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Output voltage control in the Class E ZVS inverter by frequency or reactance regulation

Abstract. *The paper presents results of an analysis of the Class E ZVS inverter in which output voltage is controlled by regulation of its operating frequency or by variations of the reactance of its series resonant circuit at the nominal frequency. Regulation characteristics of the inverter output voltage and output power versus operating frequency or reactance variation have been found. The boundaries of ZVS operation of the inverter have also been identified. Theoretical results have been validated by simulation and experimental results.*

Streszczenie. *W artykule prezentowane są wyniki analizy falownika klasy E ZVS, w którym napięcie wyjściowe regulowane jest poprzez zmianę częstotliwości pracy lub zmianę wartości reaktancji szeregowego obwodu rezonansowego przy stałej częstotliwości pracy. Dla analizowanego układu wyznaczono charakterystyki regulacji napięcia wyjściowego, mocy wyjściowej oraz warunki pracy ZVS. Otrzymany opis teoretyczny parametrów falownika zweryfikowano poprzez symulację i pomiary układu eksperymentalnego.* ***Falownik klasy E ZVS, w którym napięcie wyjściowe regulowane jest poprzez zmianę częstotliwości pracy***

Słowa kluczowe: falownik klasy E, regulacja mocy, kluczowanie ZVS.
Keywords: Class E inverter, output power regulation, ZVS switching.

Introduction

Miniaturization and the need for energy efficient operation of electronic equipment have made Class E resonant inverters attractive for applications in power electronics. High-efficiency conversion of dc energy to h.f. current or voltage in the inverters is ensured by resonant shaping of the voltage and current in the transistor switch to achieve its ZVS (zero-voltage switching) operation. This reduces both switching losses in the circuit and decreases the level of emitted electromagnetic interferences. In many practical applications e.g. induction heaters, WPT (wireless power transfer) systems or dc/dc converters frequency regulation is used to control output voltage or output power in the Class E inverter while its load impedance is varied. The operating frequency and load impedance changes can each cause NZVS (non-zero-voltage switching) operation of the switch resulting in increased power losses and reduced efficiency. To verify ZVS operation of the switch and maintain low losses in the inverter it is necessary to analyze operation of the Class E circuit in the whole span of possible loads and the whole operating frequency range. It is also important to identify regulation characteristics of the inverter to determine whether the range of its output voltage or output power control in ZVS conditions is appropriate for the predicted use. Such analyses are crucial for reliable operation and high performance of a system or equipment where the Class E inverter is applied to.

Numerous papers on the Class E ZVS inverter concentrate their analyses on optimizing the circuit operation in nominal or off-nominal (sub-optimum) conditions with a constant operating frequency and/or constant load impedance [1-13]. Only some publications theoretically analyze the operation of Class E inverter with its frequency and load impedance varied [14-24]. Such analyses are indispensable in designing Class E inverters for practical applications.

Using equations and parameters introduced in [24] the paper presents an expanded analysis and new results for the basic Class E ZVS SRC (series resonant circuit) inverter with normalized switch on-time D_T=0.5 and a sinusoidal output current. Analyzing the inverter designed for nominal operation (ZVS and ZDS- zero-derivative switching) its operating frequency and the impedance of its series resonant circuit have been modified in some range to obtain its off-nominal operation (ZVS only) and to identify boundaries of ZVS operation. The analysis have also been used to find design-essential parameters of the inverter such as: maximum normalized values of voltage and current waveforms of the switch, regulation characteristics of output voltage and output power vs. series reactance or operating frequency variations etc. Also a simple design procedure for an off-nominal Class E inverter has been proposed. Obtained theoretical equations describing the inverter operation have been validated by simulation and measurements of the laboratory model of the inverter. The experimental circuit operated in the frequency range 137-151 kHz with dc supply voltage 24 V achieving efficiency 96% for nominal operation at the output power 50 W.

Analysis of the inverter

The basic Class E ZVS SRC inverter depicted in Fig. 1a is analyzed for the given assumptions:

1) Bi-directional switch S comprises transistor T and anti-parallel diode D_S. The switch current is $i_S=i_T+i_{DS}$. The components are ideal and have zero switching times,

2) voltage v_{GS} driving the transistor T is a square wave with frequency $f=1/T$ and duty cycle D_T=0.5. The normalized conduction time of the switch is $D=D_T+D_D$, where D_D - normalized conduction time of diode D_S, $\varphi_S= 2\pi \cdot D_D$,

3) reactive components such as capacitors C_1, C_{SR}, inductor L_{SR} and choke L_{CH} are linear and lossless. The value of inductance L_{CH} is high enough to neglect ac component in its current.

4) Output current $i_O(\omega t)=I_O \sin(\omega t+\varphi)$ of the inverter loaded with R_O is a sine wave.

The nominal operation of the Class E inverter with D_T=0.5 occurs for strictly defined parameters of the inverter, and it is well described in publications [1]. To simplify the presented analysis of the inverter in off-nominal conditions its parameters have been expressed by means of parameters of the nominal reference inverter. It it assumed in the analysis that the nominal circuit operates at the frequency f_{nom} with D_T=0.5, output power P_{nom}, load resistance R_{nom}, loaded quality factor Q_{SRnom} and dc supply voltage E_{SUP}. The off-nominal circuit is obtained from the nominal inverter by modifying its operating frequency f or load impedance at the nominal frequency f_{nom}. Hence, for the off-nominal circuit its dc supply voltage is also E_{SUP}, but load resistance is $R_O \neq R_{nom}$ and/or series reactance of L_{SR}-C_{SR} is $X_{SR} \neq X_{SRnom}$.

Reactance of capacitor C_1 at operating frequency f is given by [1, 24]

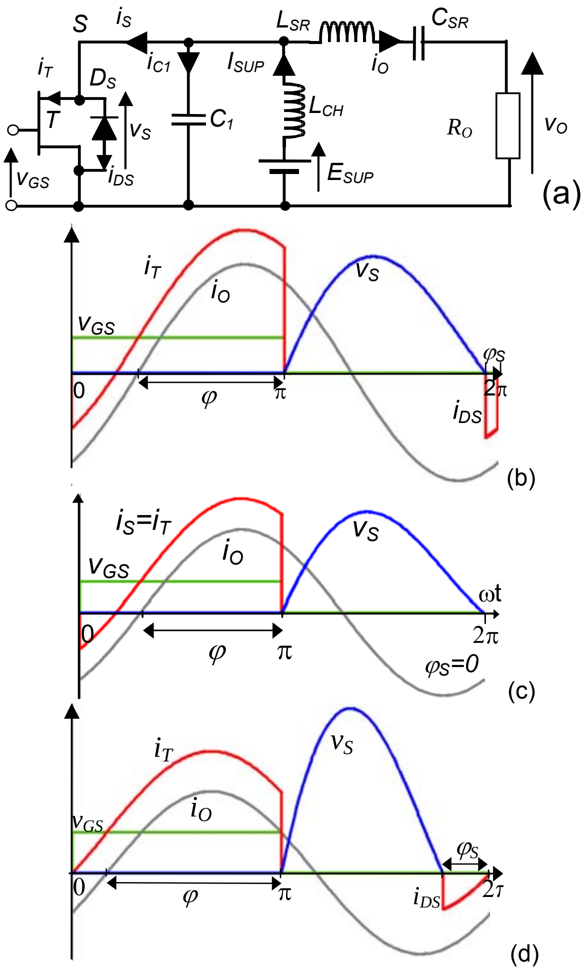

Fig. 1 Simplified diagram of the Class E ZVS inverter (a) and waveforms for its off-nominal operation at D_T=0.5 for: b) $\varphi_S \neq 0$ and $i_S(\omega t=0)<0$; c) $\varphi_S=0$ and $i_S(\omega t=0)<0$; d) $\varphi_S \neq 0$ and $i_S(\omega t=0)=0$.

$$X_{C1}=\frac{\pi\left(\pi^2+4\right)}{8A}R_{nom} \tag{1}$$

where $A=f/f_{nom}$ - a normalized frequency, R_{nom}- nominal load resistance.
Normalized output power p_O and load resistance r_O for off-nominal operation of the inverter at load R_O is expressed as [24]

$$p_O=\frac{P_O}{P_{nom}} \tag{2}$$

$$r_O=\frac{R_O}{R_{nom}} \tag{3}$$

where P_{nom} -nominal output power at R_{nom} and E_{SUP}.
For the nominal inverter with D_T=0.5 the product of its output power P_{nom} and load resistance R_{nom} is given by [1]

$$P_{nom}R_{nom}=\frac{8E_{SUP}^2}{\pi^2+4} \tag{4}$$

Normalized reactance x_{SR} of the series branch L_{SR}-C_{SR} at frequency f is [1, 24]

$$x_{SR}=\frac{X_{SR}}{R_{nom}}=\frac{X_{LSR}-X_{CSR}}{R_{nom}}=Q_{SRnom}\left(A-\frac{1}{A}\right)+\frac{\pi\left(\pi^2-4\right)}{16A} \tag{5}$$

where Q_{SRnom}- loaded quality factor for nominal operation at load resistance R_{nom}.

Off-nominal operation of the inverter can be described by a set of equations {(6), (7), (8)} derived from equations given in [24]. The set of equations yields the solution for nominal operation of the circuit only for A=1, r_O=1 and $x_{SR}=\pi(\pi^2-4)/16$ (point P_{nom} in Fig. 2) [1]. To find parameters for the off-nominal inverter values of some normalized parameters are assumed as input variables (e.g. r_O, x_{SR}, A) and the system {(6), (7), (8)} is solved by Newton's method for three parameters (e.g. {p_O, φ, φ_S}) depending on a characteristic that is needed.

$$\frac{2\sqrt{r_O p_O}}{\sqrt{\pi^2+4}}\left(\varphi_S-\pi\right)+\cos\left(\varphi_S+\varphi\right)+\cos\varphi=0 \tag{6}$$

$$\frac{1}{4A}\sqrt{\frac{p_O}{r_O}}\left(\begin{array}{l}\sqrt{r_O p_O}\left(\pi-\varphi_S\right)^2\\ +\sqrt{\pi^2+4}\left(\begin{array}{l}\left(\varphi_S-\pi\right)\cos\varphi\\ +\sin\varphi+\sin\left(\varphi+\varphi_S\right)\end{array}\right)\end{array}\right)=1 \tag{7}$$

$$\begin{array}{l}\left(\frac{\pi^2+4}{16A}\right)^2\times\\ \left(\begin{array}{l}\left(\begin{array}{l}4\sqrt{\frac{r_O p_O}{\pi^2+4}}\left(1+\cos\varphi_S+\left(\varphi_S-\pi\right)\sin\varphi_S\right)\\ +\left(\varphi_S-\pi\right)\cos\varphi\\ +\sin\left(\varphi+\varphi_S\right)-\sin\left(\varphi-\varphi_S\right)\\ +\frac{1}{2}\left(\sin\left(\varphi+2\varphi_S\right)-\sin\varphi\right)\end{array}\right)^2\\ +\left(\begin{array}{l}4\sqrt{\frac{r_O p_O}{\pi^2+4}}\left(\left(\pi-\varphi_S\right)\cos\varphi_S+\sin\varphi_S\right)\\ +\left(\pi-\varphi_S\right)\sin\varphi-\cos\left(\varphi-\varphi_S\right)\\ -\cos\left(\varphi+\varphi_S\right)-\frac{1}{2}\left(3\cos\varphi+\cos\left(\varphi+2\varphi_S\right)\right)\end{array}\right)^2\end{array}\right)\\ =r_O^2+x_{SR}^2\end{array} \tag{8}$$

To limit the scope of calculations only to ZVS operation of the inverter it is necessary to identify boundaries for the input variables of the set {(6), (7), (8)}. This can be done by analyzing the waveforms of the switch voltage $v_S(\omega t)$ and current $i_S(\omega t)$. From (1), (2), (3) and the basic equations describing the circuit operation [1, 24] the normalized waveforms of the switch voltage $v_S(\omega t)$ and current $i_S(\omega t)=i_T(\omega t)+i_{DS}(\omega t)$ as well as capacitor C_1 current $i_{C1}(\omega t)$ are given by

$$\frac{i_T(\omega t)}{I_{SUP}}=\begin{cases}1-\sqrt{\frac{\pi^2+4}{4r_O p_O}}\sin\left(\omega t+\varphi\right);\text{for } 0\le\omega t<\pi\\ 0; \qquad \text{for } \pi\le\omega t<2\pi\end{cases} \tag{9}$$

$$\text{(10)}\quad \frac{v_S(\omega t)}{E_{SUP}}=\begin{cases} 0; & \text{for } 0\le \omega t<\pi \\ \dfrac{\pi\cdot\sqrt{\pi^2+4}}{2A}\times \\ \sqrt{\dfrac{p_O}{r_O}}\begin{pmatrix} 2(\omega t-\pi)\sqrt{\dfrac{r_O p_O}{\pi^2+4}} \\ +cos(\omega t+\varphi) \\ +cos\,\varphi \end{pmatrix} & ;\text{ for } \pi\le\omega t<2\pi \end{cases}$$

$$\text{(11)}\quad \frac{i_{DS}(\omega t)}{I_{SUP}}=\begin{cases} 0; & \text{for } 0\le\omega t<2\pi\text{-}\varphi_S \\ 1\text{-}\sqrt{\dfrac{\pi^2+4}{4r_O p_O}}sin(\omega t+\varphi) & ;\text{for } 2\pi\text{-}\varphi_S\le\omega t<2\pi \end{cases}$$

$$\text{(12)}\quad \frac{i_{C1}(\omega t)}{I_{SUP}}=\begin{cases} 0; & \text{for } 0\le\omega t<\pi \\ 1\text{-}\sqrt{\dfrac{\pi^2+4}{4r_O p_O}}sin(\omega t+\varphi); & \text{for } \pi\le\omega t<2\pi\text{-}\varphi_S \\ 0; & \text{for } 2\pi\text{-}\varphi_S\le\omega t<2\pi \end{cases}$$

where $I_{SUP}=P_O/E_{SUP}$ - dc supply current of the inverter.
The peak values V_{SMAX} and I_{SMAX} of transistor switch T voltage and current waveforms occur at ωt_{VSMAX} and ωt_{ISMAX} instants, respectively

$$\text{(13)}\quad \begin{aligned} \omega t_{VSMAX} &= 2\pi-\varphi+arcsin\left(2\sqrt{\frac{r_O p_O}{\pi^2+4}}\right), \\ \omega t_{ISMAX} &= 3\pi/2-\varphi. \end{aligned}$$

where φ - can be found from the set {(6), (7), (8)}. The inverter operates in off-nominal conditions within the region enclosed by the boundary curve P_1-P_{nom}-P_2 (Fig. 2a). The exact shape of the curve depends on the chosen method of output power regulation. For A=1 (constant operating frequency f=f_{nom}) only X_{SR} (by varying L_{SR}, C_{SR} or both) and R_O are modified to regulate output power (reactance/impedance regulation). Then the ZVS region is the biggest. For frequency regulation also changes of reactance X_{C1} influence the operation of the inverter, which results in a smaller area of ZVS region. But as loaded quality factor Q_{SRnom} increases the range of the frequency variations for ZVS operation of the inverter decreases, and the ZVS regions for both methods of output power control become identical. Beyond the boundary curve NZVS conditions occur for the operation of the inverter with D_T=0.5=const. To find values of parameters and conditions for the ZVS operation of the inverter on the boundary curve P_1-P_{nom}-P_2 one can notice the following relations in the switch voltage v_S and current i_S waveforms (Fig. 1c, Fig.1d)

(14) for curve P_2-P_{nom} is ϕ_S=0

(15) for curve P_1-P_{nom} is $i_S(\omega t=0)$=0

Using (14) and (15) equations can be derived that define parameters of the inverter for its operation on the boundary curves.

Operation of the inverter on the boundary curves

Off-nominal operation of the inverter with ϕ_S=0 means that there is no current conduction by diode D_S. Because diode D_S is usually an internal antiparallel diode of a MOSFET transistor the voltage threshold and the series resistance of the diode can be quite high. This can result in increased power losses if diode D_S conducts current with a significant average or RMS value. Hence, operation of the inverter without diode D_S conducting can potentially offer higher efficiency of energy conversion. It is therefore, worthwhile to analyze this case more closely. Another advantage of this operation is it can be obtained for a range of load resistance only by adjusting the operating frequency or series reactance x_{SR}. Therefore, for D_T=0.5=const. this off-nominal operation is more flexible in practical application over the nominal operation. By substituting (14) into the set {(6), (7), (8)} and solving the set for r_O, x_{SR} and φ one obtains the equations (16), (17), (18) for the normalized resistance r_O, reactance x_{SR} and phase shift φ of the output current for the inverter operating on the boundary curve P_2-P_{nom}.

$$\text{(16)}\quad r_O=\frac{\left(\pi^2+4\right)p_O}{\pi^2 p_O^2+4A^2},$$

$$\text{(17)}\quad x_{SR}=\frac{\pi\left(\pi^2+4\right)\left(\left(\pi^2-8\right)p_O^2+4A^2\right)}{16A\left(\pi^2 p_O^2+4A^2\right)},$$

$$\text{(18)}\quad \varphi=\pi-\arccos\left(\pi p_O/\sqrt{\pi^2 p_O^2+4A^2}\right).$$

Using (16) - (18) the off-nominal inverter can be easily designed for A=1 and given E_{SUP}, Q_{SRnom}, R_O, $P_O\le P_{nom}$, because all its parameters and waveforms are given by simple equations with p_O, r_O, and φ as constant. Then reactance components of the inverter are calculated from equations (1) - (5). This gives some improvement over the proposed in [3] design method with a data table. The parameters r_O, x_{SR} and φ given by (16) - (18) depend on the normalized frequency A, which can be found by equating (5) and (17) as

$$\text{(19)}\quad A=\frac{1}{2\sqrt{2}}\sqrt{\begin{array}{l} 4-\pi^2 p_O^2+\dfrac{2\pi}{Q_{SRnom}}+ \\ \dfrac{1}{Q_{SRnom}}\sqrt{\begin{array}{l}\left(\pi^2 p_O^2 Q_{SRnom}-2\pi-4Q_{SRnom}\right)^2 \\ -16\pi p_O^2 Q_{SRnom}\left(2-\pi\cdot Q_{SRnom}\right)\end{array}}\end{array}}$$

With (19) it is now possible to find the equation for the curve P_2-P_{nom} (x_{SR} vs. r_O) for the frequency regulated inverter. For point P_r of the curve P_2-P_{nom} its coordinate r_O achieves its maximum value r_O=r_{OMAX} for which the inverter operates in ZVS conditions. The expression for r_{OMAX} is obtained from dr_O/dp_O=0 and (19) as

$$\text{(20)}\quad r_{OMAX}=\frac{\pi^2+4}{2\sqrt{\pi\dfrac{\pi^2-4+4\pi Q_{SRnom}}{Q_{SRnom}}}}$$

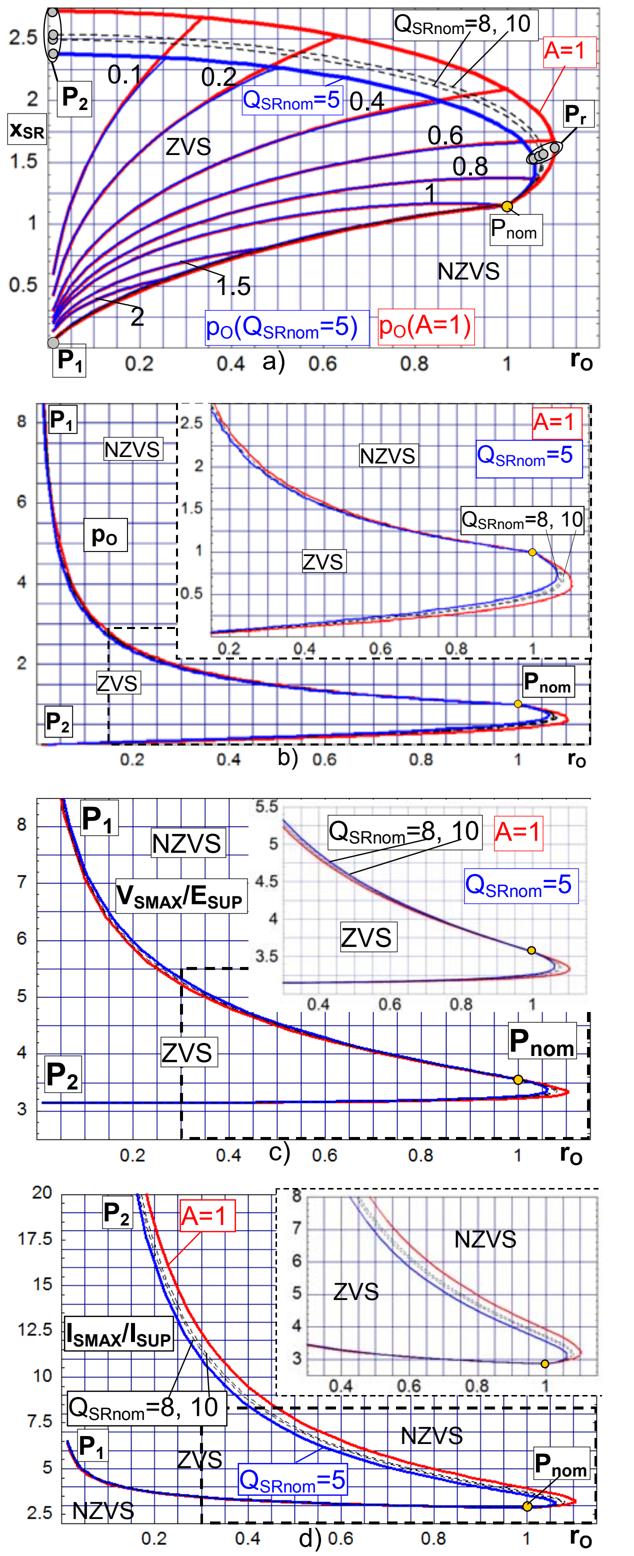

Fig. 2 Normalized parameters of Class E ZVS SRC inverter vs. load resistance r_O; a) series reactance x_{SR} vs. r_O (p_O={0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0}=const. - series reactance x_{SR} vs. r_O for constant normalized output power p_O), b) output power p_O vs. r_O, c) peak voltage of the switch V_{SMAX}/E_{SUP}, d) peak current of the switch I_{SMAX}/I_{SUP}

From (16) - (18) also the expressions for other normalized parameters occurring at r_{OMAX} are found: frequency $A(r_{OMAX})$, reactance $x_{SR}(r_{OMAX})$ and output power $p_O(r_{OMAX})$

$$A\left(r_{OMAX}\right)=\frac{1}{2}\sqrt{4+\frac{\pi-\dfrac{4}{\pi}}{Q_{SRnom}}} \tag{21}$$

$$x_{SR}\left(r_{OMAX}\right)=\frac{\pi^4-16}{8\sqrt{\pi\dfrac{\pi^2-4+4\pi Q_{SRnom}}{Q_{SRnom}}}} \tag{22}$$

$$p_O\left(r_{OMAX}\right)=\sqrt{\frac{\pi^2-4+4\pi Q_{SRnom}}{\pi^3 Q_{SRnom}}} \tag{23}$$

For $Q_{SRnom}\rightarrow\infty$ the normalized frequency $A(r_{OMAX})\rightarrow 1$, and the operating frequency equals to the nominal frequency $f=f_{nom}$. Then the parameters for point P_r achieve their values as for the impedance regulated inverter:

$$r_{OMAX}\left(A=1\right)=\frac{1}{\pi}+\frac{\pi}{4}\approx 1.1037 \tag{24}$$

$$p_O\left(r_{OMAX},A=1\right)=\frac{2}{\pi} \tag{25}$$

$$x_{SR}\left(r_{OMAX},A=1\right)=\frac{\pi^4-16}{16\pi} \tag{26}$$

The expression (24) demonstrates that the basic Class E ZVS SCR inverter with D_T=0.5 can operate in ZVS conditions with load resistance above its nominal value R_{nom} without a matching circuit if only reactance x_{SR} is in the limits given by (17). Power losses in the transistor switch are found assuming that the losses are small and do not affect waveforms in the circuit. From (9), (16), (18) and (4) conduction losses in transistor T for the off-nominal inverter operating with ϕ_S=0 can be estimated for the nominal frequency A=1 as

$$\begin{aligned}P_{Cond}&=r_{DSon}\int_0^{\pi} i_T^2\left(\omega t\right)d\omega t\\&=\frac{r_{DSon}\cdot p_O^2}{16}\left(\frac{P_{nom}}{E_{SUP}}\right)^2\left(24+\pi^2+\frac{4}{p_O^2}\right)\end{aligned} \tag{27}$$

To compare conduction losses in the off-nominal inverter to the conduction losses in the nominal inverter (p_O=1) normalized conduction losses are defined as p_{Cond}

$$p_{Cond}=\frac{P_{Cond}}{P_{Condnom}}=p_O^2\,\frac{24+\pi^2+\dfrac{4}{p_O^2}}{28+\pi^2} \tag{28}$$

Switching losses at the turn-off instant in the inverter are found assuming that the switch current $i_S(\omega t)$ falls linearly during time t_f<<T_{nom}=1/f_{nom} [1]. Then, capacitor C_1 is charged by current i_S linearly falling from its start value $i_S(\omega t=\pi)=I_{SUP}(1+1/p_O)$.

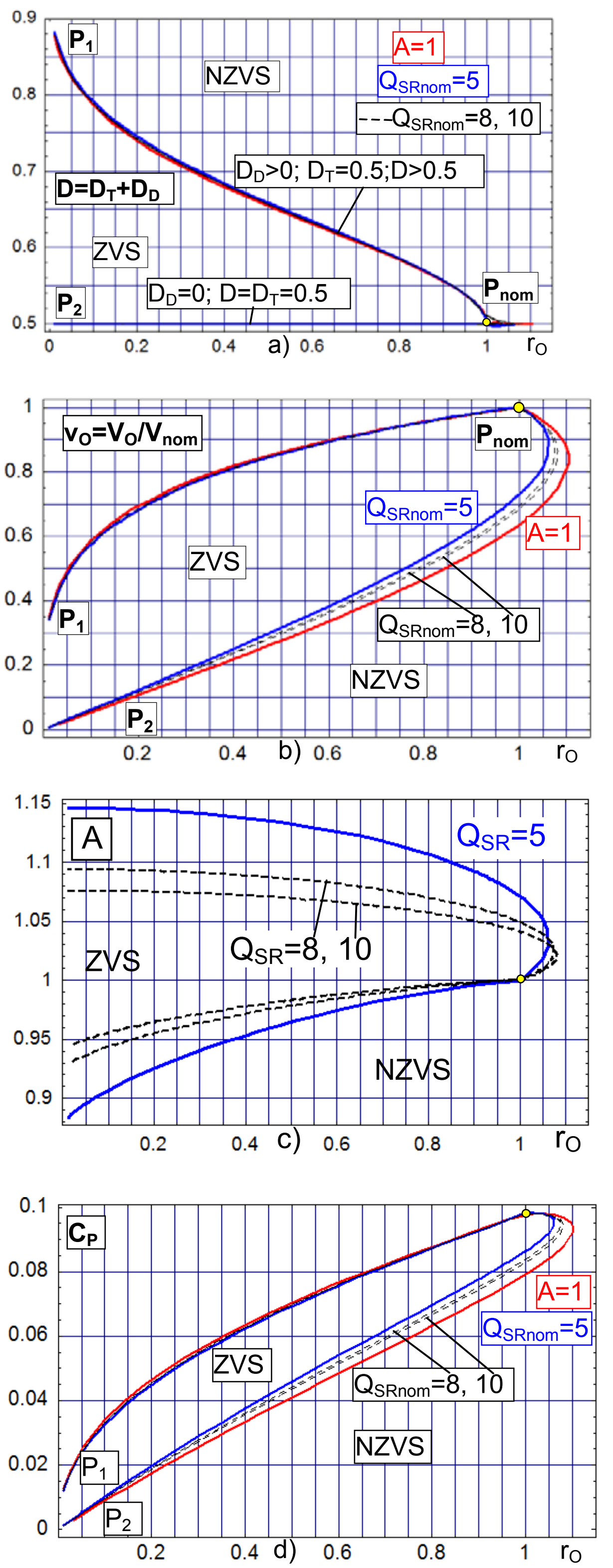

Fig. 3 Normalized parameters of Class E ZVS SRC inverter vs. load resistance r_O; a) on-time D vs. r_O, b) output voltage v_O vs. r_O, c) operating frequency A vs. r_O, d) power capability c_P vs. r_O

(29)
$$P_{SW}=\frac{1}{2\pi}\int_{\pi}^{\pi+\omega t_f} i_T v_S \, d\omega t =\frac{p_O^2\left(1+\frac{1}{p_O}\right)^2}{4}\frac{P_{nom}\left(\omega t_f\right)^2}{12}$$

Normalized switching losses p_{SW} for the off-nominal inverter are given by

(30)
$$p_{SW}=\frac{P_{SW}}{P_{SWnom}}=\frac{p_O^2\left(1+\frac{1}{p_O}\right)^2}{4}.$$

From (28) and (30) it can be deduced that both conduction and switching losses in the transistor T in the off-nominal inverter (with ϕ_S=0) decrease as its output power is reduced ($0 \le p_O \le 1$). This suggests that efficiency of the off-nominal circuit can be maintained high even at reduced output power.

For the boundary curve P_1-P_{nom} (Fig. 2a) defined by the equation (15), this condition can also be expressed using (9) as

(31)
$$\sqrt{r_O \cdot p_O}=\frac{\sqrt{\pi^2+4}\cdot\sin\varphi}{2}.$$

With (31) normalized output power p_O can be eliminated from the set {(6), (7), (8)}, which then can be solved for φ, φ_S and x_{SR} (or A if (5) is substituted) with $0 \le r_O \le 1$ as an input variable. Found parameters for the inverter operating on the curve P_1-P_{nom} can next be used in equations (1)- (13) to compute its waveforms and other data. Having found the ranges of basic parameters (r_O, x_{SR}, p_O, φ, φ_S, A) of the inverter by analyzing its operation on the boundary curve P_2-P_{nom}-P_1 it is now possible to calculate practically any of its regulation characteristics or find the range of its parameters versus selected input variables for the whole ZVS region (for D_T=0.5). Figure 2 and Fig. 3 present plots of theoretically found selected normalized parameters of the inverter vs. its load resistance r_O for the circuit operating on the boundary curve P_2-P_{nom}-P_1. In Fig. 2a plots of reactance x_{SR} vs. r_O show that ZVS region depends on the method of output power regulation. Plots of x_{SR} vs. r_O for p_O=const. identify the required characteristic of the impedance of the series branch L_{SR}-C_{SR}-R_O of the inverter necessary to stabilize its output power at a given level as r_O is changed. The plots also give approximate estimation of the level of output power p_O for given x_{SR} and r_O. The plot of output power p_O vs. r_O in Fig. 2b shows that the regulation range of p_O is almost independent of the method of regulation. It can also be noticed that the output power can be effectively controlled by varying x_{SR} for given r_O. Normalized maximum peak voltage of the switch V_{SMAX}/E_{SUP} vs. r_O in Fig. 2c demonstrates a rapid rise of its value as output power is increased (curve P_1-P_{nom}), but when the inverter operates on curve P_{nom}-P_2 (φ_S=0) then V_{SMAX} varies much less decreasing slightly as r_O becomes smaller. ($V_{SMAX}/E_{SUP}\to\pi$ as $r_O\to 0$). The increase of the normalized peak current of the switch I_{SMAX}/I_{SUP} vs. r_O depicted in Fig. 2d (P_2-P_{nom}) results from decreasing the output power of the inverter as r_O decreases. It consequently leads to reduction of the dc supply current ($I_{SMAX}\to I_O$ and $I_{SUP}\to 0$ then $I_{SMAX}/I_{SUP}\uparrow$). Normalized conduction time D of the switch depicted in Fig. 3a is constant ($D=D_T$=0.5) when the inverter operates on curve P_2-P_{nom}, but D increases above 0.5 for the rest of ZVS region. The increase results from diode D_S current conduction. The plot of output voltage v_O vs. r_O in Fig. 3b shows that v_O achieves its maximum value for nominal operation of the inverter. Changes of x_{SR} or r_O that shift operation of the inverter from point P_{nom} always decrease v_O. But then the output power p_O is increased if both x_{SR} and r_O decrease (P_{nom}-P_1) or the output power p_O is decreased if x_{SR} increases and r_O decreases (P_{nom}-P_2). If the inverter is frequency regulated then its frequency range and output power range both increase as r_O is reduced (Fig. 3c).

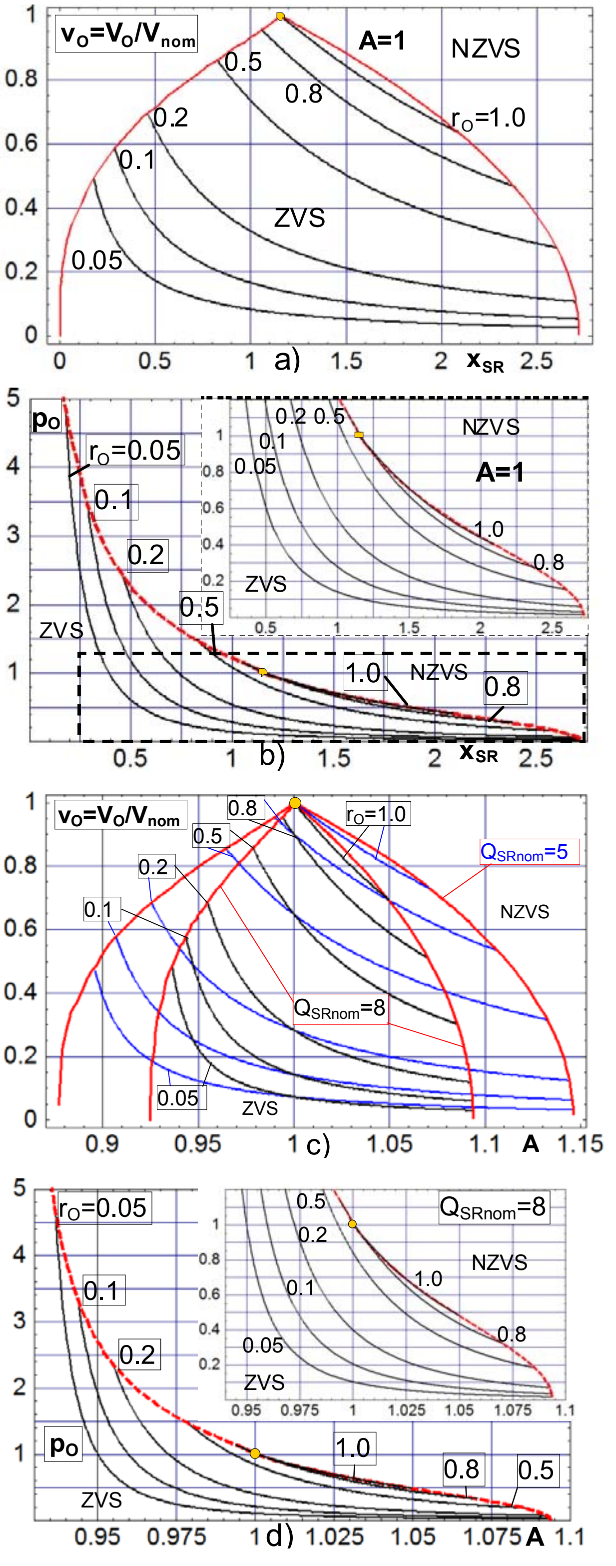

Fig. 4 Normalized regulation characteristics of Class E ZVS SRC inverter; a) output voltage v_O vs. series reactance x_{SR}, *A=1*, b) output power p_O vs. series reactance x_{SR}, *A=1*, c) output voltage v_O vs. frequency *A*, d) output power p_O vs. frequency *A*

As shown in Fig. 3d output power capability is the highest for the nominal operation, but it also remains high for the operation of the inverter with load resistance $R_O>R_{nom}$.

Figure 4 presents normalized regulation characteristics of output voltage and output power for the inverter for given values of load resistance r_O. The characteristics are non-linear and both the level of output voltage (output power) and its regulation range depend on the value of load resistance r_O. Using presented equations it is therefore, also possible to design the level and the regulation range of output voltage (output power) in the inverter according to requirements for given load.

Experimental verification

To verify obtained equations an experimental nominal Class E inverter was designed, simulated with LTSpice and then built and tested. The experimental inverter was designed for the following parameters: f_{nom}=140 kHz, E_{SUP}= 24 V, P_{OR}= 50 W - rated output power, η= 0.95 - efficiency for P_{OR}, R_{OR}= 6 Ω - rated load resistance for P_{OR}, $P_{nom}=P_{SUPnom}=P_{OR}/\eta$=50/0.95=52.63 W - the input power (dc supply power) of the inverter, the total resistance in the L_{SR}-C_{SR} branch of the inverter including equivalent loss $R_{nom}=R_{OR}/\eta$= 6/0.95 = 6.316 Ω, loaded quality factor for nominal operation Q_{SRnom}= 8, fall time of the switch current t_f= 50ns. As the switch *S* Inifineon's MOSFET transistor irfb4228 was used with r_{DSon}= 12 mΩ, V_{DSMAX} =150V. The values of calculated [1] and used (given in brackets) components were as follows: shunt capacitance - $C_1=1/(2\pi\cdot f_{nom}\cdot R_{nom}\cdot\pi(\pi^2+4)/8)$ = 33.06nF (33.0nF), series capacitance - $C_{SRnom}=1/2\pi\cdot f_{nom}\cdot R_{nom}(Q_{SRnom}-\pi(\pi^2-4)/16)$= 26.30nF (26.26nF), series inductance - $L_{SRnom}=Q_{SRnom}\cdot R_{nom}/2\pi\cdot f_{nom}$= 57.41 μH (57.63 μH), choke inductance - $L_{CH}>7\cdot R_{nom}/f_{nom}$= 315.6 μH (740 μH), peak voltage across the switch $V_{SMAXnom}=3.562\cdot E_{SUP}$= 85.49 V. Theoretical estimations of conduction losses in transistor *T* $P_{Condnom}=P_{Cond}(p_O=1)=r_{DSon}\cdot(P_{SUPnom}/E_{SUP})^2(28+\pi^2)/16$ = 0.136 W and switching losses $P_{SWnom}=P_{SW}(p_O=1)=P_{SUPnom}\cdot(\omega t_f)^2/12$=8.484 mW.

To demonstrate the efficacy of the proposed theoretical equations in designing and finding the parameters of the Class E inverter for its off-nominal operation an example of calculations is presented for R_O= 6 Ω. Calculations are carried out for the nominal operating frequency (*A*=1) and off-nominal operation is achieved by modification of C_{SR} (reactance regulation). To include power losses in the design example normalized load resistance r_O is defined for total load resistance (equivalent losses included) $r_O=(R_{OR}/\eta_P)/R_{nom}$=1, and from (16) normalized output power is $p_O=P_{SUP}/P_{SUPnom}=(\pi^2+4-(16-8\pi^2+\pi^4-16r_O^2\pi^2)^{0.5}/(2r_O\pi^2)$= 0.4053. The parameters for off-nominal operation of the inverter are as follows: output power $P_O=p_O\cdot P_{SUP}\cdot\eta$=20.26 W, output voltage $V_{ORMS}=(P_O\cdot R_O)^{0.5}$= 11.03 V, from (18) output current phase shift - $\varphi=\pi-\arccos(\pi\cdot p_O/(\pi^2 p_O^2+4)^{0.5})$= 2.138 rad, from (13) $\omega t_{VSMAX}=2\pi-\varphi+\arcsin(2(r_O\cdot p_O/(\pi^2+4))^{0.5})$= 4.494 rad, from (10) peak voltage across the switch $V_{SMAX}=3.222\cdot E_{SUP}$= 77.33 V, from (27) conduction losses in transistor *T* $P_{Cond}=r_{DSon}\cdot p_O^2(P_{nom}/E_{SUP})^2\times(24+\pi^2+4/p_O^2)/16$= 0.0344 W, from (29) switching losses $P_{SW}=p_O^2(1+1/p_O)^2\cdot P_{nom}\cdot(\omega t_f)^2/48$= 4.189 mW, from (17) series reactance of the branch L_{SR}-C_{SR} is $X_{SR}=2.087\cdot R_{nom}$= 12.52 Ω, modified value of capacitance C_{SR} to obtain off-nominal operation (φ_S=0) of the inverter $C_{SR}=1/(2\cdot\pi\cdot f_{nom}\cdot(Q_{SRnom}\cdot R_{nom}-X_{SR}))$=30.44 nF, relative change of series capacitance C_{SR}/C_{SRnom}= 30.44/26.30= 1.157. Comparing theoretical estimations of power losses in transistor *T* for nominal and off-nominal operation of the experimental inverter one can find p_{Cond}=0.2525 and

p_{SW}=0.4937. This means that conduction losses decrease almost 4 times and switching losses fall about 2 times in the example inverter whereas its output power is reduced only 2.5 times if compared to its nominal operation. This indicates high efficiency of the inverter in the off-nominal operation mode. Depicted in Fig. 5b experimental waveforms for the example demonstrate a good agreement with the calculations. Similar calculations can be carried out for the off-nominal operation of the inverter obtained by frequency regulation.

The operation of the designed inverter was further tested at the constant operating frequency f_{nom} with varied load resistance R_O (Fig. 5c) as well as with varied reactance X_{SR} (by modifying C_{SR}) for constant load resistance R_O=4 Ω and R_O=6 Ω (Fig. 6a, 6c). The inverter was also measured for frequency regulation of its output voltage for constant load resistance R_O=4 Ω and R_O=6 Ω (Fig. 6b, 6d). Measured regulation characteristics of output voltage V_{ORMS} and peak voltage V_{SMAX} of the transistor switch are compared with theoretical results and simulations in Fig. 6.

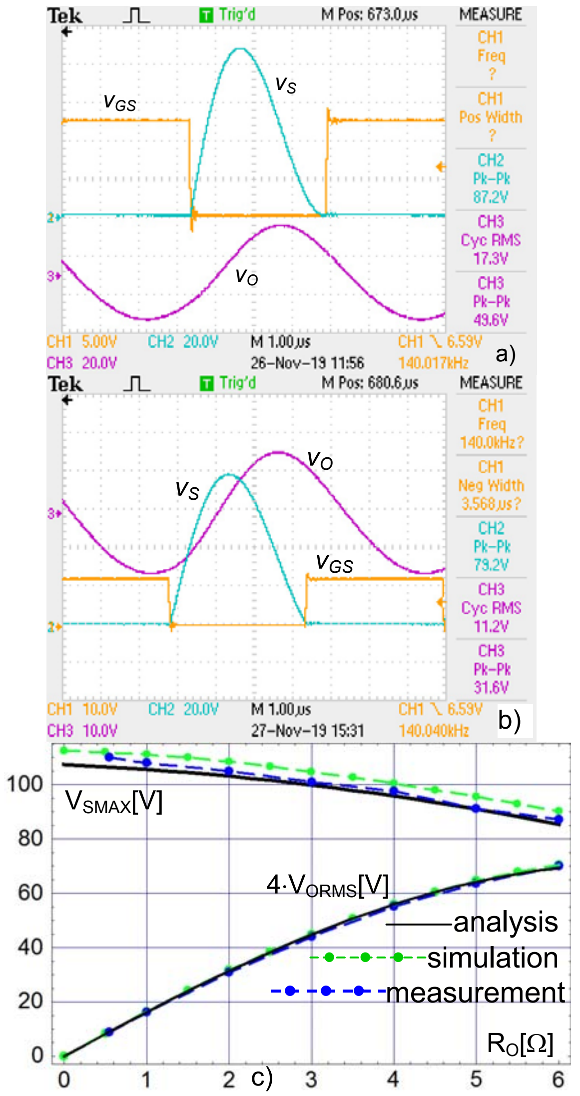

Fig. 5 Measured voltage waveforms and regulation characteristics of the experimental inverter; a) waveforms for nominal operation, b) waveforms for off-nominal operation (φ_S=0), c) peak voltage of the switch V_{SMAX} vs. load resistance R_O, d) output voltage V_{ORMS} vs. load resistance R_O

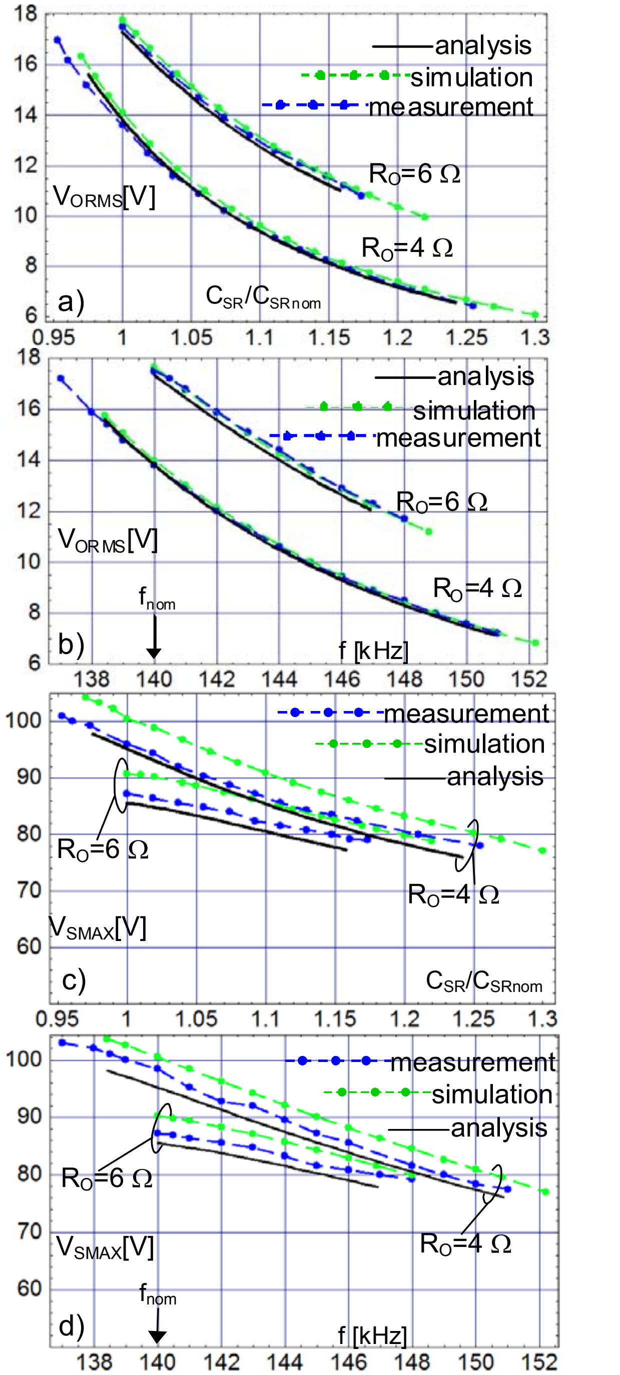

Fig. 6 Measured regulation characteristics of the experimental Class E inverter for constant load resistance R_O={4, 6} Ω: a) output voltage V_{ORMS} vs. normalized series capacitance C_{SR}/C_{SRnom}; b) output voltage V_{ORMS} vs. operating frequency f, c) switch peak voltage V_{SMAX} vs. normalized series capacitance C_{SR}/C_{SRnom}; d) switch peak voltage V_{SMAX} vs. operating frequency f

The efficiency of the experimental circuit for nominal operation at P_O= 50.0 W was 96%, and it exceeded 98% for off-nominal operation at P_O= 20.9 W and R_O=6 Ω. The measurements confirm theoretical predictions that the reduced losses in the off-nominal inverter can result in high efficiency [5, 25, 26]. Some observed differences between theoretical results and measurements or simulations resulted mainly from finite values of Q_{SR} and inductance L_{CH} as well as non-linear output capacitance of the transistor switch.

Conclusions

The paper presented results of an extended analysis for the basic nominal Class E ZVS SRC inverter with a sinusoidal output current and normalized switch on-time D_T= 0.5. To identify boundaries of ZVS operation of the analyzed nominal inverter its operating frequency as well as the reactance and resistance of the series branch of its resonant circuit have been modified. Various normalized parameters of the inverter have been found and plotted to demonstrate the ranges of their values for ZVS operation of the inverter. Simple analytical equations have been derived to determine the parameters of the inverter for its high-efficiency off-nominal operation without current conduction of the antiparallel diode. A design example for the off-nominal operation of the inverter has also been given. Regulation characteristics of output voltage and output power versus operating frequency and versus the reactance of the series branch of the inverter have been found. The regulation characteristics can be very useful in designing Class E inverter for industrial applications where it is often necessary to determine parameters of the circuit under regulation. Simulation and measurements of the laboratory model validated theoretical results. The analysis and presented results provide in-depth insight into the operation of the inverter. The results of the analysis allow the designer to calculate the circuit parameters at any point of its steady state ZVS operation as well as to examine various regulation strategies. Derived equations for the inverter can also be used e.g. to analyze its operation with various impedance loads such as matching circuits, heating coils or WPT coils.

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