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The features of the determination and use of instantaneous power components

Abstract *The problem of determining the instantaneous power when it is used in the process of diagnosing nonlinear objects of electrical systems is formulated. Generalized calculation relations for instantaneous power components are obtained and presented with different methods for their determination. The results of their comparative analysis made it possible further determine the conditions of applicability of the obtained relations from the standpoint of measurement accuracy, minimizing the volume of calculations, as well as increasing the visibility and information content of the results.*

Streszczenie. *Analizowano problem określania mocy chwilowej przy diagnostyce obiektów nieliniowych. Przedstawiono różne metody określania składowych mocy oraz analizowano dokładność.* ***Analiza metod określania chwilowych składowych mocy***

Key words: electrotechnical system, measurement, diagnostics, instantaneous power, nonlinear load.
Słowa kluczowe: moc chwilowa, składowe mocy, nieliniowość,

Introduction

Providing the required level of accuracy or reliability is one of the main tasks that are posed in the development of industrial complexes for measuring and diagnosing electrical systems. In its turn, it determines the requirements for the quantitative and qualitative composition of independent diagnostic parameters, on which the definition of diagnostics criteria is based. In this case, it is understood that independent parameters are such parameters that cannot be obtained by a simple linear combination of other parameters.

The posed problem is somewhat different for conditionally linear and nonlinear objects. In the former case, the symbolic method is mainly used for calculations, using the integral parameters of electrical signals. In the latter case, this approach gives results that are valid only for each individual value of the load from the object. They cannot be generalized for the arbitrary nature of the alternating nonlinear load.

The progress of computer and microprocessor measurement technology has led to the use of such diagnostic problems, along with the integral parameters of electrical signals, their time dependences. This necessitated the development of methods for mathematical representation and approaches to the interpretation of the physical meaning of their individual components [1].

Today, there are a large number of different ways of finding and representing forms of components of instantaneous electrical parameters, primarily instantaneous power [2, 3]. However, their comparative analysis and generalization in the context of the task were not carried out.

The purpose of this paper consists in obtaining generalized expressions for the components of the instantaneous power of non-sinusoidal signals symmetrical with respect to the time axis, which can include almost all electrical signals in diagnostic systems with alternating current. An additional task is to carry out their comparative analysis in terms of measurement accuracy, minimizing the amount of calculation, increasing the clearness and information content of the results.

Theoretical provisions

Let us consider the task of forming a set of independent diagnostic parameters by the example of electrical parameters characterizing a conditional diagnosed object of the "black box" type, which is a passive two-terminal device, as shown in Fig. 1.

If this object is linear and the power is sinusoidal, at informative signals of supply voltage $u_1(t)$ (task) and load current $i_1(t)$ (response) the number of independent integral diagnostic parameters is limited to three. Fig. 2 explains this.

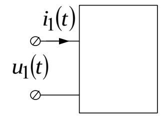

Fig. 1. The diagnosed object of the "black box" type

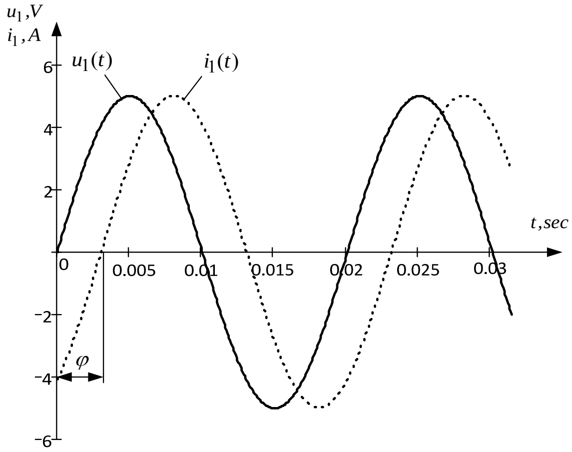

Fig. 2. The special features of the diagnostics of linear objects at sinusoidal power supply

Really, having effective values of voltage U_1, current I_1 and the value of phase shift φ between them, it is rather easy to obtain all the other integral parameters characterizing this object: average values – $U_{1av} = U_1 / 1{,}11$, $I_{1av} = I_1 / 1{,}11$; amplitude values – $U_{1max} = U_1\sqrt{2}$, $I_{1max} = I_1\sqrt{2}$; active, reactive and total power – $P_1 = U_1 I_1 \cos\varphi$, $Q_1 = U_1 I_1 \sin\varphi$, $S_1 = U_1 I_1$, circuit resistance – $R_1 = Re\left(\dfrac{U_1 e^{j\cdot\varphi_{u1}}}{I_1 e^{j\cdot\varphi_{i1}}}\right)$,

$$X_1 = Im\left(\frac{U_1 e^{j\cdot\varphi_{u1}}}{I_1 e^{j\cdot\varphi_{i1}}}\right),\ Z_1 = \sqrt{R_1{}^2 + X_1{}^2}\ .$$

For the case when the diagnostic object is non-linear and the supply voltage is not sinusoidal in shape, which corresponds to the actual operating modes of almost all industrial electrical systems, the number of independent diagnostic parameters increases significantly.

Therefore, for this case (Fig. 3) particular numerical relations with the corresponding average and amplitude values do not unambiguously determined the effective values of voltage U_1 and current I_1, and the value of phase shift angle φ does not have clear physical interpretation in fact.

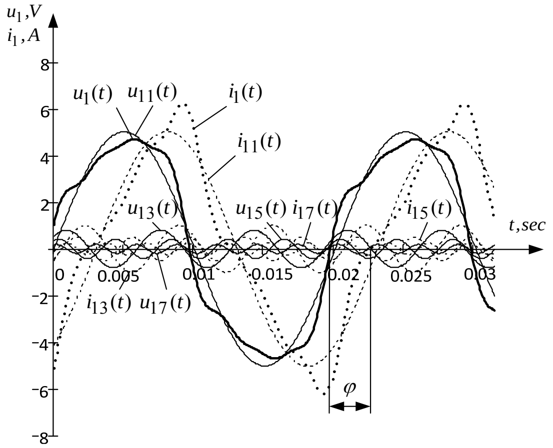

Fig. 3. The special features of the diagnostics of nonlinear objects at non-sinusoidal power supply

Instead, a number of new parameters are determined for the harmonic components that are marked in the initial signal: amplitude and initial phase obtained as a result of the discrete or fast Fourier transform – $A_\nu = \left|a_\nu + jb_\nu\right|$, $\varphi_\nu = arg\left(a_\nu + jb_\nu\right)$, where a_ν, b_ν – quadrature components of voltage and current signals; active, reactive and total powers – $P = \sum\limits_\nu P_{q\nu}$; $Q = \sum\limits_\nu Q_{q\nu}$; $S = \sum\limits_\nu \sqrt{P_{q\nu}^2 + Q_{q\nu}^2}$, where $P_{q\nu}, Q_{q\nu}$ – harmonic powers calculated from quadrature components:

(1) $$P_{q\nu} = \frac{a_{i1\nu}a_{u1\nu} + b_{i1\nu}b_{u1\nu}}{4}\ ;$$

(2) $$Q_{q\nu} = \frac{a_{i1\nu}b_{u1\nu} - a_{u1\nu}b_{i1\nu}}{4}\ ,$$

where $a_{u1\nu}, b_{u1\nu}, a_{i1\nu}, b_{i1\nu}$ – voltage and current quadrature components obtained based on the known relations; resistance and reactance for the harmonics in the electric circuit – $R_\nu = Re\left(\frac{U_{1\nu} e^{j\cdot\varphi_{u1\nu}}}{I_{1\nu} e^{j\cdot\varphi_{i1\nu}}}\right)$, $X_\nu = Im\left(\frac{U_{1\nu} e^{j\cdot\varphi_{u1\nu}}}{I_{1\nu} e^{j\cdot\varphi_{i1\nu}}}\right)$; the angles of the phase shifts between the harmonics of curves $u_1(t)$ and $i_1(t)$ – $\varphi_{u1i1\nu} = arccos\left(\frac{P_{q\nu}}{I_{1\nu}U_{1\nu}}\right)$; the coefficients of nonlinear distortions – $THD = \sqrt{\sum\limits_{\nu=2}^{k} A_\nu^2} \Big/ A_1$, where k – the number of harmonics.

That is, we can distinguish three independent components for each of the harmonics and, in addition, have a number of integral parameters that characterize the signal as a whole. The latter, most often, are devoid of physical meaning, but, under certain conditions, are sensitive to diagnosed damages.

Unfortunately, in practice, the harmonic components of the current and voltage of the order exceeding the fifth cannot be reliable diagnostic parameters due to the high measurement error, which for them usually exceeds the zone of sensitivity to defects. Therefore, the topical problem is to obtain an additional number of independent diagnostic parameters.

In this direction, the authors developed refined algorithms for determining each of the components of instantaneous power, greatly simplifying the task of its calculation, which makes it possible to talk about the value and relative composition in the formation of the corresponding coefficients of the Fourier series. In addition, this increases the reliability of the diagnostic process, since each of the instantaneous power components carries information about real physical phenomena and processes that occur in the researched electrical system [4].

In the course of research, it was found that, despite the variety of existing approaches, only two main methods for determining the components of instantaneous power can be distinguished. The first one consists in the expansion of the instantaneous power curve directly in the Fourier series; the second involves the determination of its components by multiplying two harmonic series obtained from the expansion of the time dependences of voltage and current. Each of these methods has its own forms of presentation and the algorithms determined by them [1].

The fundamental difference between the proposed methods and conventional solutions consists in taking into account the single-frequency components of the instantaneous power of the power harmonics from the combination frequencies. It is proved in [2] that the energy balance is observed for instantaneous powers only with this approach.

Let us consider the solution to the posed problems taking into account the above said.

1. Expansion using a discrete or fast Fourier transform into components of the instantaneous power curve directly. In this case, the following is obtained:

$$P_0 = \frac{1}{N}\sum_n P_n\ ;\ p_{a\nu} = \frac{2}{N}\sum_n P_n\, sin\left(2\nu\omega t_n\right);$$

$$p_{b\nu} = \frac{2}{N}\sum_n P_n\, cos\left(2\nu\omega t_n\right);$$

$$P_{max\nu} = \sqrt{p_{a\nu}{}^2 + p_{b\nu}{}^2}\ ;\ \varphi_\nu = arctg\left(\frac{p_{b\nu}}{p_{a\nu}}\right).$$

In the given expressions N – the number of reference points for the measured curve for the period of the fundamental frequency of the signal; P_n – the values of the instantaneous power at the discretization points; ν – the harmonic number; ω – the angular frequency of the first harmonic; $p(t), P_0, p_{a\nu}, p_{b\nu}$ – respectively instantaneous power and its quadrature constant, sine and cosine components.

The result can be presented both in the form of a harmonic series via the amplitudes and phases of components

$$p(t) = P_0 + \sum_{\nu} P_{max\nu}\, sin(\nu\omega t + \varphi_{\nu}),$$

and via the instantaneous power quadrature components

$$p(t) = P_0 + \sum_{\nu} p_{a\nu}\, sin(\nu\omega t) + \sum_{\nu} p_{b\nu}\, cos(\nu\omega t).$$

This method is the simplest and most efficient in use. However, for its use it is necessary to obtain simultaneous readings for voltage and current signals, which is often not feasible in modern computer measuring systems due to the presence of one analog-to-digital converter (ADC). It is also not always possible accurately take into account the time shift between non-sinusoidal signals of complex shape. In addition, the error of recovery of the considered high-frequency components significantly increases due to their twice as high frequency as compared with the initial current and voltage signals. Another big drawback of this method is the lack of information on the composition of each of the quadratures and the integral power as a whole.

2. The multiplication of the series of voltage and current, presented as the sum of the quadratures of the corresponding signals. This method is described in detail in [4].

Its drawbacks include, first, a large amount of calculations and the inability fully identify the obtained sine and cosine harmonics of power.

3. Multiplication of voltage and current series, presented in the form of total harmonic components:

$$u(t) = \sum_{\nu=1}^{(N-1)/2} U_{m\nu}\, sin(\nu\omega t + \varphi_{u\nu}); \tag{3}$$

$$i(t) = \sum_{\nu=1}^{(N-1)/2} I_{m\nu}\, sin(\nu k t + \varphi_{i\nu}), \tag{4}$$

where the amplitudes and phases of the harmonics are determined by relations:

$$X_{m\nu} = \sqrt{a_{x\nu}{}^2 + b_{x\nu}{}^2}\ ; \tag{5}$$

$$\varphi_{x\nu} = arctg\left(\frac{b_{x\nu}}{a_{x\nu}}\right). \tag{6}$$

In this case, the calculation expressions will be of the form:

– constant component

$$p(t)\big|_{const} = \sum_{\substack{j,k=1\\ j=k}}^{\infty} U_j I_k\, cos(\varphi_{Uj} - \varphi_{Ik}) = \sum_{j=1}^{\infty} P_j\ ;$$

– sine components:

of single-frequency harmonics

$$p(t)\big|_{sin} = -\sum_{\substack{j,k=1\\ j=k}}^{\infty} U_j I_k\, sin(\varphi_{Uj} + \varphi_{Ik})\, sin\, 2j\omega t =$$

$$= -\sum_{j=1}^{\infty} Q'_j\, sin\, 2j\omega t;$$

of combined-frequency harmonics

$$p\big|_{sin} = -\sum_{\substack{j,k=1\\ j+k=m\\ j\neq k}}^{\infty} U_j I_k\, sin(\varphi_{Uj} + \varphi_{Ik})\, sin\, m\omega t -$$

$$-\sum_{\substack{j,k=1\\ j-k=n\\ j\neq k}}^{\infty} U_j I_k\, sin(\varphi_{Uj} - \varphi_{Ik})\, sin\, n\omega t =$$

$$= -\sum_{\substack{j,k=1\\ j+k=m\\ j\neq k}}^{\infty} Q'_{jk}\, sin\, 2m\omega t - \sum_{\substack{j,k=1\\ j-k=n\\ j\neq k}}^{\infty} Q_{jk}\, sin\, 2n\omega t;$$

– cosine components:

of single-frequency harmonics

$$p(t)\big|_{cos} = \sum_{\substack{j,k=1\\ j=k}}^{\infty} U_j I_k\, cos(\varphi_{Uj} + \varphi_{Ik})\, cos\, 2j\omega t =$$

$$= \sum_{j=1}^{\infty} P'_j\, cos\, 2j\omega t;$$

of combined-frequency harmonics

$$p(t)\big|_{cos} = \sum_{\substack{j,k=1\\ j+k=m\\ j\neq k}}^{\infty} U_j I_k\, cos(\varphi_{Uj} + \varphi_{Ik})\, cos\, m\omega t +$$

$$+ \sum_{\substack{j,k=1\\ j-k=n\\ j\neq k}}^{\infty} U_j I_k\, cos(\varphi_{Uj} - \varphi_{Ik})\, cos\, n\omega t =$$

$$= \sum_{\substack{j,k=1\\ j+k=m\\ j\neq k}}^{\infty} P'_{jk}\, cos\, 2m\omega t + \sum_{\substack{j,k=1\\ j-k=n\\ j\neq k}}^{\infty} P_{jk}\, cos\, 2n\omega t.$$

When using the presented method, the pattern of the formation of the sine and cosine components of instantaneous power becomes more clear and understandable. However, the accuracy of determining the components of instantaneous power due to the need to use additional indirect measurement procedures is slightly lower compared with the previous method.

The obtained expressions can be most effectively used in advanced modern methods of accounting for energy consumption and compensation for low-quality power supply in networks with a high coefficient of non-linear distortion.

Despite the high information content, the considered methods do not allow controlling the mechanism of instantaneous power signal formation, isolating the most significant parts of its components, analyzing the nature of the processes separately at each harmonic, etc. To increase the clearness of the presentation of the obtained results in the course of research, the use of matrix forms of representation of both the power signal and the current and voltage signals forming it is substantiated.

In matrix form for the second and third methods, the considered relations will be of the form of a tensor product:

$$P = U \otimes I^T,$$

where U, I^T, P – respectively voltage matrix column, current transposed matrix column (row matrix) and power matrix.

The difference between the methods is that each of the elements of the voltage and current matrices is represented either by the sum of the quadrature of the signals [4], or by the total components in accordance with (3) – (4).

The final form of the matrices, respectively, for the active and reactive components in the first case will be as follows

$$P = \frac{1}{2}\left(\begin{vmatrix} b_{i1} & \dots & b_{i\nu} \end{vmatrix}^T \cdot \begin{vmatrix} b_{u1} & \dots & b_{u\nu} \end{vmatrix} + \begin{vmatrix} a_{i1} \\ \dots \\ a_{i\nu} \end{vmatrix} \cdot \begin{vmatrix} a_{u1} \\ \dots \\ a_{u\nu} \end{vmatrix}^T \right) =$$

$$= \begin{vmatrix} P_{11} & \dots & P_{1\nu} \\ \dots & \dots & \dots \\ P_{\nu 1} & \dots & P_{\nu\nu} \end{vmatrix};$$

$$Q = \frac{1}{2}\left(\begin{vmatrix} a_{u1} \\ \dots \\ a_{u\nu} \end{vmatrix} \cdot \begin{vmatrix} b_{i1} & \dots & b_{i\nu} \end{vmatrix} - \begin{vmatrix} a_{i1} \\ \dots \\ a_{i\nu} \end{vmatrix} \cdot \begin{vmatrix} b_{u1} & \dots & b_{u\nu} \end{vmatrix} \right) =$$

$$= \begin{vmatrix} Q_{11} & \dots & Q_{1\nu} \\ \dots & \dots & \dots \\ Q_{\nu 1} & \dots & Q_{\nu\nu} \end{vmatrix},$$

and in the second case – analogous to it, only through full forms of recording the components of voltage and current.

At the same time, on the main diagonal of the power matrix there will be the results of the product of single-frequency components, and other possible products will be located in their places in accordance with indices i and j, where i – the column number corresponding to the voltage harmonic number, j – the row number corresponding to the current harmonic number.

It is easy to obtain the required values $p(t)|_{const}, p(t)|_{cos}, p(t)|_{sin}$ from the above relations.

The matrix form of presentation of the results can be effectively used in monitoring and diagnostics systems. The relationship of the final results with the intermediate calculation data, which are the products of single-frequency and combination components, is obvious. It allows quite simple substantiation of the changes in the shape of the instantaneous power curve and the diagnostic criteria on this basis.

Experimental research

When comparing the obtained relations, three factors were evaluated: information content, the amount of calculation and the accuracy of the results.

An accuracy assessment was performed for the second and third methods, because the low accuracy of the first one was confirmed in the paper [5].

As the amplitudes and phases of harmonics are determined in accordance with relations (5) – (6), additional procedures when switching from quadrature components to the amplitude and phase of harmonics are summing, squaring, multiplying, extracting the square root and finding the arc tangent, etc., relative errors for which were calculated using classical metrological ratios [6].

Taking into account approximately half the amount of calculations when multiplying the series of current and voltage, made up of quadrature, as compared with multiplying the series of total components, the measurement error in the second case was higher due to additional components of the error of indirect measurements.

In the calculations, was taking into account the real range of variation of the measured parameters, due to acceptable level of the input voltages of the standard input / output modules of computerized measuring systems.

Because of the calculations, it was found that the measurement error of the instantaneous power components grows from 0.5 to 3.9%, where a lower value corresponds to the harmonic of the fundamental frequency, and a larger value corresponds to the high-frequency components.

Conclusions

1. A number of generalized expressions for the instantaneous power components are obtained and presented, taking into account the products of harmonics of the combination frequencies and their presentation forms. The obtained expressions can be used in theoretical calculations of energy processes in nonlinear circuits with non-sinusoidal effects. They can be practically used in promising modern methods of accounting for energy consumption and compensation for low-quality power supply in networks with a high coefficient of non-linear distortion and diagnostic systems for electrical and electromechanical objects.

2. For the proposed options for determining the components of instantaneous power, the conditions for their most effective use are formed. With strong requirements for the measurement error of the instantaneous power components, the expressions obtained because of multiplying the quadrature of the original signals should be used. It is expedient to use expressions for the total components of the voltage and current harmonics if there are restrictions on the volume and time of calculations, when a slight decrease in the accuracy of measurements is permissible. It is advisable to use matrix forms of presenting the results when researching the mechanism of formation of instantaneous power components.

3. Taking into consideration the increase in the crockitude of the obtained generalized expressions with an increase in the number of harmonic components taken into account, the use of matrix forms of their representation is substantiated and confirmed.

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