A comparative study between PID and PD-SMC and PD-ASMC control applied on a delta robot

Abstract. This paper presents the modelling and control of a delta robot. The software SOLIDWORKS is used in this work to get a performing model that is very close to real system. The proportional integrator derivative (PID) control is used in this proposal. The results are compared with PD-Sliding mode (PD-SMC) and PD a robust SMC (PD-ASMC). This is an important comparative study where the advantages of each controller are presented: the PD-SMC improve the performance of the trajectory tracking, where the control signal performances and the robustness was improved by the PD-ASMC. Results presented are done with matlab-simulink and with Solidworks.

Keywords: Delta parallel robot, Dynamic model, trajectory tracking, PD-ASMC, RMSE.

Introduction

Robots are now occupying lot of fields, including industrial, medical, etc. This importance prompted many researchers to work on solving the problem of tracking the trajectory and the increase in the accuracy of its work. In the old days, PID (proportional-integrator-derivative) was used due to its ease of application and acceptable performances [1]. But due to the complex dynamics of parallel robots and the requirement to optimize performances to follow complex trajectory, PID produces control signals not physically feasible. This has led many researchers to develop and improve PID results. The philosophy of nonlinear PD was to make the constants P and D variable by changing the error and this gives acceptable results but lacks robustness, due to its closed structure. This resulted in the insertion of robust controllers on PD including FUZZY LOGIC [2,3]and sliding mode control SMC. The PD-SMC controller gave better results in terms of control, performance, path following and strength.

The goal of automatic control is to decrease control signals and increase performances. It is known that the mathematical model is somewhat far from the real system, which leads to a lack of robustness in the sliding mod control SMC. The SMC is known to have high vibrations in the control signals due to the “sign” function. The role of the constant I in the PID controller is to completely cancel the modeling error and the error signal, but in return it reduces the phase margin and this greatly affects the robots because they move along paths and need a large phase margin, which leads to destabilization of the system. This is what led to his absence. The absence of the constant I in the PID control and the presence of the sign function in the SMC makes the PD-SMC less robustness and the control signals are vibrational.

3dof Delta parallel robots are one of the most popular robots in the industry, which led to the application of many controllers, and due to their complex dynamics, it was the passion of many researchers, which prompted them to work on it a lot and apply linear and nonlinear controls, such as PD control [4], and linear control that was synthesised around an operating point [5,6], Adaptive control [7], PD-SMC and NPD-SMC [8], however, the above mentioned controllers are considered insufficient, and this was shown in the first paragraph in relation to PID, PD-SMC, NPD-SMC.

In this paper, we will create a model of the robot on SOLIDWORKS in order to represent the real system mechanically, and we will introduce a new controller, is called PD-ASMC, to solve the problem of tracking the trajectory, robustness and control signals. SMC is known to drag the model to the sliding surface, if they find a model error that affects a robustness of control, we will compensate for this by entering the error signals multiplied by the ratio of the modelling error in the sliding surface function (S), and after pulling the system to the sliding surface, the SMC takes it to the original with the “sign” function, and we will replace the “sign” with a “tanh” to reduce the vibrations of the control signals.

Finally, we conduct a comparative study between PID, PD-SMC and PD-ASMC in order to demonstrate the efficacy and high tracking performance of the PD-ASMC.

The structure of the article is as follows: In Section 2 a kinematics analysis of the robot will be studied, from which the inverse kinematics will be extracted and at the end of...
the section we will see the dynamic model of 3-DOF parallel robots. In Section 3, the PID and PD-SMC controller will be studied and designed, and we will address the errors of the controllers, and from them we will design the PD-ASMC controller. In Section 4 simulation tests are performed on the SOLIDWORK model to assess the accuracy and robustness of the controller. We will also study a comparison between the three controllers and the strength of the proposed controller will be demonstrated. We’ll finish with the conclusion

Model of the DPR

Kinematics analysis

Delta parallel robots are known as complex systems due to their closed form, which affects the field of work where we find it small and limited and has a lot of singularities. In analytical kinematics we external the relation between the coordinates of x and θ [9,10]. We see in figures (1,2) [11] that the platform is connected with the arms l in the points P. We will take advantage of the length of the arms l and set it equal to the Euclidean norms as shown in the equation (1).

\[
J = \left\| \mathbf{I} \right\| = l_{ix}^2 + l_{iy}^2 + l_{iz}^2
\]

(1)

when analysing Equation 1, we extract the following

2

\[
E_1 \cos \theta_i + F_i \sin \theta_i + G_i = 0
\]

(2)

where

\[
E_1 = 2L(y + a)
\]

\[
E_2 = -L(\sqrt{3}(x + b) + y + c)
\]

\[
E_3 = -L(\sqrt{3}(x + b) - y - c)
\]

\[
F_i = F_2 = F_3 = 2L
\]

\[
G_1 = x^2 + y^2 + z^2 + a^2 + L^2 + 2ya - l^2
\]

\[
G_2 = x^2 + y^2 + z^2 + b^2 + c^2 + L^2 + 2(xb + yc) - l^2
\]

\[
G_3 = x^2 + y^2 + z^2 + b^2 + c^2 + L^2 + 2(-xb + yc) - l^2
\]

And

\[
a = w_B - u_p, b = \frac{1}{2}s_p + \frac{\sqrt{3}}{2}w_B, c = w_p - \frac{1}{2}w_B.
\]

The flowing figure 2 represent the Base and platform for the DELTA robot

Fig.2. Base and platform design of the delta robot

Inverse kinematics

In this part, we will study the reverse movements of the robot. We will calculate \( \theta \) by means of the x coordinates. In the equation (2) we can calculate the value of \( \theta \) [12,13], by changing a variable in this equation, as follows

\[
q_i = \tan \theta_i \text{ then } \cos \theta_i = \frac{1 - q_i^2}{1 + q_i^2} \text{ and } \sin \theta_i = \frac{2q_i}{1 + q_i^2}
\]

We will replace \( \cos \theta \) and \( \sin \theta \) with what they are equal to and substitute it into equation (2) and get the following equation

\[
(G_i - E_i)q_i^2 + (2F_i)q_i + (G_i + E_i) = 0
\]

(3)

The solution of equation (3) is

\[
q_i = \frac{F_i \pm \sqrt{E_i^2 + F_i^2 - G_i^2}}{G_i - E_i}
\]

(4)

And from it we extract \( \theta \)

\[
\theta_i = 2\tan^{-1}(q_i)
\]

(5)

We notice that the equation (5) has two solutions for each angle, one is positive and the other is negative, and from the combination of the three angles we extract 8 positions for the robot and all of it is corrects. But the dynamic of the robot forces us to choose one position, which is the positive angle.
Dynamic model of DPR

The dynamic model is the relationship between the torques (and \( I \) or forces) applied to the actuators and the articular positions, speeds and accelerations [14,15]. We represent the dynamic model by a relation of the form (6). Fig. 1 shows the schema of a 3-DOF delta parallel robot and in table 1 we find the architectural parameters of the robot extracted from SOLIDWORKS.

\[
\tau = M(\dot{\theta}) \ddot{\theta} + C(\dot{\theta}, \ddot{\theta}) \dot{\theta} + G(\theta)
\]

where: \( \tau \) – vector torque \( \tau = [\tau_1, \tau_2, \tau_3] \), and \( \theta \) - the joint vector \( \theta = [\theta_1, \theta_2, \theta_3] \) and \( \dot{\theta}, \ddot{\theta} \) - the speed and acceleration joint vector respectively.

\[
M(\theta) = Ib + (mn + 3mab + 3mb) J^T J
\]

M–the inertia matrix and \( Ib = \frac{L^2}{3} \left[ \frac{1}{3} mb + \frac{2}{3} mab \right] l \) denotes the upper links inertial matrix.

\[
C(\dot{\theta}, \ddot{\theta}) = \dot{J}(mn + 3mab + 3mb) \frac{dJ}{dt} \dot{\theta}
\]

C–vector resulting from Coriolis and centrifugal forces, \( I \) – represent the 3x3 identity matrix,

\[
G(\theta) = J^T (mn + 3mab) g - L(mb/2 + mc + mab/2) g \cos(\theta)
\]

is the gravity torque vector, and \( g \) the gravity acceleration, \( J \) is the Jacobin matrix that represents the relation between the articular and operational speed \( \chi = J \dot{\theta} \)

Table 1. architectural parameters of the robot

<table>
<thead>
<tr>
<th>name</th>
<th>meaning</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mn</td>
<td>Moving platform</td>
<td>187.3(g)</td>
</tr>
<tr>
<td>mab</td>
<td>Lower arm</td>
<td>53.1(g)</td>
</tr>
<tr>
<td>mb</td>
<td>Upper arm</td>
<td>88(g)</td>
</tr>
<tr>
<td>L</td>
<td>Upper legs length</td>
<td>150(mm)</td>
</tr>
<tr>
<td>l</td>
<td>Lower legs parallelogram length</td>
<td>300(mm)</td>
</tr>
<tr>
<td>UB</td>
<td>planar distance from (0) to near base side</td>
<td>239.6(mm)</td>
</tr>
<tr>
<td>SB</td>
<td>Base equilateral triangle side</td>
<td>415(mm)</td>
</tr>
<tr>
<td>UP</td>
<td>planar distance from (P) to a platform vertex</td>
<td>49.65(mm)</td>
</tr>
<tr>
<td>WP</td>
<td>planar distance from (P) to near platform side</td>
<td>24.8(mm)</td>
</tr>
<tr>
<td>SP</td>
<td>platform equilateral triangle side</td>
<td>86mm</td>
</tr>
</tbody>
</table>

The inverse dynamic model represented by the relation of the form (6). The direct dynamic model is that which expresses the articular accelerations as a function of the positions, speeds and couples of the joints. It is then represented by the following relation:

\[
\ddot{\theta} = M(\theta)^{-1} (\tau - C(\dot{\theta}, \ddot{\theta}) - G(\theta))
\]

Controller design

PID control design

We will convert the robot system to a linear model as shown in Equation 11. And we will control its joints by the PID controller [16] as shown in Figure 3.

\[
\tau = m_{max} \dot{\theta} + c_{max} \ddot{\theta} + g_{max}
\]

where \( m_{max}, c_{max}, g_{max} \) – the maximum value of the element \( M_{ii}, C_{ij}, G_{ij} \) Respectively

The controller law is shown as follows

\[
\tau_c = K_p (\dot{\theta}_d - \dot{\theta}) + K_i (\theta_d - \theta) + K_d (\ddot{\theta}_d - \ddot{\theta})
\]

The calculation of the three constants \( (K_p, K_i, K_d) \) will be based on Khalil Domber method

\[
K_{pi} = 3m_{max} W^2, K_{ij} = m_{max} W^3, K_{dj} = m_{max} W
\]

Where

\[
m_{max} = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}
\]

The frequency \( W \) is selected at \( 50 \text{rad/s} \) Where

\[
K_p = \begin{bmatrix} 375 & 0 & 0 \\ 0 & 375 & 0 \\ 0 & 0 & 375 \end{bmatrix}, K_i = \begin{bmatrix} 6250 & 0 & 0 \\ 0 & 6250 & 0 \\ 0 & 0 & 6250 \end{bmatrix}, K_d = \begin{bmatrix} 7.5 & 0 & 0 \\ 0 & 7.5 & 0 \\ 0 & 0 & 7.5 \end{bmatrix}
\]

Fig. 3. Block diagram of the PID controller

PD-SMC control design

The PD control in this part will represent the sliding function, so as to ensure the trajectory tracking and the reduction in the modelling error [17], as shown in the following equation.

\[
S = K_p e(t) + K_i \dot{e}(t)
\]

The goal of the controller is to following the trajectory which means \( \lim_{t \to \infty} e(t) = 0 \). Based on this, and from the derivative of the sliding function we will extract the control unit as follows

\[
\dot{S} = K_p \dot{e}(t) + K_d \ddot{e}(t)
\]

where \( \dot{e}(t) = \dot{\theta}_d - \dot{\theta} \)

Substituting equation (10) into equation (14), we obtain the continuous control unit shown as follows

\[
\tau_c = (K_d M^{-1}) \dot{e}(t) + K_i \dot{\theta}_d + K_d M^{-1} (c(\theta, \dot{\theta}) + G(\theta))
\]

In order to ensure the stability of the system along the path and in difficult situations, we will do this on through the LAYAPONOV function and we will extract the law of the discontinuous controller \( \tau_d \) as shown.
Guardian to ensure equation 2 is less than zero, \( \dot{S} \) must be opposite to the \( S \) signal. Thus, we will add to the continuous control unit the discontinuous controller as shown as follows

\[
U = \tau_c + \tau_d
\]

where \( \tau_d = K \text{sign}(S) \).

**PD-ASMC control design**

In this part we will process \( \tau_c \) and this is to compensate for the modeling error and to increase the robot performance, we will process \( \tau_d \) and this is in order to improve the control signal[17], as shown below.

\[
S = K_p e(t) + K_d \ddot{e}(t) + \xi(t)
\]

where \( \xi(t) = \beta \epsilon(t) \delta \) \( \beta \) and \( \delta \) are positive constants, we will increase the missing information by inserting the error in the sliding function which makes it more stable, looking for the constants representing the modeling error rate.

\[
\dot{S} = K_p \dot{e}(t) + K_d \ddot{e}(t) + \xi
\]

\[
\dot{S} = K_p \dot{e}(t) + K_d \ddot{e}(t) - M(\dot{\theta})^{-1}(\tau - C(\dot{\theta}, \ddot{\theta}) - G(\theta)) + \xi
\]

where \( \dot{\xi}(t) = \delta^{-1} \beta \epsilon(t) \delta \)

the continuous control unit shown as below

\[
\dot{S} = (K_d M^{-1})^{-1} [K_p \dot{e}(t) + K_d \ddot{e}(t) - K_d M^{-1} (C(\dot{\theta}, \ddot{\theta}) - G(\theta))] + \xi
\]

In the discontinuous controller, we will replace the “sign” function with the “tanh” function to reduce the vibrations of the control signal the discontinuous control unit shown as follows

\[
\tau_d = K \text{tanh}(S)
\]

**Simulation and results**

In this part we will study and clarify the difference between controllers. We will choose a complex trajectory so that we can clearly see the difference between the three controls. To create the trajectory, we followed the quintic method and it would be flower shaped, as shown in Figure 5. We will design the controllers basing on mathematical models and apply them to the SOLIDWORKS as shown in Appendices. model in order to get an excellent study which is closer to reality.
(23) \[ RMES_\beta = \sqrt{RMES_{\beta_1}^2 + RMES_{\beta_2}^2 + RMES_{\beta_3}^2} \]

With
\[ RMES_{\beta_i} = \sqrt{n^{-1} \sum_{i=1}^n e_{\beta_i}} \]

where \[ e_{\beta_i} = (\beta_{i+1} - \beta_i) \]

As for the operating area representing in the figure (10,11,12), we notice that the three controls have similar results along the trajectory, but the performance of PD-ASMC remains better than that of PD-SMC and this is clearly shown in Z-axes and is better than PID in the three coordinates.

Table 2. The parameters of PID PD-SMC PD-ASMC

<table>
<thead>
<tr>
<th></th>
<th>( K_p )</th>
<th>( K_d )</th>
<th>( K_I )</th>
<th>( \beta )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>375</td>
<td>7.5</td>
<td>6250</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
<tr>
<td>PD-SMC</td>
<td>375</td>
<td>7.5</td>
<td>( _ )</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
<tr>
<td>PD-ASMC</td>
<td>375</td>
<td>7.5</td>
<td>( _ )</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

From the graphs (13,14,15), is clear that the PID controller produces a control signal that exceeds 100 Nm in terms of amplitude, to achieve the performance that we have seen in graphs 4,5 and 6, and this signal can be physically produced and from it we conclude that this performance achieved by the PID can be seen in reality. On the other hand, PD-SMC produced an excellent control signal in terms of amplitude. We see that it ranges between -15Nm and 15Nm, but has very large vibration, which is a result of the "sign" function. Similarly, due to the complexity of the trajectory, PD-ASMC produces an excellent control signal in terms of amplitude and vibration, where the amplitude of the control signal exceeds 20 and the vibrations are almost non-existent.
CONCLUSION
We have seen in this paper a new path in simulation where we have created a real model of a DELTA parallel robot on SOLEDWORKS, and this may help us to know the power of the controller and the performance resulting from it. It is known that the nonlinear mathematical model of a system is far from the real system, this leads researchers to adjust the parameters in the control unit to make it work on the truth system. We also saw a new control method in the field of DELTA ROBOT, which is PD-ASMC, where the results were good in terms of performance and control signal. PD gave good results in terms of performance, but it cost a lot of control signal in term of amplitude, where PD-SMC was better than it, but it contains big vibrations in the control signal, to correct this disadvantage we created the PD-ASMC controller.

Appendix

![Fig.16. fixed base](image1)
![Fig.17. upper arm](image2)
![Fig.18. lower arm](image3)
![Fig.19. moving platform](image4)

Fig.16. fixed base
Fig.17. upper arm
Fig.18. lower arm
Fig.19. moving platform

Fig.20. Delta Robots design
Fig.21. Model of delta robot in Simulink MATLAB

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