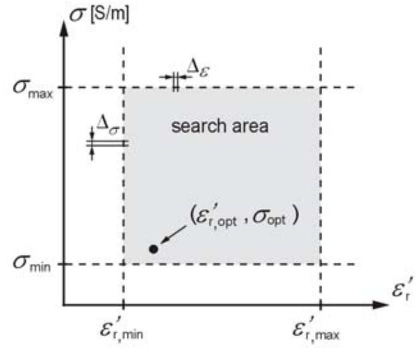

Fig.6. Limitations of the search area when determining the equivalent electric parameters

A calculation algorithm was used to generate variants, which due to the short calculation time allowed the analysis of all real variants. This stage includes:

- determination of the relative error value (δ_A) by reference to an analytical calculated value obtained on the base of numerical calculations calculated using the Equation 4;
- using a sorting algorithm to create and refresh a list of calculated variants with the lowest approximation error value δ_A.

Calculation results

At frequency 2.4 GHz, the conductivity range $\sigma_{FDTD} \in \langle 0.01, 0.04 \rangle$ S/m, which is usually used to describe ceramic materials [1-4] was used to calculate the equivalent electric parameters. Figure 7 presents the relative error distributions δ_A depending on the value of ε_r' and σ. The location of the optimal variant ($\varepsilon_r'_{,opt}$, σ_{opt}), in the global sense described by the minimum value of the objective function, is presented as a point marked with a white dot. The figures also present the whole range of local minima, which give solutions with a greater error.

Regardless of the conductivity material, variants with the smallest error (δ_A) were in the area $\sigma_{opt} \in \langle 0.01, 0.03 \rangle$ S/m. Along with the increase the conductivity describing the numerical model (i.e. $\sigma_{FDTD} \in \langle 0.01, 0.04 \rangle$ S/m) the point determining the equivalent parameters of the ceramic material is moved towards higher values σ_{opt}.

Table 2 summarizes the best solutions (min δ_A) obtained using the developed optimization algorithm. The calculated values of $\varepsilon_r'_{,opt}$ and σ_{opt} were given, while indicating a relative error.

The obtained results indicate that for analysed wall the equivalent value of $\varepsilon_r'_{,opt}$ average equal 3.3 and conductivity is in range $\sigma_{opt} \in \langle 0.004, 0.02 \rangle$ S/m.

Table 2. Results of calculation the equivalent electric parameters for analysed cases taking into account the variable conductivity

Electric parameters of clinker brick adopted in numerical calculations (FDTD)		Designated equivalent electric parameters for the ceramic material		Relative error
$\varepsilon_r'_{,FDTD}$	σ_{FDTD} [S/m]	$\varepsilon_r'_{,opt}$	σ_{opt} [S/m]	δ_A [%]
4.44	0.01	3.63	0.005	0.13
	0.02	3.27	0.009	0.80
	0.03	3.52	0.015	0.25
	0.04	3.15	0.022	0.42

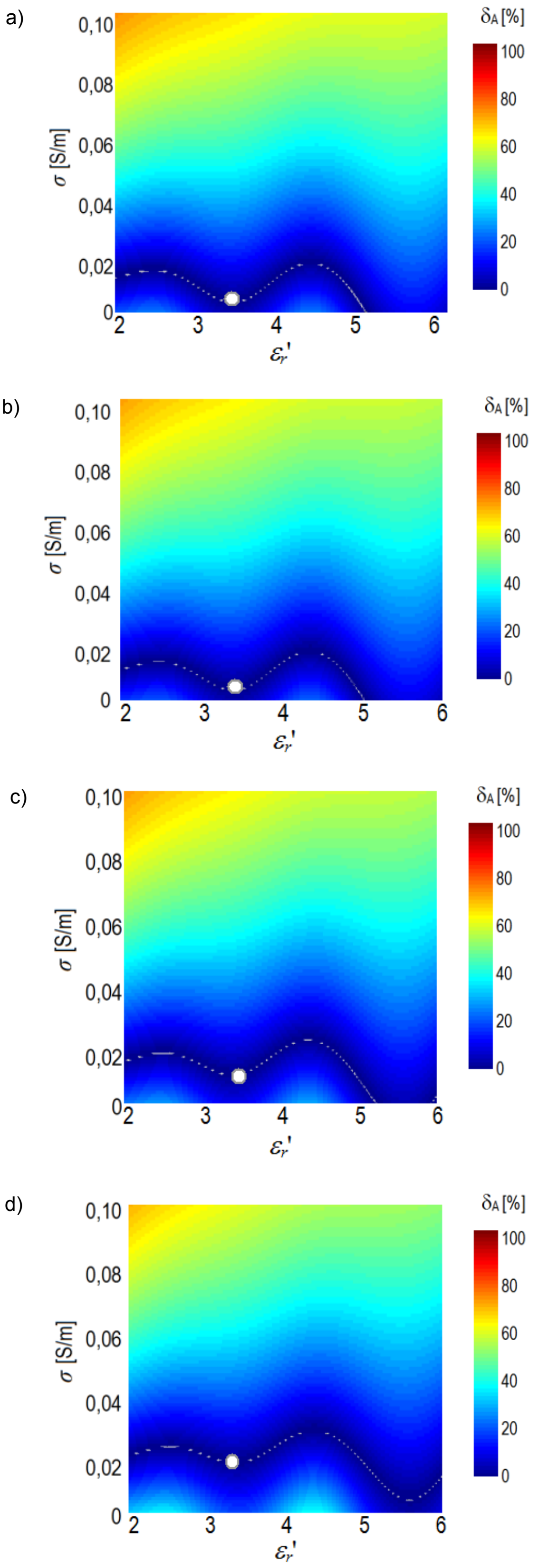

Fig.7. Calculated substitute relationship between relative electric permittivity and conductivity in a model with a wall made of hollow bricks, with the initial assumption: a) σ_{FDTD}=0.01 S/m, b) σ_{FDTD}=0.02 S/m, c) σ_{FDTD}=0.03 S/m, d) σ_{FDTD}=0.04 S/m

The analysis showed that the calculated equivalent value of σ_{opt} was twice lower than the conductivity assumed for the calculations σ_{FDTD}. However, irrespective of the variant of the accepted input data, the determined electric permittivity $\varepsilon_r'_{,opt}$ was much lower than the value assumed in the numerical calculations $\varepsilon_r'_{,FDTD}$=4.44.

Conclusions

Equivalent electric parameters values describing the heterogeneous material, calculated using the developed algorithm could be used in modelling large-scale systems, when it is not possible to map the complex structure of walls made of e.g. hollow bricks.

The developed algorithm is universal and can be used to determine substitute parameters for various complex structures (e.g. clinker bricks, concrete with admixtures) taking into account different values of ε_r', σ and their range, size of hollows or inclusions (admixtures), as well as frequencies or wall thickness.

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