1. F.Ezzahra KASSID¹, 2. Essaâdia AZELMAD², 3. Lahbib BOUSSHINE³, 4. Abdelmajid BERDAI⁴

National Higher School of Electricity and Mechanics of Casablanca (ENSEM), Hassan II University. Casablanca, (1,2,3 and 4) Morocco ORCID. 1. 0000-0002-3686-106X, 2. 0000-0002-05755, 3. 0000-0003-1019-215X, 4. 0000-0003-3960-885X

doi:10.15199/48.2021.10.06

Magnetic Force computation for a magnetic separator based on permanent magnet

Abstract. Suspended permanent magnet separator is widely used in many industrial fields. Magnetic force generated by the magnet should be higher than total value of all other competitor forces to attract the particle towards the magnet. In this paper, Approaching the value of magnetic force generated by the magnet and that required to pull the particle is of a great importance for optimum design of separator. a program simulating numerically nodal magnetic force generated by the magnet was proposed and compared with required magnetic force.

Streszczenie. Separator z magnesami trwałymi jest szeroko stosowany w wielu dziedzinach przemysłu. Siła magnetyczna generowana przez magnes powinna być wyższa niż całkowita wartość wszystkich innych sił konkurencji, aby przyciągnąć cząstkę do magnesu. W niniejszej pracy duże znaczenie dla optymalnego zaprojektowania separatora ma przybliżenie wartości siły magnetycznej generowanej przez magnes i potrzebnej do odciągnięcia cząstki. Zaproponowano program symulujący numerycznie węzłową siłę magnetyczną generowaną przez magnes i porównano go z wymaganą siłą magnetyczną. (**Obliczanie siły magnetycznej dla separatora magnetycznego wykorzystującegomagnesy trwałe**)

Keywords: magnetic force,magnetic separation,finite element method. Programming. **Słowa kluczowe**: separator magnetyczny, sioła przyciągania, magnesy trwałe

Introduction

Suspended magnetic separator devices are widely used in magnetic separation in many industries. They rely on permanent magnets to ensure the separation between ferromagnetic particles and the mixed fees.

The ability to generate enough force index and torque of rotation are very essential for a suspended magnetic separator based on permanent magnet.to judge its efficiency [1]. Knowing then, the magnetic force generated by the suspended magnet makes the control of those factors easier and allows to choose the optimum design of the separator from the wild range of manufactory products in the market.

Thereupon, the purpose of this paper is to present an algorithm based on finite element method that can be implemented and used in any commercial software. This algorithm allows to calculate the nodal magnetic force and helps choosing the optimum distance between the suspended magnet separator and the feeds of particles to achieve a better separation.

First, a light briefing about different ways of magnetic force calculation is presented. Then a presentation of the chosen method of virtual work and the algorithm to be followed is done.

Afterwards, the paper discusses how to calculate needed physical elements (*A.B*), then elaborates the importance of the 'layers' in achieving a good precision of the program, and later finishes by introducing some cases to compare the analytical results with the program findings.

Even if the model used for the entire whole work is a two dimensional parallelepiped permanent magnet. The program can be used for any shape of magnet and even with three dimensional elements.

Magnetic force

Magnetic force generated by the magnet and acting on the ferromagnetic particle can be calculated based on the equation below :

(1)
$$F = V_p(M.\Delta B)$$

where: *B*: magnetic induction; V_P : volume of the particle; *M*: magnetization

From the previous expressions , it can be concluded that:

- Magnetic force depends solely on the shape of the particle rather that its size and on its magnetic properties.

Also, a higher gradient of magnetic field or induction will generate a higher force. That was said, using a strong magnet helps increasing the force density and attract weakly magnetic particles.

One of the above equations can be used to verify mathematical accuracy of the program later.

Different methods

To simulate the magnetic force numerically, several methods had been presented and explained in many literatures, as equivalent magnetizing currents, Maxwell stress tensor, virtual work methods by using surface or volume integration. A Comparison of those methods and their results can be found in [2, 3,4]

It has been shown that methods using volume integration are the most accurate, (virtual work method, Maxwell stress tensor).

It was also shown that Virtual work method with volume integration can be done using the same type of mesh for distorted area. Thus, requiring less integration time as only one integration is needed for field computation and volume integration. Also accuracy of the local force is very high, therefore this method was used to develop the finite element analysis and program the force in this paper.

Virtual work Method

Magnetic force can be obtained based on the derivation of magnetic energy (W) or coenergy (W').. General expressions are as follows:

(2)
$$F = -\partial W / \partial s$$
 and $F = \partial W' / \partial s$

Based on magnetic energy ,Coulomb had identified the expression of the force in (2) .By using magnetic induction and the Jacobian of isoparametric elements the Formula is

(3)
$$F = -1/\mu 0 \sum_{ek} \int_{\Omega ek} \frac{\partial B}{\partial s} (B - Br) + \frac{\partial B}{\partial s} (B - Br) + (B - Br)(B - Br) \left| \frac{\partial G}{\partial s} \right|^{T} \left| \frac{\partial G}{\partial s} \right|^{T}$$

where: *B*: magnetic induction; *G*: Jacobian of mesh element *Br*: Remanent magnetic induction

Programming magnetic force by VWM Introduction

Global declaration section: where the model is presented, characteristic of the environment, magnet and particle are defined as the number of elements, nodes, magnetic induction, type of elements..... This section can be updated with each specific study case.

Main function: this section is considered as the heart of the program. It includes all equations and formulas related to the calculation of all necessary parameters.

Sub-program function: this section is only optional as it can be included in the Main function section.Besides, it is considered to be essential for a clean and a visible program creation.Mainly if there is a change in the type of elements and dimensions.(ex: quadratic instead of triangular). It includes all the necessary sub-functions for the final equation calculation as: Stiffens matrix ,Determinant ,Jacobian and Source Matrix.

Results view section: in this suction, graphs, tables and curves can be obtained to visualise the output of the program

Flow chart presentation

The steps of the simulating program are expressed in the following chart. The force is computed in two dimensions, yet the same flow chart can be used in three dimensions.

The Magnetic induction calculation was done using the vector potential method. This method was used for its accuracy and easiness to compute [5].Mathematic model will be presented later in the article.

Fig .1 presents a detailed flow chart expressing main

A detailed discussion regarding how to calculate every needed parameter of the program will follow starting by the magnetic induction

Vector potential:

Magnetic field vectors can be expressed in terms of either the magnetic field strength H or the magnetic induction B[3]. The field vectors are related by:

$$(4) B = \mu_m H$$

With the magnetic permeability μ_m .The proportionality constant. In a vacuum, the permeability has the value of

$$\mu_m = \mu_0.$$

When a magnetic field passes through a material, the material acquires an induced magnetization M, given by:

$$H.X_m = M$$

Magnetic field vectors can be expressed in terms of either the magnetic field strength H or the magnetic induction B[3] the field vectors are related by:

$$B = \mu m.H$$

With the magnetic permeability μm the proportionality constant.

The magnetic induction can also be expressed as

$$B = \mu_0 (H+M)$$

Eq (6),(7) and (8) show that

(9)
$$\mu_m = \mu_{0.}(l + \chi_m)$$

Which relates the permeability to the susceptibility. Following Maxwell 's equation for magnetostatic,magnetic field is expressed as follow: (10)

(11) $\nabla B = 0$

where J is the density charge.Via a magnetic vector potential approach. The flux density may be written in terms of the vector potential, A as

 $\nabla XH = J$



Fig .1. Flow chart presentation

 $BX\nabla = A$

As *A* is the vector potential. The vector potential is defined in a way that it's circulation integral round any closed loop is equal to the total flux enclosed by that loop. By using Lorentz gauge , the Alembert equation is as follow:

(13)
$$\mu J = \Delta A - \mu \frac{\partial^2 A}{\partial t^2}$$

In quasistaionary terms:

(14)
$$\mu J = \Delta A$$

Therefore, vector potential is expressed by the Poisson's equation formulation

Boundary value conditions

The permanent magnet is a suspended one in the air,it's flux lines are parallel to the boundary edge .Hence the Dirichlet conditions are the best to express this problem. Below the formulation:

A=0 at the permanent magnet space of study boundary

Domaine critisation :

Triangular elements are used in this case ,with the following interpolation equation :

(15)
$$\phi^{e}(x,y) = \sum_{j=1}^{3} N_{j}^{e}(x,y)\phi_{j}^{e}$$

By using variation formulation we get the expressions of the matrices

Set of matrices are represented as follow:

(16)
$$K_{i,j}^{e} = \int \left(\left(\frac{\alpha \, dN_{i}^{e}}{dx} \frac{dN_{j}^{e}}{dx} \right) + \beta N_{i}^{e} N_{j}^{e} \right) dx$$
$$b_{i}^{e} = \int N_{i}^{e} f dx$$
$$g_{i}^{e} = \alpha N_{i}^{e} d\phi / dx$$
$$(\phi^{e}) = \begin{pmatrix} \phi_{1}^{e} \\ \phi_{2}^{e} \end{pmatrix}$$
$$(K)(\Phi) = (b) + (g)$$

Afterwards, Magnetic induction can be computed by using the general definition of rotation under Cartesian coordinates.

$$rot A = B$$

Stiffness matrix based on isoparametric elements.

For the particle to be separated from the feed, it has to move towards the magnet after being held captive under its force of attraction. In terms of finite element discretization, the particle is going to bend the surrounding medium. And deform the shape of its elements.

This is why it is very important to work with isoparametric elements .Using the Jacobian is essential to calculate the transformation between natural coordinates and physical ones .Many expressions can be found in [3]

Importance of layers

As it was stated before, the movement of the particle deforms the space around .To describe the distorted area, many layers of free space surrounding the particle are used. Each layer thickness is one range of elements.

For one layer around the particle, nodal derivatives are assigned "1" for internal boundary and "0" for the external boundary.

In case of using many layers, the external boundary of the last layer is assigned to "0" and the internal boundary of the layer surrounding the particle is assigned to "1".for layers In between, values are assigned based on boundary layers methodology as expressed in below formula (16). The accuracy of obtained results depends mainly on the number of layers used. According to [6,7] ,the sensitivity to mesh quality can be greatly reduced and the accuracy can be improved by adding more layers.

So to present this in the program, a vector is needed as below:

(18)
$$(\frac{\partial xi}{\partial x})i = P_i$$

where: Pi : Colum matrix ; *i* :number of layers; *Size* : number of nodes of the Environment; Values =(1,2/i,3/i,,,,i-1/i)



Fig .2. Multi-layers surrounding the particle

Applications:

Case I:

To evaluate the effectiveness of the proposed program, the first example aims to calculate the magnetic potential generated by a magnet (PM) NdFeB.Its characteristics are taken directly from IBSMagnet catalog [8].

the magnet size is 6 mm X 6 mm X1 mm and the magnetization direction is set to be along the axis Y with Br = 1.4 tesla and μ m =1,45. Simulation of the magnet is done using femm [9]



Fig.3:Disribution of flux lines around the magnet

Fig.3 shows the distribution of magnetic flux lines around the magnet. The lines tend to reach the terminal and the pole spacing.The descritisation of the Finite element environment was done using 1353 nodes and 2560 elements. Fig. 4 shows the results of the variation of nodal magnetic vector potential obtained by Femm , MATLAB



Fig.4 : Trend of the magnetic potential around y direction

Fig.4 presents the tendency of the magnetic potential following the Y direction for both analytical and programmed values.

As it can be seen, the difference between the two curves is hardly perceptible. The estimated margin of error between both curves is less than 2%. Therefore, it can be considered that the proposed program is valid for magnetic potential.

Also program's results were verified with cases mentionned in [10].

It is clear that the values of the potential vector decrease as the distance from the magnet increases.

While the magnetic vector potential can be interpreted as potential energy per unit element of current, the direction of vector potential will determine the needed energy required by magnet to create an attraction force with the particle (Fig 6)

Case II:

The second case study is to calculate the magnetic force on one ferromagnetic part placed below a permanent magnet, as it's shown in Fig 5: Magnet dimensions are: 10 mm X 2mm X 1mm . Br =1,17T The iron paticle dimensions are: 2 mm X 2mm X 1mm

The magnetic force is calculated at the surrounding layers around the iron particle.



Fig.5 : Model structure



Fig.6 : Disribution of flux lines around the magnet

Fig .6 shows that lines are contours of equal magnetic induction in the verticle direction.Taking into consideration the variation of the magnetic induction between both poles of the magnet,this creates a field gradient between the poles.

Because that magnetic force is proportional to the product of the magnetic induction and the field gradient, this helps predicting the direction of the magnetic force.

Table 1 shows the results of magnetic force for each layer. Taking the average of 9 layers is enough to converge to the solution.

| Table 1. Nodal force variation based on layer's value | | | | |
|-------------------------------------------------------|--------|---------|--------|--|
| N of layers | Fy (N) | Fx (N) | F (N) | |
| 1 st Layer | 0,0050 | -0,0021 | 0,0054 | |
| 2 st Layer | 0,0135 | 0,0159 | 0,0209 | |
| 3 st Layer | 0,0090 | 0,0123 | 0,0152 | |
| 4 st Layer | 0,0067 | 0,0105 | 0,0125 | |
| 5 st Layer | 0,0054 | 0,0094 | 0,0108 | |
| 6 st Layer | 0,0045 | 0,0087 | 0,0098 | |
| 7 st Layer | 0,0038 | 0,0081 | 0,0089 | |
| 8 st Layer | 0,0034 | 0,0078 | 0,0085 | |
| Average (2-8) | 0.0066 | 0.0103 | 0.0123 | |

Table 1. Nodal force variation based on layer's value

The table below shows that the error value between the analytical and proposed method is only (2%) for the X direction and 3% for teh Y direction.As for the total force ,the values are the same for both methodes.

Table 2. Comparison of nodal force

| Nodal Force | classical virtual method | Used method | Analytical method |
|----------------|--------------------------|-------------|-------------------|
| Fx | -0,0021 | 0,0103 | 0,0105 |
| Fy | 0,0050 | 0,0066 | 0,0064 |
| F | 0,0054 | 0,0103 | 0,0123 |

The classical virtual work method computes the magnetic force by integration on one layer of elements around the objects. The proposed program allows using as many layers as needed to get more accurate results compared to the analytical values.

Even if the structure of the programme allows to calculate the magnetic force for any type of particle (paramagnetic, ferromagnetic,,,) based on the general formulas of the magnetic force.Another flowchart can be used by using force index .according to Hopstock . The magnetic force for ferromagnetic particle depends on the magnetic force index index till reaching the magnetic saturation. Afterwards ,only the gradient determines the force.

A helpful guide to program the force index using the vector potential for suspended magnetic separator has been presented in [11].

Conclusion :

This paper reviewed the different existing methods to calculate the magnetic force generated by a suspended magnetic separator using permanent magnet.

To move forward with proposing a program allowing the calculus of magnetic force by the mean of finite element method.

The program is relatively simple to implement and computationally inexpensive. Especially if it is used with macro structure. Its effectiveness depends on using the required number of layers around the particle.

The program proved its efficiency with many study cases using different number of elements (starting from 200 until 7000).

Furthermore, du to its subprogram function section , the program allows a quick switch to a different type of elements and gives the possibility to compare results for each simulation.

It worth mentioning also that although the used approach has been applied to a rectangular magnet, it's equally applicable to the analysis of the magnetic field and force index a calculation of a cylindrical magnet, by considering the correct formulas.

This is make for the program able to cover several types of magnetic separators and to help better understanding the real process of separation.

The force error estimation is relatively weak reaching a maximum of 3% compared with analytical results.

Knowing the required magnetic force to be generated by the magnet helps tremendously to calculate all the other competing forces to have more understanding to the environment of separation.

Thus, this program is not limited to help modelling the magnetic force generated by a magnetic separator, It can also be used to assist in the design phase of the separator to obtain optimal dimensions while simulating its force behaviour towards the magnetised particle under different circumstances.

Authors:

Fatima Ezzahra Kassid , National Higher School of Electricity and Mechanics of Casablanca (ENSEM), Hassan II University. Casablanca Morocco, Material and Structure Mechanics Team (MSMT), E-mail: fatima-ezzahra.kassid@ensem.ac.ma; Essaâdia Azelmad National Higher School of Electricity and Mechanics of Casablanca (ENSEM), Hassan II University. Casablanca Morocco, Material and Structure Mechanics Team (MSMT), Prof dr Lahbib Bousshine, National Higher School of Electricity and Mechanics of Casablanca Morocco (ENSEM), Hassan II University. Casablanca, Material and Structure Mechanics Team (MSMT), E-mail: Ibousshine@yahoo.fr;Prof Dr Abdelmajid Berdai, National Higher School of Electricity and Mechanics of Casablanca (ENSEM), Hassan II University. Casablanca Morocco , Electrical Systems and Energy Laboratory (LESE), E-mail <u>a.berdai@gmail.com</u>

REFERENCES

- [1] Svoboda, J., Magnetic Techniques for the Treatment of Materials, Kluwer Academic Publishers, USA, (2004)
- [2] Z. Ren; Comparison of Different Force Calculation Methods in 3D Finite Element Modelling, IEEE TRANSACTIONS ON MAGNETICS, VOL. 30, NO. 5. SEPTEMBER 1994
- [3] virtual work principle for magnetic or electric force and torque computation ,IEEE transactions on magnetics, vol. mag-20, no. 5, september 1984
- [4] J Coulomb ,A methodology for the derivation of global electromechanical quantities from a finite element analysis and its application to the evaluation of magnetic forces, torques and stiffness, IEEE transactions on magnetics, vol. mag-19, no. 6, november 1983
- [5] W. N. Fu and S. L. Ho ,Error Estimation for the Computation of Force Using the Virtual Work Method on Finite Element Models, IEEE TRANSACTIONS ON MAGNETICS, VOL. 45, NO. 3, MARCH 2009
- [6] Heinz E.Knoepfel,'Magnetic Field',pp 79-82.
- [7] A Benhama, Computation of electromagnetic forces from finite element field solutions, UMIST, UK, 1996
- [8] www.ibsmagnet.com/pdf/en/ibs_e.pdf
- [9] www.femm.info.
- [10] Finite element for electrical engineering, universitat Politecnica De Catlunya, Ramon bargallo, 2006
- [11] K. F, S. A, L. B and A. B, Force index computation for a magnetic separator based on permanent magnet, National Higher School of Electricity and Mechanics of Casablanca (ENSEM),CSF 2021 to be published, 2021