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# The Assessment of the Distortion of Electric Power in Alternating and Pulse Current of a Rectifier Circuit

***Abstract**. The spectra of currents and powers of circuit elements with a single-phase rectifier are determined by the method of harmonic analysis. The presence of two power components oscillating with a frequency of two hundred Hertz and compensating for each other is noted. It is proposed to use an indicator called "the degree of power distortion" which is determined by the ratio of the root-mean-square value of power with the exclusion of its canonical components to the root-mean-square value of power.*

***Streszczenie.**. Analizowano widmo prądu i mocy układu z jednofazowym prostownikiem. <Moc ma dwie składowe o częstotliwości 200 Hz. Zaproponowano wskaźnik zniekształcenia mocy określony przez stosunek rms mocy z wyłączeniem jest podstawowej składowej do rms mocy.* ***Analiza zniekształcenia mocy w obwodzie z prostownikiem***

**Key words:** electric energy power, root-mean-square value of power, quality of electric energy, power distortion.
**Słowa kluczowe:** obwód z prostownikie, zniekształcenie mocy.

## Introduction

Power converting equipment is rapidly spreading in industry, transport and other fields of human technical activity. At present, scientists are solving complex problems of providing the high quality of the input flow of electricity, on condition that the requirements for the output flow of electricity are met. The use of specific connecting circuits of semiconductor elements and the corresponding control system enables reduction in power coefficient [1]. The corresponding indicator is calculated by the relation of active power to total power. The use of high-frequency modulation of control pulses to control the converter and adjust the power coefficient in conditions of voltage asymmetry is insufficient and causes incorrect current generation [2]. Elimination of the defect is ensured by the formation not of current or voltage, but of the instantaneous power curve. A similar control strategy for matrix AC / DC converter is used in [3]. However, in this case, in addition to the formation of a given instantaneous power, the authors divide it into instantaneous active and instantaneous reactive power. The latter is maintained at zero level. The issue of instantaneous power control is also important for DC / AC converters [4] providing AC power of industrial frequency. The proposed solution reduces the pulsation of the instantaneous power on the capacitor storage in a direct voltage circuit.

The above solutions are promising and are currently only being implemented. Thus, distortions of electricity continue to exist and deserve special attention [5].

## The analysis of the recent research

Electric energy is used by researchers as a general indicator that allows one to link the parameters of the mode in the DC circuit of the converter with the parameters of the mode in the AC circuit [6]. Instantaneous electric power reflects the process of electric energy transfer considering it as a signal. In [7] the authors formulate a method for assessing the controllability of electromechanical complexes. In the search for an indicator of the quality of energy processes in the electromechanical system the use of effective power is substantiated in [8]. The index of the capacity of the power channel of an electromechanical complex is proposed.

The connection of additional power losses in power lines with reactive power and pulsations of instantaneous active power is determined in paper [9]. To determine the level of losses the concept of root-mean-square load useful power and root-mean-square reactive power is used. Similar indicators of the energy process were used in [10], where the characteristics of the change in the efficiency of the electric power transmission system were determined.

Thus, the instantaneous power qualitatively reflects the process of electricity transmission, and the results of its analysis can be used to assess this process. In this case, root-mean-square values are used as integral indicators of power components. However, the existence of a single order of the assessment of instantaneous power for alternating and direct current circuits is not revealed.

## The purpose of the paper

Determining the basis for the formation of a single procedure for calculating the distortion of electric power in alternating and direct current circuits.

## The basic material and results of the research

Rectifier is a device that converts alternating current electrical energy into constant-sign pulsating current electrical energy. A structurally simple device provides a change in the nature of the flow of electrical energy. In the controlled mode, the change of the control angle results in higher harmonics in the instantaneous power and significantly complicates the power consumption mode.

Consider the change in time parameters of the mode of the circuit with a rectifier, shown in Fig. 1. We will consider the semiconductor elements of the rectifier as idealized, i.e. the resistances in the open state are zero, and the resistances in the closed state are infinitely large. The resistance and voltage of the source are zero

$$u_{gr} = U_{m.gr}\sin(\omega t) = 220\sqrt{2}\sin(2\pi 50t),$$

where $t$ − time; $\omega$ − angular frequency; $U_{m.gr}$ − voltage amplitude of the power supply.

Lumped resistance $R_{gr} = 1$ Ohm represents the network. The load is resistance of $R_{ld} = 10$ Ohm. Assume that the angle of the rectifier valves control is zero.

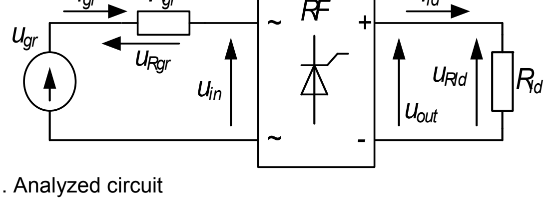


Fig 1. Analyzed circuit

Under such conditions, the following current will flow in the mains circuit of the rectifier:

$$i_{Rgr} = I_m \sin(\omega t) = \frac{U_m}{R_{ld} + R_{gr}} \sin(\omega t),$$

where $I_m$ – current amplitude. The current has one fundamental harmonic (Fig. 2 a).

The rectified current is constant-sign pulsating [11]:

$$i_{Rld} = \frac{4I_m}{\pi}\left(\frac{1}{2} + \frac{\cos(2\omega t)}{2^2-1} - \frac{\cos(4\omega t)}{4^2-1} + \ldots\right) = $$
$$= I_{Rld.0} + \sum_h I_{Rld.h} \cos(h\omega t),$$

where $I_{Rld.0}$ – the constant component of the load current; $I_{Rld.h}$ – the amplitude of the *h*-th harmonic of the load current and its spectrum is more complex (Fig. 2 b)

The acting values of the alternating and pulsating current

$$I_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i_{Rld}^2 dt} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i_{Rgr}^2 dt},$$

are equal in this case and are 20A.

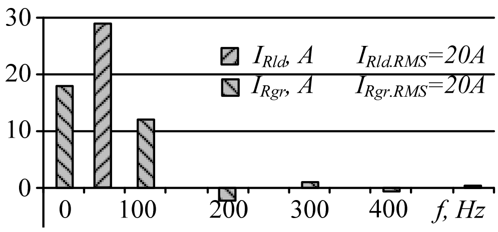


Fig 2. The spectrum of the network current and the load current

For all elements of the circuit, the power change occurs according to the time diagram of Fig. 3 a.The spectrum of this power consists of two components whose frequencies are determined by the sum and difference of current and voltage frequencies (Fig. 3 b):

$$p = P_0 - P_{100}\cos(2\omega t),$$

where $P_0$ – the constant component of power; $P_{100}$ – the amplitude of the alternating component of power.

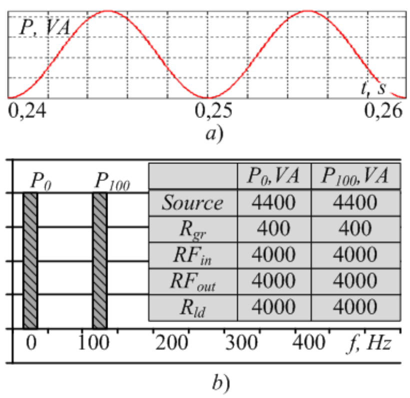


Fig 3. Time diagram (a) and the spectrum (b) of the circuit elements power

We analyze the powers of the active resistances in an alternating ( $R_{gr}$ ) and direct current ( $R_{ld}$ ) circuits:

$$p_{Rgr} = i_{Rgr}^2 R_{gr}; \quad p_{Rld} = i_{Rld}^2 R_{ld}.$$

The power of resistance in the alternating current circuit:

$$p_{Rgr} = \frac{1}{2}\left(\frac{U_m R_{gr}}{R_{ld} + R_{gr}}\right)^2 \left[1 - \cos(2\omega t)\right],$$

which confirms the previously obtained result.

The power of resistance in the direct current circuit [11] will be considered limiting the number of current harmonics to ten (*h* = 2, 4, 6, 8, 10):

$$p_{Rld} = P_{Rld.0} + \sum_s P_{Rld.s}\cos(s\omega t) = P_{Rld.0} + \sum_s P_{Rld.a.s}\cos(s\omega t).$$

where *s* – the number of the power harmonic (0…2*h*); $P_{Rld.s} = P_{Rld.a.s}$ – the amplitude of the power harmonic, cosine, orthogonal component.

As mentioned above, the harmonic composition of the power harmonics of all circuit elements coincides [12]. However, in the general case, if there are 10 harmonics of current, the power of the load resistance will have 20 harmonics without taking into account the constant component. It should be noted that, because of squaring the current, the power will have only even cosine components. Determine analytically the amplitudes of power harmonics:

$$P_{Rld.0} = \frac{R_{ld}}{2}\left[2I_{Rld.0}^2 + \sum_{h=2,4,6,8,10} I_{Rld.h}^2\right];$$

$$P_{Rld.a.2} = R_{ld}\left[2\left(I_{Rld.0}I_{Rld.2}\right) + \sum_{h=4,6,8,10} I_{Rld.h}I_{Rld.(h-2)}\right];$$

$$P_{Rld.a.4} = \frac{R_{ld}}{2}\left[I_{Rld.2}^2 + 4\left(I_{Rld.0}I_{Rld.4}\right) + 2\sum_{h=6,8,10} I_{Rld.h}I_{Rld.(h-4)}\right];$$

$$P_{Rld.a.6} = R_{ld}\left[2I_{Rld.0}I_{Rld.6} + \sum_{h=8,10} I_{Rld.h}I_{Rld.(h-6)} + \sum_{h=4} I_{Rld.h}I_{Rld.(6-h)}\right];$$

$$P_{Rld.a.8} = \frac{R_{ld}}{2}\left[I_{Rld.4}^2 + 4\left(I_{Rld.0}I_{Rld.8}\right) + 2\sum_{h=10} I_{Rld.h}I_{Rld.(h-8)}\right];$$

$$P_{Rld.a.10} = R_{ld}\left[2I_{Rld.0}I_{Rld.10} + \sum_{h=2,4} I_{Rld.h}I_{Rld.(10-h)}\right];$$

$$P_{Rld.a.12} = \frac{R_{ld}}{2}\left[I_{Rld.6}^2 + 2\sum_{h=2,4} I_{Rld.h}I_{Rld.(12-h)}\right];$$

$$P_{Rld.a.14} = R_{ld}\left[\sum_{h=4,6} I_{Rld.h}I_{Rld.(14-h)}\right];$$

$$P_{Rld.a.16} = \frac{R_{ld}}{2}\left[I_{Rld.8}^2 + 2\sum_{h=6} I_{Rld.h}I_{Rld.(16-h)}\right];$$

$$P_{Rld.a.18} = R_{ld}\left[\sum_{h=8} I_{Rld.h}I_{Rld.(18-h)}\right];$$

$$P_{Rld.a.20} = \frac{R_{ld}}{2} I_{Rld.10}^2.$$

For this example, the average value of power, active power:

$$P_{Rld} = \frac{1}{T}\int_{t_0}^{t_0+T} p dt = P_{Rld.0} = \frac{R_{ld}}{2}\left[2I_{Rld.0}^2 + \sum_{h=2,4,6,8,10} I_{Rld.h}^2\right],$$

is 4 kW. Determine the value of each of the harmonics of the load resistance power. In addition, we separately determine the value of the power determined by the square of the corresponding current harmonic $P_{Rld.c.s} = 0.5R_{ld}I_{Rld.2h}^2$, $s = 2h$. These power components are called canonical in [13]. According to [13] the difference $P_{Rld.pc.s.} = P_{Rld.s} - P_{Rld.c.s}$, is a pseudo-canonical component of power harmonics (Fig. 4). The constant component is formed by the canonical power component; the second power harmonic is formed by the pseudo-canonical component. Canonical and pseudo-canonical components in the case of the fourth harmonic are the same and opposite in sign, when added give zero.

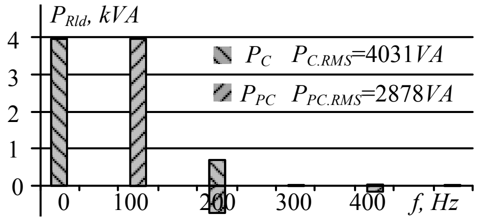


Fig. 4. Representation of the spectrum of power of load resistance using canonical and pseudo-canonical power components

Assume that in the general case, voltage and current are periodic non-sinusoidal functions:

$$u=\sum_k u_k=\sum_k \sqrt{2}U_k\cos\left(k\omega t-\psi_{uk}\right);$$

$$i=\sum_n i_n=\sum_n \sqrt{2}I_n\cos\left(n\omega t-\psi_{in}\right),$$

where, $k,n$ − the number of the harmonic of voltage and current; $U_k, I_n$ − the acting values of the voltage and current harmonic; $\psi_{uk}, \psi_{ik}$ − the initial phase of the voltage and current harmonic. Power function [14] (hereinafter power), in this case:

$$p=ui=\sum_s P_{a.s}\cos\left(s\omega t\right)+\sum_s P_{b.s}\sin\left(s\omega t\right)=\sum_s p_{a.s}+\sum_s p_{b.s},$$

where, $s$ – the number of the power harmonic; $P_{a.s}$, $P_{b.s}$ − the amplitudes of the cosine and sine quadrature components of the power harmonic; $P_s$ − the amplitude of the power harmonic; $\psi_s$ − the initial phase of the power harmonic.

According to the power decomposition proposed in [11]:

$$p=\left(p_{a.0}+p_{b.0}\right)+\sum_{\substack{s=k+n;k=n;\\ s=2h}}\left(p_{a.c.s}+p_{b.c.s}\right)+$$

$$+\sum_{\substack{s=k\pm n;k\neq n;\\ s=2h}}\left(p_{a.pc.s}+p_{b.pc.s}\right)+\sum_{\substack{s=k\pm n;k\neq n;\\ s\neq 2h}}\left(p_{a.nc.s}+p_{b.nc.s}\right)$$

where $p_{a.0}, p_{b.0}$ − the orthogonal components of zero-frequency power; $p_{a.c.s}, p_{b.c.s}$ − the orthogonal components of the power canonical component $s=k\pm n,\ k=n;$ $p_{a.pc.s}, p_{b.pc.s}$ − the orthogonal components of the pseudo-canonical component of power $s=k\pm n,\ k\neq n,$ $s$ − even; $p_{a.nc.s}, p_{b.nc.s}$ − the orthogonal components of the non-canonical component of power $s=k\pm n,\ k\neq n,$ $s$ – odd. We analyze the circuit power (Fig. 1). As shown in [11], canonical components are associated with active and reactive power, and other components reflect the distortion of the instantaneous power of electrical energy.

Assume that the angle of control of the rectifier thyristors $\alpha$ changes from 0° to 120°. We determine the following integral indicators for both the alternating current circuit and the pulsating current circuit:

1. Active power

$$P=P_{a.0}=\frac{1}{T}\int_{t_0}^{t_0+T} ui dt=\sum_{h=k=n} U_h I_h\cos\left(\psi_{uh}-\psi_{ih}\right);$$

2. Reactive power

$$Q=P_{b.0}=\sum_{h=k=n} U_h I_h\sin\left(\psi_{uh}-\psi_{ih}\right);$$

3. Total power

$$S=\sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2 dt}\sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} u^2 dt};$$

4. Power of distortion

$$D=\sqrt{S^2-\left(P^2+Q^2\right)};$$

5. Root-mean-square value (norm) of power

$$P_{RMS}=\sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} u^2 i^2 dt};$$

6. Root-mean-square value of power with exclusion of its canonical components

$$P_{RMS.wc}=\sqrt{\frac{1}{T}\int_{t_0}^{t_0+T}\left(p-\sum_{\substack{s=k+n\\ s=2k=2n}} p_{c.s}\right)^2 dt}$$

7. The degree of distortion of the instantaneous power

$$q_D=\frac{P_{RMS.wc}}{P_{RMS}};$$

8. The degree of active power

$$q_P=\frac{P}{P_{RMS}}.$$

It should be noted that indicators 2-4 are not used for circuits of constant-sign pulsating current. However, there are papers [13], in which the authors research the relationship of reactive power with processes in the direct current circuit. Due to the fact that in the circuit (Fig. 1) the power generated by periodic currents and voltages is analyzed, we will determine the indicators. The results are summarized in Fig. 5 for each element of the circuit (Fig. 1), as a function of the rectifier control angle $\alpha$. Fig. 5 contains the graphs of indicators $q_D$ and $q_P$ for the elements of the circuit in Fig. 1. Due to the absence of reactive elements in the circuit (Fig. 1), the reactive power is available at the input of the rectifier and is balanced by the power supply (dependence *Q* Fig. 5 a, b). As the control angle increases, the level of reactive power *Q* and distortion power *D* of the power supply and the rectifier changes (Fig. 5 a, b), which results in the increase of the difference between active *P* and total *S* powers. The said difference is practically unnoticeable in case of the analysis of active *P* and total *S* powers of the circuit active resistances (Fig. 5 c, d). This is caused by the natural lack of reactive power *Q* and the insignificant level of distortion power *D*. One should note the same nature of the change in the power of distortion *D* (Fig. 5 c, d). The distortion power at zero control angle is zero in all cases $D=0$ . However, this contradicts the arguments given above, in particular the diagram of Figs. 3, on which the current has higher harmonics, which obviously cause distortion.

Dependences of relative indicators of power on a rectifier control angle (Fig. 5 e, f) show coincidence of the degree of active power $q_P$ at the whole interval of change of the angle of control of the rectifier (Fig. 5 e) for all elements of the circuit. The degree of distortion of the instantaneous power $q_D$ for the circuit elements changes in a different way (Fig. 5 f). The dependence of the degree of distortion of the instantaneous power of the load resistance $q_{D.Rld}$ differs significantly, which is caused by the difference in the current resistance spectrum of the load $R_{ld}$. The curves of the degrees of the power supply instantaneous power distortion

$q_{D.Ugr}$ and the rectifier input circuit power $q_{D.RF}$ coincide. This confirms the previous statements regarding the group of graphs presented in Figure 5 a and b. The dependence of the degree of distortion of the network resistance instantaneous power $q_{D.Rgr}$ occupies a certain intermediate place.

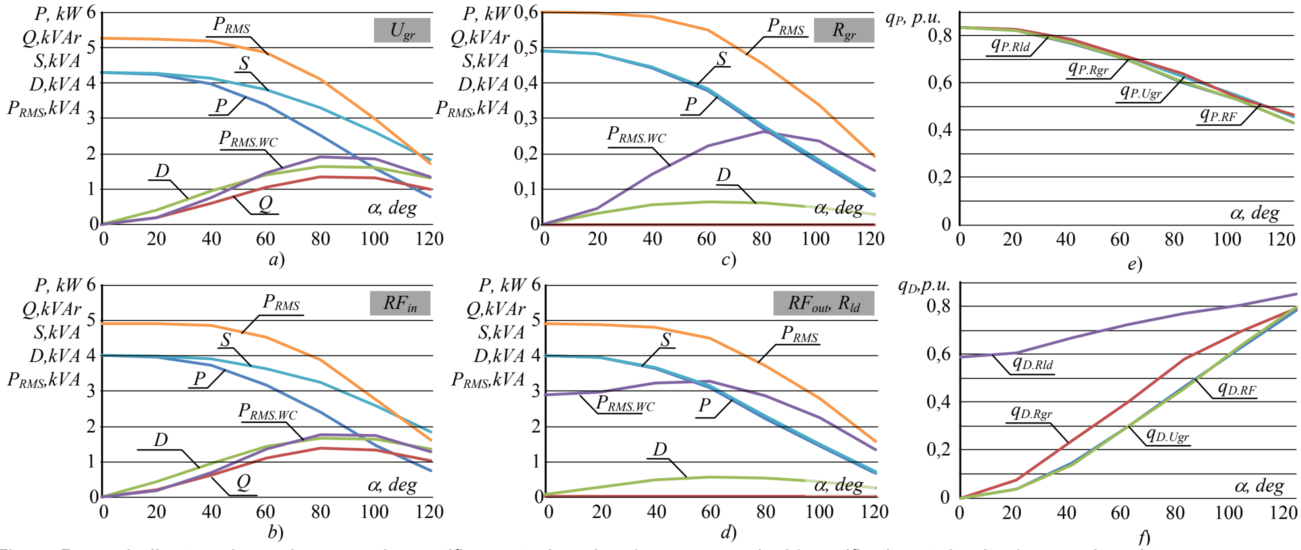


Fig. 5. Power indicators dependence on the rectifier control angle: a) power supply; b) rectifier input circuit; c) network resistance; d) rectifier output circuit, load resistance; e) active power degree; f) the degree of instantaneous power distortion

**Conclusions and lines of further research**

1. It has been proved that the spectral composition of electric power in a circuit with a single-phase uncontrolled rectifier in all elements of the circuit is the same. Taking into account that the circuit has only active elements, the constant component of power and the amplitude of the pulsating component are the same. It is noted that the spectral composition of the current at the input and output of the rectifier differs.

2. As a result of the analysis of the components that form the power harmonics in the pulsating current circuit, the presence of two power components oscillating with a frequency of 200 Hz has been noted. These components oscillate in antiphase and compensate for each other; as a result, a power harmonic of a given frequency has zero amplitude.

3. It has been proposed to use an indicator called "the degree of power distortion" which is determined by the relation of the root-mean-square value of power with the exclusion of its canonical components to the root-mean-square value of power.

4. As a result of a series of experiments, the dependences of the degree of power distortion on the control angle of the rectifier have been obtained. It has been determined that the dependence of this indicator for the active resistance in the pulsating current circuit differs from similar dependences for other elements of the experimental circuit.

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