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doi:10.15199/48.2020.08.18

Principle of the least action in models and algorithms optimization of the conditions of the electric power system

***Abstract**. The possibility and expediency of application of the principle of least action (PLA) in the optimum control of conditions of the electric power system (EPS) is shown. The loss of active power and electricity was taken as the criterion of optimality. The algorithm for determining the optimal condition of the EPS is presented. Its peculiarity is that, in accordance with the LPA, the ideal condition of the EPS is first determined, and then parametric restrictions are taken into account. The ideal condition of the system is characterized by the lowest possible losses, and taking into account active restrictions on the parameters transposes the EPS to the optimal condition. In the optimal condition of the EPS, the losses increase depending on the degree of heterogeneity of the system.*

Streszczenie.** Pokazano możliwość i celowość zastosowania zasady najmniejszego działania w optymalnej kontroli stanów systemu elektroenergetycznego. Utratę mocy czynnej i energii elektrycznej przyjęto za kryterium optymalności. Przedstawiono algorytm określania optymalnego stanu systemu elektroenergetycznego. Jego szczególną cechą jest to, że zgodnie z zasadą najmniejszego działania najpierw określa się idealny stan systemu elektroenergetycznego, a następnie uwzględnia się ograniczenia parametryczne. Stan idealny systemu charakteryzuje się najniższymi możliwymi stratami, a wzięcie pod uwagę aktywnych ograniczeń parametrów przenosi system elektroenergetyczny do stanu optymalnego. W optymalnym stanie systemu elektroenergetycznego straty rosną w zależności od stopnia niejednorodności układu. **Celowość zastosowania zasady najmniejszego działania w optymalnej kontroli stanów systemu elektroenergetycznego

Keywords: optimal control, electric power system, least action principle, minimum power losses.
Słowa kluczowe: optymalne sterowanie, system elektroenergetyczny, zasada najmniejszego działania, minimalne straty mocy.

Introduction

One of the root reasons for the nonoptimal conditions of the EPS modes is their heterogeneity as a constructive parameter of an electric network [1]. In [2], the relation of reactive and active resistances of all *i-th* branches of the EPS $X_i/R_i \neq idem$ was introduced as a sign of heterogeneity. Optimization of inhomogeneous EPS modes can be performed by reducing the degree of heterogeneity due to the reconstruction of existing networks or by using flow control to approximate it to the flows in a homogeneous EPS. Purposeful reconstruction of power grids, construction of new power lines can allow supply the optimality of the conditions s of the EPS regime regardless of the technological conditions of operation. However, this approach requires significant capital investment. There is another way - to improve the optimal control of the normal modes of EPS in order to improve its efficiency [3, 4]. As a result, technical losses of electricity in the EPS should be reduced and electricity is quality improved.

Effective compensation for the negative influence of inhomogeneity on the optimality of EPS modes is primarily related to the increase of the level of Automation of Operational Dispatcher Control (ASDC) as an important task of improving the control of EPS. Further development of ASDC is carried out on the principles of Smart Grid and envisages the transition from centralization to decentralization of control and full automation of the basic functions of the control process: gathering and processing of information, making decisions and forming control influences in order to maintain the normal modes of EPS taking into account their hierarchical structure [3, 4]. It should be noted that the automation of control involves not only the automatic implementation of one or more control functions, but the creation of a system that organically enters a person, a computer, a set of methods and techniques that ensure the effective execution of a large number of control and creation functions based on the dispatching expertal system.

The most cumbersome, given the limitations on the reliability and speed of the solution, are the computational procedures for determining the set of optimal independent parameters of the EPS, especially when it comes to real large-scale electrical systems. The fundamental difference between modelling the optimal conditions of these systems based on the principle of least action is the ability to reliably obtain the extremum of the objective function by simple calculations, followed by the refinement of independent parameters. Therefore, this approach offers significant advantages in solving the problems of operational control of the normal modes of the EPS.

The main goal of the research is the creation of a method and algorithm for the development of a system of automatic control by normal modes using principle last action.

The principle of the least action (PLA) as a method of generating optimizing influences on the EPS normal modes

As shown in [5, 6], PLA determines the optimality of the functioning of any system, as well as the development of a system that is aimed at increasing the degree of its ideality. For a system at any moment of its existence, the norm is a qualitative optimum, the depth of which is determined by the degree of ideality of the system. This feature of LPA can be used as an idea to build a computational process to find the optimal mode of the EPS, where the criterion of optimality is the loss of electricity during its transportation.

PLA is effectively used to solve a number of tasks in the power industry. In [7], the authors use the modified Hamilton-Ostrogradsky principle for the analysis of transients in the electrical network. This article describes a model of a high-voltage electrical network, where the transmission lines are presented as equivalent circuits with distributed parameters. In [8], an algorithm for ensuring the stability of the electrical system was created using the theory of analytical mechanics. The proposed approach was tested on the scheme IEEE 14, was modeled the control effect of the system at a three-phase short circuit. A controller has been developed that can stabilize the system asymptotically in the equilibrium point. In [9], the balance of power in the power system is presented in the form of a Hamiltonian function, which can be useful in the problems of calculating the parameter areas of stable and unstable conditions of the system. The article describes the method of determining the energy balance for the direct analysis of the transient stability in the grid. The method is based on the assumption that the Hamiltonian is a bounding surface that the trajectory of a system must pass through, when energy enters it, energy is dissipated inside, and energy is stored inside the system as time progresses.

In the considered problems, where the effective use of PLA is characteristic, all of them are related to the analysis of energy balance in the power system. However, a PLA approach can be useful for algorithmizing tasks that minimize the components of electricity in its overall balance.

It is shown in [10] that known energy approaches, such as the representation of an object of control and of all automated systems such as the Euler-Lagrange system or in the form of a controlled Hamiltonian system, can be used to simplify the synthesis of the control system of the power generation. The article presents a control system for autonomous wind installation. [11, 12] shows the efficiency of the use of PLA for optimal control of the condition of the local electric system with renewable energy sources, in particular with photovoltaic power plants. The automated control system is practically implemented and a positive experience has been gained. Implementation of the steady-state mode of the electric grid according to its alternate R-scheme provides not only the economical mode of the power sources, but also achieves minimal losses of power in the local electrical system. It is therefore advisable to summarize the experience gained and to extend the use of LPA for complex power systems.

Changing the condititon of the system between two fixed positions from the moment $t = t_1$ to $t = t_2$ always occurs in such a way that a stationary value (usually a minimum) of the functional is given [5]

$$minQ = \int_{t1}^{t2} L(g,\dot{g},t)dt \tag{1}$$

where g is some generalized parameter; $L = K - P + D$; K is the kinetic energy of the system; P is the potential energy of the system; D is the energy of the forces of external and internal dissipation. The necessary and sufficient conditions for the stationarity of this functional (the "integral of action"), determined by the methods of variational calculus, are to fulfill the Euler-Lagrange equation:

$$\frac{\partial L}{\partial g} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{g}}\right) = 0. \tag{2}$$

To obtain the Euler equation for the steady-state of the EES, we write the energy functional. According to the Hamilton-Ostrogradsky integral variational principle [13], in accordance with (1):

$$Q = \int_{t1}^{t2} L(q,\dot{q},t)dt = \int_{t1}^{t2} (W_L - W_C + W_R + W_e)dt \tag{3}$$

where $L(q,\dot{q},t) = W_L - W_C + W_R + W_e$ is the linear density of the modified Lagrange function; q – generalized parameter - electric charge; W_L is the magnetic field energy of inductive elements (kinetic); W_C – energy of electric field of capacitive elements (potential); W_R – energy that is converted to heat in the active elements (kinetic); W_e – energy that is (absorbed) from sources of electric current (kinetic).

The energy functional becomes stationary value in the when his variation equals zero. Simultaneously, the stationary value of the function of the action is obtained by using the principle of Hamilton-Ostrogradsky. This is possible when the modified Lagrange function satisfies the equation (2).

In the steady-state mode of the EPS, when the balanced the generation by energy sources and consumption by electric consumers, the variant part is only the energy of internal and external dissipation, that is dissipated in the form of heat. Zeroing this variation in a complex closed-loop system is possible by redistributing electricity between sources and consumers. To do this, the dissipation energy function must reach a stationary (minimum) value. Since the integrand in (3) is additive, then the stationary value, in this case, becomes functional as a whole. Note that the exchange of energy between inductive and capacitive elements occurs with losses included in the W_R.

Accordingly, the Lagrange function, which includes only dissipation energy, in order to obtain the stationary value of the functional Q, must satisfy the following Euler equation transformed from (2):

$$-\frac{\partial}{\partial t}\left(\frac{\partial W_R}{\partial I}\right) = 0. \tag{4}$$

Equation (4) need take into account only for condition, that we have accordance of currents in EPS to Kirchhoff's first law is necessary. If we don't take into account Kirchhoff's first law and act formally, using expression (4), consequently, we will have the trivial result. Exactly, minimum of energy dissipation (power losses) will be for a condition, that currents in the branches equal zero. So, the problem of minimization power losses need to consider like the variational task of contingent minimum with using indeterminate Lagrange multipliers. Respectively, the system of equations (4) is formatted.

Optimal current distribution in the power system

The problem of determining current distribution in the EPS, that supply a minimum of power losses for its transmission, can be formulated as follows:
minimize

$$\Delta P = \dot{\mathbf{I}}_t \mathbf{R} \hat{\mathbf{I}} \tag{5}$$

provided

$$\left.\begin{aligned} \mathbf{M}\mathbf{I}_a &= \mathbf{J}_a \\ \mathbf{M}\mathbf{I}_r &= \mathbf{J}_r \end{aligned}\right\}, \tag{6}$$

where $\dot{\mathbf{I}}_t$, $\hat{\mathbf{I}}$ – the vectors of currents in the branches are transposed and conjugated; $\mathbf{I}_a$, $\mathbf{I}_r$ – vectors of active and reactive component currents in the branches; $\mathbf{R}$ is the diagonal matrix of active resistances of the branches; $\mathbf{M}$ is the matrix of connections of circuits of nodes in nodes in which the lines corresponding to the generating nodes are crossed out (this is equivalent to combining all power sources into one calculated balancing node); t – hereinafter as the lower index is a symbol of transposition.

The corresponding Lagrange function for problem (5) – (6) is written as follows:

$$L = \dot{\mathbf{I}}_t \mathbf{R} \hat{\mathbf{I}} + \left[\boldsymbol{\lambda}_{at} \boldsymbol{\lambda}_{rt}\right] \begin{bmatrix} \mathbf{M}\mathbf{I}_a - \mathbf{J}_a \\ \mathbf{M}\mathbf{I}_r - \mathbf{J}_r \end{bmatrix}, \tag{7}$$

where $\left[\boldsymbol{\lambda}_{at} \boldsymbol{\lambda}_{rt}\right]$ is the transposed vector of indefinite Lagrange multipliers.

The Lagrange function (7) satisfies the following system of Euler equations [5]:

$$\left[\begin{array}{cc|cc} 2\mathbf{R} & 0 & \mathbf{M}_t & 0 \\ 0 & 2\mathbf{R} & 0 & \mathbf{M}_t \\ \hline \mathbf{M} & 0 & \multicolumn{2}{c}{\multirow{2}{*}{0}} \\ 0 & \mathbf{M} & & \end{array}\right] \begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{r0} \\ \lambda_a \\ \lambda_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{J}_a \\ \mathbf{J}_r \end{bmatrix} \tag{8}$$

From the system of equations (8), the optimal currents in the branches and the Lagrange multipliers are generally defined as:

$$(9)\qquad \begin{pmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{r0} \\ \lambda_a \\ \lambda_r \end{pmatrix} = \begin{pmatrix} \mathbf{C}_r & 0 \\ 0 & \mathbf{C}_r \\ -2\mathbf{R}_{ij} & 0 \\ 0 & -2\mathbf{R}_{ij} \end{pmatrix} \begin{pmatrix} \mathbf{J}_a \\ \mathbf{J}_r \end{pmatrix},$$

where $\mathbf{C}_r = \mathbf{R}^{-1}\mathbf{M}_t\left(\mathbf{M}\,\mathbf{R}^{-1}\mathbf{M}_t\right)^{-1}$ is the matrix of current distribution coefficients of the EPS calculation scheme, in which the branch supports are represented only by their active components (equivalent R-scheme of the EPS); $\mathbf{R}_{ij} = \left(\mathbf{M}\,\mathbf{R}^{-1}\mathbf{M}_t\right)^{-1}$ – nodial resistances matrix of the equivalent R-scheme of the EPS.

Generalized indicator of EPS heterogeneity

In inhomogeneous EPS, steady-state current placement can be represented in its sums by two current vectors:

$$(10)\qquad \dot{\mathbf{I}} = \dot{\mathbf{I}}_e + \dot{\mathbf{I}}',$$

where $\dot{\mathbf{I}}_e$ is the vector of economic (ideal) currents in the branches, found as a result of the calculation of the EPS mode by its equivalent R-scheme; $\dot{\mathbf{I}}' = \mathbf{N}\dot{\mathbf{I}}_{eq}$ – the vector of additional currents in the branches, the imposition of which leads to the implementation of Kirchhoff's second law; N is the second matrix of connections; $\dot{\mathbf{I}}_{eq}$ – the vector of contour equalization currents.

Determined in EPS power losses calculated by using expression (5) with tale into account expression (10), are was rewritten:

$$(11)\qquad \begin{aligned} \Delta P &= \left(\widehat{\mathbf{I}}_{et} + \widehat{\mathbf{I}}'_t\right)\mathbf{R}\left(\dot{\mathbf{I}}_{et} + \dot{\mathbf{I}}'_t\right) = \widehat{\mathbf{I}}_{et}\mathbf{R}\dot{\mathbf{I}}_e + \\ &+ \left[\widehat{\mathbf{I}}'_t\mathbf{R}\dot{\mathbf{I}}_e + \widehat{\mathbf{I}}_{et}\mathbf{R}\dot{\mathbf{I}}' + \widehat{\mathbf{I}}'_t\mathbf{R}\dot{\mathbf{I}}'\right] = \Delta P_e + \Delta P', \end{aligned}$$

where ΔP_e, $\Delta P'$ are respectively the power losses calculated from the R-scheme of the EPS and the additional losses caused by the heterogeneity of the EPS.

The value of currents $\dot{\mathbf{I}}_e$ corresponds to the current distribution and the active power losses in a homogeneous EPS. The task of optimizing of power losses in the EPS is to reduce currents $\dot{\mathbf{I}}'$ and bring it to zero. These currents can be defined as follows:

$$(12)\qquad \dot{\mathbf{I}}' = \dot{\mathbf{I}} - \dot{\mathbf{I}}_e = \mathbf{C}\dot{\mathbf{J}} - \mathbf{C}_r\dot{\mathbf{J}} = (\mathbf{C} - \mathbf{C}_r)\dot{\mathbf{J}},$$

where $\dot{\mathbf{J}}$ is the vector of the defining currents in the units of the EPS; $\mathbf{C} = \mathbf{Z}^{-1}\mathbf{M}_t\mathbf{Y}_z^{-1}$ – current distribution matrix in EPS; $\mathbf{C}_r = \mathbf{R}^{-1}\mathbf{M}_t\mathbf{Y}_r^{-1}$ – current distribution matrix in the equivalent R-scheme of the EPS; $Z = R + jX$ – diagonal matrix resistances of branches of EPS; M_t is the transposed branch-node matrix, is known as the incidence matrix; Y_z, Y_r are nodal conductivity matrices, respectively, for the equivalent Z-scheme and R-scheme of the EPS.

So, the problem of the decreasing power losses in EPS can be formulated as

$$(13)\qquad \dot{\mathbf{I}}' = (\mathbf{C} - \mathbf{C}_r)\dot{\mathbf{J}} \Rightarrow 0.$$

Since the matrix C is complex, and the matrix $\mathbf{C}_r$ is real, then relation (13) holds by take into account condition $\mathbf{C} \Rightarrow \mathbf{C}_r$, namely

$$(14)\qquad \mathbf{C}_{reac} = 0,\ \mathbf{C}_a = \mathbf{C}_r,$$

where $\mathbf{C}_a$, $\mathbf{C}_{reac}$ – active and reactive components of currents distribution matrix $\mathbf{C}$.

Note that the first condition in (14) is necessary and the second condition is sufficient.

Let us write the matrix $\mathbf{C}$ according to its determination and designation through the active and reactive resistances of the branches and conduction of the units of the EPS:

$$\begin{aligned} \mathbf{C} &= (\mathbf{G} - \mathrm{j}\mathbf{B})\mathbf{M}_t(\mathbf{R}_N + \mathrm{j}\mathbf{X}_N) = \\ &= (\mathbf{G}\mathbf{M}_t\mathbf{R}_N + \mathbf{B}\mathbf{M}_t\mathbf{X}_N) + \mathrm{j}(\mathbf{G}\mathbf{M}_t\mathbf{X}_N - \mathbf{B}\mathbf{M}_t\mathbf{R}_N), \end{aligned}$$

where $\mathbf{G}$, $\mathbf{B}$ – active and reactive components of branches conductivity matrix; $\mathbf{R}_N$, $\mathbf{X}_N$ – active and reactive components of nodal resistances matrix.

The latter expression shows that:

$$\mathbf{C}_{reac} = (\mathbf{G}\mathbf{M}_t\mathbf{X}_N - \mathbf{B}\mathbf{M}_t\mathbf{R}_N)$$

or

$$(15)\qquad \mathbf{C}_{reac} = \mathbf{G}(\mathbf{M}_t\mathbf{X}_N\mathbf{R}_N^{-1} - \mathbf{X}\mathbf{R}^{-1}\mathbf{M}_t)\mathbf{R}_N.$$

Denote the expression in parentheses in (15)

$$(16)\qquad \gamma = \mathbf{M}_t\mathbf{X}_\text{в}\mathbf{R}_\text{в}^{-1} - \mathbf{X}\mathbf{R}^{-1}\mathbf{M}_t.$$

The last expression is a matrix of generalized indicators of EPS heterogeneity. From expression (16) it is easy to make sure that for a homogeneous EPS, when for all branches $X_i/R_i = idem$, $\gamma = 0$. So, regardless of the load of EPS $\dot{\mathbf{I}}' = 0$, the additional power losses are absent, caused by unbalanced currents in the EPS. In other cases, when $X_i/R_i \neq idem$, $\gamma \neq 0$ and, accordingly $\dot{\mathbf{I}}' \neq 0$. In these cases, as is known, in order to compensate for the additional losses in the EPS it is necessary to enter in the circuits the equalizing EMF (Electricmotive forces).

Algoritmization of conditions of optimal control

In Fig. 1 is illustrated two approaches to constructing a computational process to determine the optimal system parameters: using gradient methods and using PLA. The essence of gradient methods is that an initial approximation $x^{(0)}[x_1^{(0)}, x_2^{(0)}]$ is chosen in the domain of admissible solutions D and further, by this or that computational scheme, is descended into the region of optimality [14]. PLA uses a different algorithm: is ideal with the lowest possible loss of power in the given conditions the state of the system $x_e\ [x_{1e}, x_{2e}]$, which is usually outside the admissible area D; the mode is then introduced into the admissible range from the point $x_e\ [x_{1e}, x_{2e}]$ to the point $x_o\ [x_{1o}, x_{2o}]$, in which all restrictions are fulfilled, but the losses are greater. Note that, when using gradient methods, the computational process is constructed in the opposite direction – not a lift, but a descent from the level of greater losses (the point of initial approach) to the level with less losses.

In [3, 15] are considered two problems of optimal control of the power system modes. Namely, the first problem is the operative determination of the optimal values of the EPS mode parameters, that provide the minimum criterion of optimality - active power losses. The second problem is the creation of conditions for the self-optimization of EPS modes by means of automation. In practice, EPS modes are constantly perturbed. Consumer loads, power generation, topology, parameters are changing. Accordingly, the optimal control system should be implemented optimizing the effects that should enter the EPS mode in the optimality area. This problem can be formulated as a problem of approximation of the current mode of EPS to economic (ideal):

$$F(\mathbf{x},\mathbf{u}) \Rightarrow F(\mathbf{x}_e,\mathbf{u}_e) \tag{17}$$

For conditions, that

$$\mathbf{x} \in \mathbf{D}_x, \quad \mathbf{u} \in \mathbf{D}_u\,, \tag{18}$$

where $\mathbf{x}$, $\mathbf{x}_e$, $\mathbf{u}$, $\mathbf{u}_e$ – parameters of condition $\mathbf{x}$ and parameters of control $\mathbf{u}$, accordingly current and economic conditions of the system, that determination of the value of optimality criterion F; $\mathbf{D}_x$, $\mathbf{D}_u$ – valid ranges of state parameters $\mathbf{x}$ and control parameters $\mathbf{u}$.

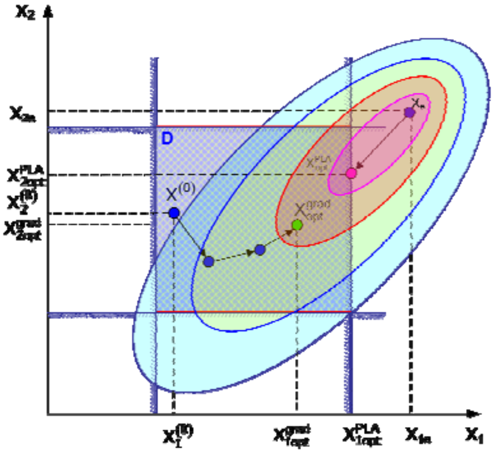

Fig.1. The levels of power losses in the EPS at different values of its conditions parameters

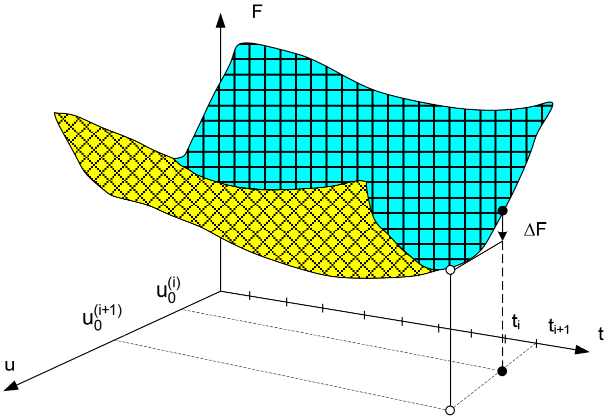

Fig. 2. Changes in the optimality criterion over time and depending on the control parameter

In Fig. 1 shows an example of changing the optimality criterion – power losses in the EPS in time and depending on the parameter of the control unit (CU) u. In the case of a homogeneous system, the trajectory of the optimality criterion F runs along the bottom of the "ravine". According to PLA, this will always be independent of the load. In other cases, when $X_i/R_i \neq idem$, the trajectory F, depending on the specific operating conditions, can pass along the sides of the ravine in any way. However, even in this case, according to the LPA, the power losses to ensure the technological process will be minimally possible. In order to approximate (optimize) power losses at each point of the trajectory of their change to ideally possible, it is necessary to carry out constantly in the system of operation optimizing actions in the system by means of regulation.

It is possible to compensate for the additional power losses in the EPS by regulating the voltage at the nodes of the EPS and introducing into the circuits the equalizing electromotive forces (EMF).

In this formulation, the control variables are the EMFs, which must be entered in all closed circuits for optimal current distribution and load of power sources. Equalizing EMFs can be introduced by changing the transform coefficients of the transformers included in the EPS circuits.

In practice, the load distribution between active and reactive power sources is performed by algorithms, that are based on numerical optimization methods. The disadvantages of these methods with regard to the optimal control of the states of the systems and in particular of the EPS are discussed in [14, 16]. Therefore, in the framework of tasks (17) - (18), the optimization of the EPS states is also possible and appropriate to be performed with the use of PLA.

In Fig. 3 is shown a block diagram of the software complex, which consists of separate blocks: optimization of the state of the EPS in terms of active power; optimization of the state of EPS by reactive power, voltage and transformation factors; complex optimization for active and reactive power and voltage.

The structural and logical scheme of PC optimization of the condition of the EPS is as follows.

1. The problem is selected and the calculation model is formed accordingly. The basic module is the steady-state calculation module, which is carried out according to the following scheme. Economic condition is calculated at given active and reactive power at the nodes (except balancing). The EMFs of unbalance are determined and a system of circuit equations is formed. As a result, solving the formed system of equations we enter equalization EMFs $\dot{E}_{eq}$. After specifying the setting currents at the nodes by the calculated voltages, the EPS mode corresponds to the first and second Kirchhoff laws.

2. If the task is to optimize the mode of the EPS, then the equations for calculating the economic resistances of Rei sources of active and reactive power, which are determined by the method described in [11], are appropriately selected:

$$R_{ei} = \frac{B_i(P_i)\cdot U_i^2 \cdot Pr_i}{P_i^2 \cdot c}\,, \tag{19}$$

$$R_{ei} = \frac{B_i(P_i)\cdot U_i^2 \cdot Pr_i}{Q_i^2 \cdot c}\,,$$

$$R_{ei} = \frac{B_i(P_i)\cdot U_i^2 \cdot Pr_i}{(P_i^2 + Q_i^2)\cdot c}\,,$$

where $B_i(P_i)$ are consumable characteristics of power sources by optimization of PS of active and reactive powers and complex optimization; P_i, Q_i – active and reactive powers of sources; U_i is voltage of nodes; Pr_i – the price of conditional fuel on i a power source; c – the cost of kWh of losses of the electric power.

In these equations, if the power source isnt included in power supply system as subject to managing (it is characteristic of renewable power sources), then instead of consumable characteristics $B_i(P_i)$ take of account the power which is released, and instead of Pr_i these is the holiday cost of kWh of β_i.

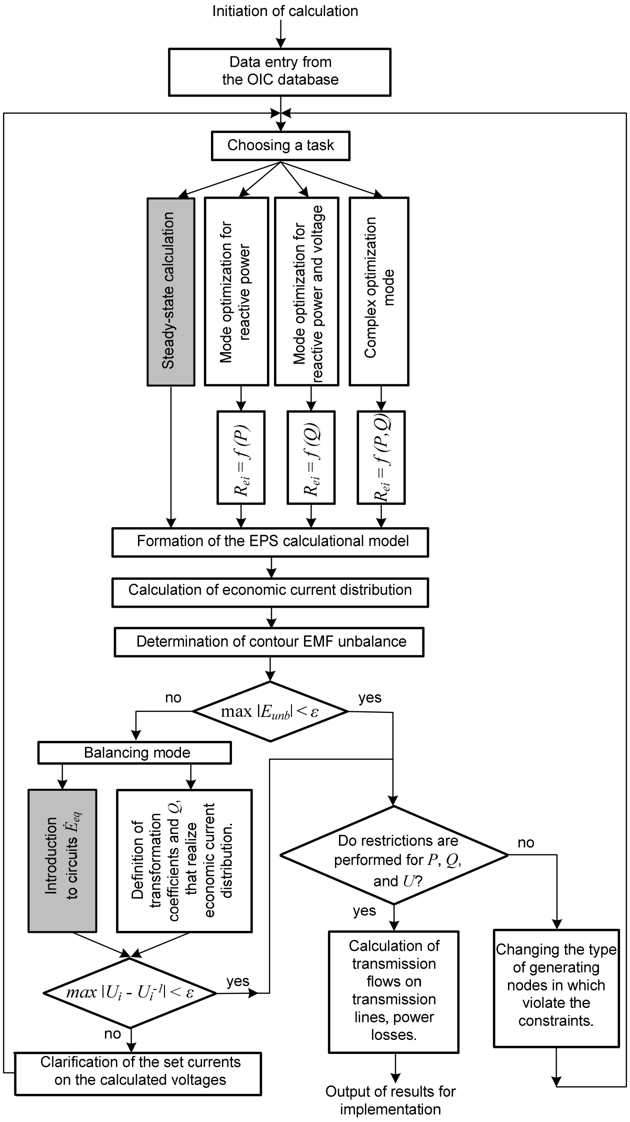

Fig. 3. Structural diagram of the software complex for optimization of the conditions of the EPS

3. The calculation model of the PS is formed and the economic current distribution corresponding to the “ideal” state of the PS is defined. The circuit EMFs of the unbalance are determined, which are converted into transform coefficients of transformers, autotransformers and volt-additional transformers. The introduction of these calculated coefficients of transformation into the PS circuits together with the change in the load of the reactive power sources leads to the balancing of the regime. This optimal state is the closest in the sense of the chosen optimality criterion to the "ideal" state.

4. Due to the fact that the transform and load coefficients of the sources of reactive power (SRP) are discrete quantities, it is possible to break the voltage limits at some nodes. In these cases, the power in the generating units is determined and fixed, leading to constraints. In this case, power losses increase in the PS.

5. The results of the calculation for their realization are displayed. With the "manual" control of the PS state, the results are given as recommendations for operational personnel. If the optimal control state is carried out using the automatic control system (ACS), then the settings of local ACS power sources, transformers and autotransformers.

Organisation of computational process in program complex of optimisation of EPS modes

For simplification of logical connections and increase of efficiency of algorithmic realization of the program complex of the analysis and optimization of EPS states, the logical scheme of its functioning (Fig. 4) is divided into a number of self-sufficient processes: calculation of the normal steady PS state (Fig. 4, block A), verification of conditions optimality of the EPS mode (Fig. 4, block B), modeling of the ideal state of the EPS and correction of independent variables to solve the optimization problem (Fig. 4, block B).

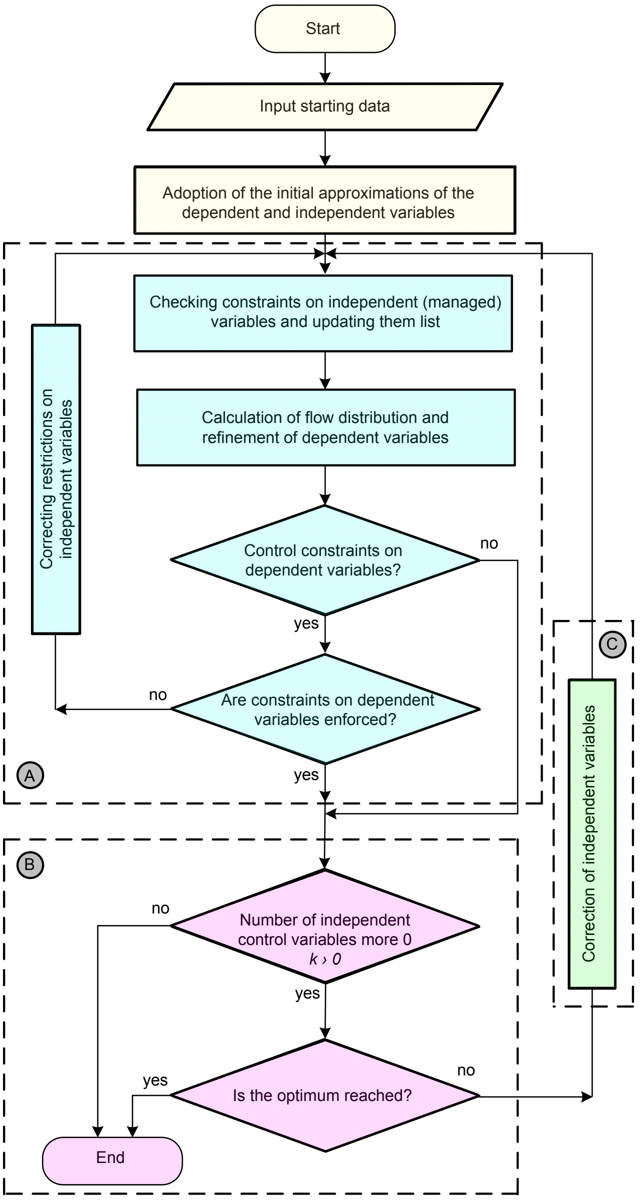

Fig. 4. Generalized the block diagram of the algorithm of analysis and optimization of the EPS normal condition

The current EPS state is represented by a set of passive parameters (resistances and conductance’s of the EPS equivalent schema), as well as dependent and independent variables. Dependent variables include modules U_i and phases δ_i of voltages in the EPS nodes without to the basic and balancing (for the balancing nodes the dependent parameter is δ_i) and the currents loads I_{ij} of the individual elements of the equivalent schema, as well as the derived parameters: power flows P_{ij}, Q_{ij}, power losses, etc. Independent variables include the voltages of the basic U_{bo},

δ_{bo} and balancing nodes U_{bi}, active P_{G1} and reactive Q_{G1} generating powers at the EPS nodes of the equivalent schema, longitudinal k'_{ij} and transverse k''_{ij} transformation coefficients of the branches of the equivalent schema (in the case of longitudinal regulating $k''_{ij} = 0$).

Depending on the formulation of the problem of optimization of the EPS normal state from the list of independent variables optimized variables are selected, the correction of which allows to enter the normal state in the optimality area by a given criterion.

In the process of solving the problem of optimization of the PS normal state, the constraints on the parameters are controlled in two stages:

- before calculating the normal steady state PS mode (Fig. 4, block A), restrictions on independent (including optimized) variables are controlled, the list of which is defined the optimization problem;
- during the calculation of the normal steady state of the PS (Fig. 4, block A), restrictions on dependent parameters are controlled – voltage modules at independent nodes U_i and modules of currents of branches I_{ij} (active power flows P_{ij} are be controlled for transmission lines, that have voltage 330 kV and more). Constraints are doing by adjusting the constraints (narrowing the ranges of acceptable values) to the corresponding independent optimized variables.

The sequence of simulation of normal and normal-optimal states of the PS is as follows (after initiating the program start manually or automatically by the appropriate means by automatic system of dispatcher control (ASDC)).

Before starting the simulation process, the control of the constraints on independent variables is monitored. In case of violation of constraints, depending on the setting, a corresponding warning-message is formed, or the value of the independent variable is fixed at the limit value, after which it loses the feature optimized. After this, independent variable cannot be considered optimized.

The calculation of current distribution in the EPS is performed on the basis of preliminary refinement of the dependent parameters of the normal mode (modules on voltage phases in independent nodes) on the basis of the Newton method. The peculiarity of its application for this problem is the method of forming the Jacobi matrix, which involves the formation of a constant (depends on the passive parameters of the EPS) and a variable by separate procedures (depending on the parameters of the state of the EPS). Independent parameters are determined by the values of the voltages at the nodes, Values of voltages at the nodes are also monitored.

According to the set program parameters in process solving problems of optimization of monitoring dependent parameters it can be carried out with providing, or without ensuring their rated values. In the first case of identification of a deviation of a certain parameter starts iterative process of adjustment of adjacent independent parameters for its introduction to area of permissible values. In other case the corresponding message, and the list of the independent (optimized) variables is formed remains without changes.

The conditions of the optimality of the current mode of EPS are checked according to the set criterion and the problem. The current condition is considered optimal in two cases: if the difference between the optimality criterion on the current and the previous iteration is within a given error (the state is in the optimality range), or if after checking the constraints on the independent optimized variables they have all been set to boundary values and no optimized signs. In the second case, a corresponding message about the insufficiency of the adjustment range for a certain parameter is issued. If the optimality conditions are not reached, the independent parameters of the EPS condition are corrected based on the principle of least action.

The transition from the extreme to optimal independent parameters of the state of the EPS is performed by controlling the constraints on the dependent and independent variables and adjusting the latter accordingly. This is done by the module of analysis of the normal states of the EPS (Fig. 4, block A).

The solution to the optimization problem is terminated if the state of the EPS is entered in the optimality region, or there are no independent optimized variables whose values are not limiting for the specified admissible intervals.

Example

The developed method is illustrated by the example of test 14 node electrical circuit IEEE. The electrical system has five sources of active and reactive power. The balancing node is node 1. In this example, we demonstrate the basic capabilities of the developed method.

According to the developed algorithm for estimation of power losses in the electrical network, the steady state was calculated. The calculations were made using the software package DigSILENT PowerFactory [17]. Active power losses amounted to 13.39 MW. The minimization of total power losses can be achieved as a result of the most favorable load distribution between power plants (PPs) and reactive power sources (RPSs). We start by calculating the economic (ideal) state of the EPS using the R-scheme. Total losses in the power plant amounted to 8.64 MW or 35,5% less than in the initial state of the system. They are achieved by the redistribution of load between the EC and the DRP and between the 132, 33 and 11 kV networks operating in parallel. The PPs and the RPSs in the EPS calculation model are represented by the characteristics of economic resistance (19). In a number of knots, the voltages went beyond acceptable limits. You can enter them into a valid range by changing the reactive power generation and transformation ratios. As this was achieved through the redistribution of power flows in the power grids compared to the economic state, the total losses increased. They amount to 10.31 MW. Thus, we have the optimal state of the EPS, which is the best (from the position of the chosen criterion of optimality – losses of active power) over the initial state by 23.0 %. Compared to the "ideal" mode, the losses increased by 19.3 %. This is the "price" of constraints on the system state parameters.

Conclusions

1. Considering the characteristics of the EPS as a management object, as well as the need to implement process control in the context of incomplete and unreliable current information on the condition of the object and external influences, it is considered necessary to move to partial decentralization of control using adaptive automatic control systems. However, the use of self-regulating control systems, built on the basis of LPA, does not lose the systemic effect of minimizing the total power losses in the electrical networks.

2. The experience that LPA is manifested not only in mechanics but also in electromechanics and electrical engineering, indicates the necessity and expediency of investigating the possibility of using it to construct a method and algorithms for optimal control of modes of inhomogeneous electric grids of power systems when the criterion of optimality is a minimum of power losses during of its transportation.

3. Summarizing what has been said about improving the efficiency of optimal control of the normal states of the EPS and applying for this purpose the principle of least action, it is advisable to use for the following tasks: mathematical modelling of the normal states of the EPS on the basis of PLA to solve the problems of optimal control by the criterion of minimum electricity losses in electric networks during its transportation; development of a method and algorithm for determining the optimal system state parameters, the values of which correspond to the minimum power loss in the EPS; development of a method and algorithm for determining the parameters of the economic (ideal) state of the EPS for the construction of a system of optimal control of the EPS modes.

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