1. NEKROUF Sadek[1], 2. CHEKROUN Soufyane[1]

1Department of Electrical Engineering, University Mustapha Stambouli of Mascara.BP.305, Route El Mamounia, 29000 Mascara Algeria
ORCID: 1. 0000-0001-9637-5330, 2. 0000-0002-7394-5879

doi:10.15199/48.2021.11.16

Optimal controller design for a birotor helicopter

Abstract. *Robust control problem for a two degree of freedom (2-DOF) lab helicopter is investigated. The helicopter dynamics involves nonlinearity, uncertainties, and coupling. A new high performance bounded (HPB) linear quadratic regulator control law has been presented that extends classical LQR by providing faster settling times, eigenstructure to optimize its performance, and has much quicker computation times than classical LQR. The robust compensator is designed to restrain the effects of uncertainties, nonlinear properties, and disturbances.*
The simulation results on the 2-DOF lab helicopter demonstrate the effectiveness of the proposed control strategy.

Streszczenie. *Zbadano problem solidnego sterowania helikopterem laboratoryjnym o dwóch stopniach swobody (2-DOF). Dynamika helikoptera obejmuje nieliniowość, niepewności i sprzężenie. Przedstawiono nowe prawo sterowania liniowym regulatorem kwadratowym o wysokiej wydajności (HPB), które rozszerza klasyczną LQR, zapewniając szybsze czasy ustalania, strukturę własną w celu optymalizacji jego działania i ma znacznie szybsze czasy obliczeń niż klasyczne LQR. Wytrzymały kompensator jest przeznaczony do ograniczania skutków niepewności, właściwości nieliniowych i zakłóceń. Wyniki symulacji na śmigłowcu laboratoryjnym 2-DOF pokazują skuteczność proponowanej strategii kontroli.* (***Optymalna konstrukcja kontrolera dla dwuwirnikowego helikoptera)***

Keywords: optimal control, robust LQR control, MIMO System, 2-DOF lab helicopter.
Słowa kluczowe: helikopter, sterownik, system MIMO.

Introduction

Unmanned helicopters have gained much attention due to their versatile functions in remote sensing, surveillance, mineral exploration, etc. The difficulties in attitude and position controller design for helicopters, in general, stem from their particular features, such as nonlinearity, coupling, and uncertainties, and have been focused on by many researchers during the last two decades.

Many theoretical techniques were already proposed to solve these problems. The article [1] presents the techniques of the modal control, and in [2],[6]), we find the Gaussian quadratic linear LQG/LTR control, the control by fuzzy logic and in [3] a non-linear H∞ approach used for handling the system coupling considering it as a disturbance that should be rejected. In [4] it is considered a sliding mode control by defining a sliding surface.

In recent years the interest for robust methods has increased. LQR is a collection of methods that try to tackle the challenging problem of nonlinear control in a divide and conquer manner.

Recently, [5] proposed a new weights selection method called ZED (ZEro addition Decoupling method) which yields desirable closed-loop responses. The method is based on asymptotic modal properties of multivariable LQR a control weights tend to zero, and it is more practical than the inverse LQ method [6] which is a practical method to design LQR with desired closed-loop properties.[7] presented a new way of pole placement in the LQR via selecting weighting matrices which gives desired closed-loop locations. One feature of the proposed method is that the weighting matrix is obtained by solving differential equations which are derived from the characteristic equation of a Hamilton matrix, and has a diagonal form.

A new control system design algorithm which has the advantages of the existing LQR and the conventional eigenstructure assignment scheme is proposed here. The method of a transformation via block controller is utilized to develop the scheme.

To illustrate this idea, the control of flight simulator is studied. Which in a first stage, we present the formulation of the model for the study of the dynamics of the system.

System Modeling

To develop the dynamic model we used a direct method based on the calculation of the forces acting on the body of the simulator [8].

The non-linear model results in a set of six non-linear differential equations.

$$\frac{di_{ah/v}}{dt} = -\frac{R_{ah/v}}{L_{ah/v}} i_{ah/v} - \frac{k_{ah/v}\,\varphi_{h/v}}{L_{ah/v}}\omega_{h/v} + \frac{k\,1/2}{L_{ah/v}} u_{h/v} \tag{1}$$

$$\frac{d\,\omega_{ah/v}}{dt} = -\frac{k_{ah/v}\,\varphi_{h/v}}{J_{tr/mr}} i_{ah/v} - \frac{B_{tr/mr}}{J_{tr/mr}}\omega_{h/v} - \frac{f_{1/4}\left(\omega_{h/v}\right)}{J_{tr/mr}} \tag{2}$$

$$\begin{aligned}\frac{d\Omega_h}{dt} &= \frac{l_t f_2(\omega_h)\cos\theta_v - k_{oh}\Omega_h - f_3(\theta_h)}{D\cos^2\theta_v + E\sin^2\theta_v + F} \\ &+ \frac{k_m\omega_v\sin\theta_v\Omega_v\left(D\cos^2\theta_v - E\sin^2\theta_v - F - 2E\cos^2\theta_v\right)}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)^2} \\ &+ \frac{k_m\cos\theta_v\left(k_{av}\varphi_v i_{av} - B_{mr}\omega_v - f_4(\omega_v)\right)}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)J_{mr}}\end{aligned} \tag{3}$$

$$\frac{d\,\theta_h}{dt} = \Omega_h \tag{4}$$

$$\begin{aligned}\frac{d\,\Omega_v}{dt} &= \frac{l_m f_5(\omega_v) + k_g\Omega_h\,\mathrm{f}_5(\omega_v)\cos\theta_v - k_{ov}\Omega_v}{J_v} \\ &+ \frac{g\left((A-B)\cos\theta_v - C\sin\theta_v\right) - 0.5\Omega_h{}^2 H\sin 2\theta_v}{J_v} \\ &+ \frac{k_t\left(k_{ah}\varphi_h i_{ah} - B_{tr}\omega_h - f_1(\omega_h)\right)}{J_v J_{tr}}\end{aligned} \tag{5}$$

$$\frac{d\,\theta_v}{dt} = \Omega_v \tag{6}$$

where:

$\theta_{h/v}$- is the azimuth /pitch angle of beam (horizontal/vertical plaen)

$J_{tr/mr}$ – is the movement of inertia in motor tail/main propeller subsystem

$K_{ah/v}\,\varphi h_{/v}$ - is the torque constant of the tail main motor

$u_{h/v}$ - is the input voltage of the tail/main motor

J_v - is the moment of inertia about the horizontal ax

m_{ms}, m_{ts} – are the masses of the main and tail shields
m_m, m_t – are the masses of the main and tail parts of the beam
m_{mr}, m_{tr} – are the masses of the main and tail DC motor with main and tail rotor
m_b, l_b - are the mass and the length of the counter weight beam
m_{cb}, l_{cb} – represent the mass of the counter weight and the distance between the counter weight and the joint
r_{ms}, r_{ts} – *are the radius of the main and tail shield*
$\Omega_{h/v}$ - is the angular velocity around the vertical/horizontal axis
$\omega_{h/v}$ - is the rotational velocity of the tail/main motor

The nonlinear functions fi into account the frictions and coupling effects between horizontal/vertical dynamics, are defined as follows:

(7) $f_1(\omega_h) = sign(\omega_h) k_{th} \omega_h^2$

(8) $f_2(\omega_h) = \begin{cases} k_{fhp}\omega_h^2 & if \quad \omega_h \ge 0 \\ -k_{fhn}\omega_h^2 & if \quad \omega_h < 0 \end{cases}$

(9) $f_3(\theta_h) = \begin{cases} k_{chp}\theta_h & if \quad \theta_h \ge 0 \\ k_{chn}\theta_h & if \quad \theta_h < 0 \end{cases}$

(10) $f_4(\omega_v) = sign(\omega_v) k_{tv} \omega_v^2$

(11) $f_5(\omega_v) = \begin{cases} k_{fvp}\omega_v^2 & if \quad \omega_v \ge 0 \\ -k_{fvn}\omega_v^2 & if \quad \omega_v < 0 \end{cases}$

(12) $f_6(\theta_v) = \begin{cases} k_{cvp}\left(\theta_v - \theta_v^0\right)^2 & if \quad \theta_v \ge \theta_v^0 \\ k_{cvn}\left(\theta_v - \theta_v^0\right)^2 & if \quad \theta_v < \theta_v^0 \end{cases}$

Where the input vector is $u = [u_h, u_v]^T$ and state vector is $x = [\omega_h, \Omega_h, \theta_h, \omega_v, \Omega_v, \theta_v]^T$

The constants of the non-linear model are defined as:

$l_1 = \left(\frac{m_t}{2} + m_{tr} + m_{ts}\right) \mathrm{l_t}$,

$l_2 = \left(\frac{m_m}{2} + m_{mr} + m_{ms}\right) \mathrm{l_m}$, $l_3 = \left(\frac{m_b}{2} \mathrm{l_b} + \mathrm{m_{cb}} l_{cb}\right)$

$l_4 = \frac{\mathrm{m_b}}{3}\mathrm{l_b^2} + \mathrm{m_{cb}}\mathrm{l_{cb}^2}$

$l_5 = \left(\frac{\mathrm{m_m}}{3} + \mathrm{m_{mr}} + \mathrm{m_{ms}}\right)\mathrm{l_m^2} + \left(\frac{\mathrm{m_t}}{3} + \mathrm{m_{tr}} + \mathrm{m_{ts}}\right)\mathrm{l_t^2}$

$l_6 = \mathrm{m_{ms}}\,\mathrm{r_{ms}^2} + \frac{\mathrm{m_{ts}}}{2}\mathrm{r_{ts}^2}$ $\qquad l_7 = Al_t + Bl_m + \frac{m_b}{2}\mathrm{l}_b^2 + m_{cb} l_{cb}^2$

Controller Conception

The conventional LQR problem is to find the optimal control input u that minimizes the following cost function under the constraints of positive semidefinite symmetric Q and positive definite R matrices [13-14-15]

(13) $$\frac{1}{2}\int_0^\infty \left(x^T Q x + u^T R u\right) dt$$

The gain matrix $K = R^{-1}B^T P$ of LQR can be obtained by solving the following matrix Riccati equation

(14) $$PA + A^T P - PBR^{-1}B^T P + Q = 0$$

If closed-loop eigenstructure information is given, however, the gain matrix can be calculated using it [13]. Let Ac be the closed-loop system matrix, then $A_c = A - BR^{-1}B^T P$ and $BR^{-1}B^T P = A - A_c$ From the above equation, if the rank of control input matrix B is N, the solution of the matrix Riccati equation is obtained by $P = \left(BR^{-1}B^T\right)^{-1}(A - A_c)$ in order to overcome the rank deficiency problem of the control input matrix B, by

$$\bar{B} \rightarrow \begin{bmatrix} I_M \\ 0 \end{bmatrix}$$

where I_m and 0 denote the (m × m)-dimensional identity matrix and (N - m)×m-dimesional zero matrix, respectively. The transformation can be achieved by using the following transformation matrix

(15) $$T = \begin{bmatrix} B & \tilde{T} \end{bmatrix}$$

Then, the given open-loop system can be transformed as follows:

$$\bar{A} = T^{-1}AT$$

(16) $$\bar{B} = T^{-1}B = \begin{bmatrix} I_M \\ 0 \end{bmatrix}$$

the gain and weighting matrices for the transformed system should be inversely transformed to those of the original one, respectively

Consider (16) the resulting closed-loop system becomes: $\dot{x} = \left(\bar{A} - \bar{B}K\right)x$

Design Algorithm

The entire procedure for the proposed algorithm is summarized below.

Step 1: The given system is transformed into the following block controller form using the transformation matrix T mentioned in Section II as follows:

(17) $$\bar{A} = T^{-1}AT$$

(18) $$\bar{B} = T^{-1}B = \begin{bmatrix} I_m \\ 0 \end{bmatrix}$$

Step 2: Determine the desired eigenvalues (λ_i) and corresponding right eigenvectors $\left(\phi_i^d\right)$.

Step 3: Find the following matrices

(19) $$S_{\lambda_i} \equiv \left[\lambda_i I_N \bar{A} \middle| \bar{B}\right],$$

(20) $$\mathrm{R}_{\lambda_i} \equiv \left[\frac{N_{\lambda i}}{M_{\lambda i}}\right].$$

Where the columns of the matrix R_{λ_i}, form a basis for the null space of S_{λ_i}.

Step 4: Construct the augmented achievable right modal matrix Φ_{aug}^a.

Step 5: Determine the coefficient vector $\bar{P}_i$ which yields the desirable right modal matrix Φ^d in Step 2 and the augmented achievable right modal matrix in step 4.

Step 6: Construct the achievable right modal matrix Φ^a

(21) $$\Phi^a = \Phi_{aug}^a \bar{P}$$

Step 7: Calculate vector chains and construct the matrix Was follows:

(22) $$w_i = M_{\lambda_i} \overline{P}_i W = [w_1, \ldots, w_i, \ldots, w_N].$$

Step 8: Calculate the state feedback gain matrix which yields the achievable right modal matrix Φ^a and matrix W satisfying following equation:

(23) $$\mathrm{K} = \mathrm{W}\left(\Phi^a\right)^{-1}$$

Step 9 Construct the following blocked unique solution P of the algebraic Riccati equation:

(24) $$\mathrm{P} = \begin{bmatrix} RK_1 & RK_2 \\ (RK_2)^T & I_m \end{bmatrix}$$

Step 10: Calculate each block of the weighting matrix Q using the obtained P matrix in Step9.

Step 11: Check the result of the previous step whether the matrix Q satisfies the conditions or not. If it is not satisfied, try again Step 10 with another T. If it is satisfied, go to the next step.

Step12: the gain and weighting matrices for the original system are obtained by the following relations:

(25) $$K_0 = KT^{-1} \qquad Q_0 = T^{-T} Q T^{-T}$$

The system under consideration is a linearized two-input third-order continuous controllable system as follows:

(26) $$x(t) = Ax(t) + Bu(t)$$

The eigenvalues of the open-loop system are $\Lambda^{open} = \{2, -2 \pm i\}$.

Let the desired eigenvalues of the closed-loop system be $\Lambda^d \{\lambda_1, \lambda_2, \lambda_3\} = \{-3, -4, -1\}$. The desired right modal matrix Φ^d and its normalized form $\Phi^d{}_{nor}$ are selected arbitrarily as follows:

(27) $$\Phi^d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \Phi^d{}_{nor} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

According to the design procedure of the proposed algorithm, the achievable normalized right modal matrix Φ^d_{nor} can be achieved in the least-square sense as follows:

(28) $$\Phi^d{}_{nor} = \begin{bmatrix} 0.5031 & 0 & -0.4300 \\ 0 & 1 & 0 \\ -0.5031 & 0 & 0.9070 \end{bmatrix}$$

However the desired closed-loop eigenvalues are assigned exactly.

The feedback gain matrix K and the weighting matrices Q and R can be obtained as follows:

$$K = \begin{bmatrix} -13.1101 & 0.1696 & -2.1936 & -0.9525 \\ -4.1811 & 0.2015 & -15.229 & 3.6610 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & -0.5687 \\ -0.5687 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1312.8 & 62 & -1279 & 247.4 \\ 62 & 0.7 & 14.7 & 41.2 \\ -1279 & 14.7 & 137.2 & -158.1 \\ 247.4 & 41.2 & -158.1 & 12.9 \end{bmatrix}$$

Simulation Results

The controller is applied to the model of the helicopter platform obtained as shown in section II, to control the elevation and azimuth trajectory.

The following results obtained by delivering a step of 0.48 rad for elevation, for azimuth we applied a step of 0.6 rad (fig 1).

Disturbance of magnitude -0.05 rad to the control input u(t) at t = 50s for robustness test.

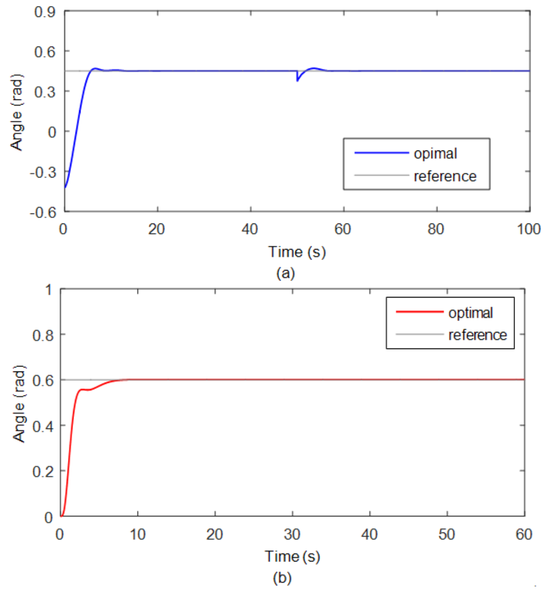

Fig.1. (a, b) show the elevation and azimuth behavior with the above mentioned controller,

In fig. 2 square reference in elevation and Azimuth simulations of closed loop performance, with initial values $\theta_h(0) = 0$ rad and $\theta_v(0) = -0.45$ rad

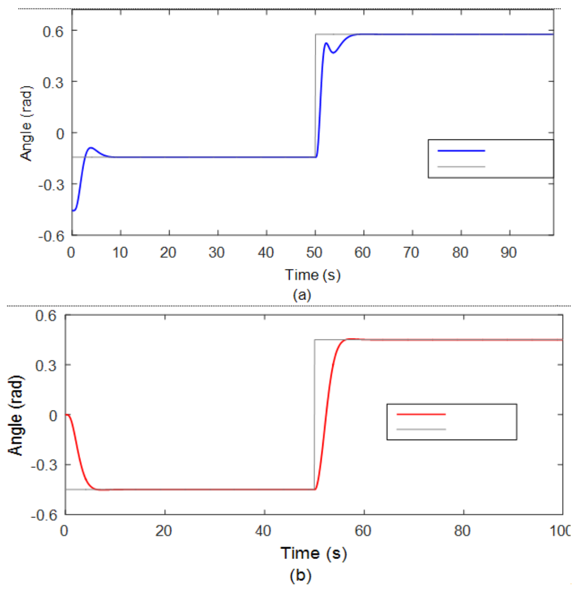

Fig.2. (a, b) square reference for the elevation and azimuth behavior with the above mentioned controller

A sinusoidal reference in elevation and Azimuth simulations of closed loop, with amplitude = 0.45 rad, frequency= 0.05 hertz for elevation and amplitude = 0.25 rad, frequency=0.1 hertz for azmuth, the initial conditions are:

$\theta_h(0)=0, \theta_v(0)=-0.45$rad.

It should be noted that the controller presented here can stabilize the system to a change of reference, and improve system performance and robustness in the presence of the disturbance and time-varying uncertainty.

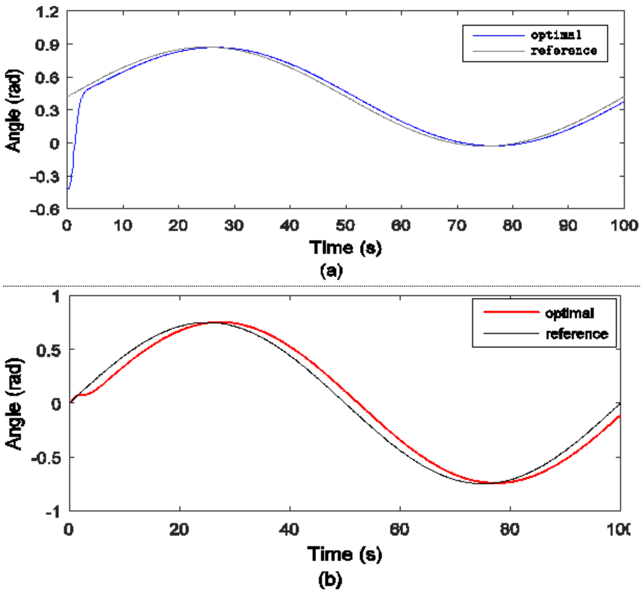

Fig.3. (a, b) Sinusoidal response for the elevation and azimuth with the above mentioned controller,

Conclusion

In this paper, we proposed a new LQR design algorithm, which guarantees that the desired eigenvalues are assigned exactly, and corresponding desired eigenvectors are assigned in the least square sense according to the conditions of the given system. Simulation results show the effectiveness and performance of the proposed controller even under disturbance and the cross coupling.

***Authors**:*
Dr NEKROUF Sadek, Department of Electrical Engineering, University Mustapha Stambouli of Mascara. BP.305, Route El Mamounia, 29000 Mascara Algeria,
E-mail: s.nekrouf@univ-mascara.dz;

Dr CHEKROUN Soufyane, Department of Electrical Engineering, University Mustapha Stambouli of Mascara. BP.305, Route El Mamounia, 29000 Mascara Algeria, E-mail: s.chekroun@univ-mascara.dz;

REFERENCES

[1] P. Apkarian, C.Champetier and J.F.Magni. Design of a helicopter output feedback control law using modal and structured-robustness techniques. Int. J. Control, 1989, Vol.50, No.4, 1195-1215.
[2] Mammar S. Robust multivariable control by the LQG/LTR and H∞ Application to a helicopter. Thesis. University of Southern Paris centers of Orsay 1990.
[3] McLean D and H Matsuda. Helicopter station keeping: comparing LQR, fuzzy-logic and neural net controllers. Engineering Application of Artificial Intelligence 11(1998) 411-418.
[4] Edgar N. Sanchez a, Hector M. Becerra a,, Carlos M. Velez: Combining fuzzy, PID and regulation control for an autonomous mini-helicopter, Information Sciences 177 (2007) 1999–2022
[5] Postlethwaite I, E.Prempain, E.Turkoglu, M.C. Turner, K Ellis and A.W. Gubbels: Design and flight of various H∞ controllers for the Bell 205 helicopter Control Engineering practice. Vol 13, No 3(2005) 383-398.
[6] Jih-Gau Juang , Ren-Wei Lin, Wen-Kai Li: Comparison of classical control and intelligent control for a MIMO system Applied Mathematics and Computation 205 (2008)
[7] Edgar N. Sanchez a, Hector M. Becerra a, Carlos M. Velez (2007) "Combining fuzzy, PID and regulation control for an autonomous mini-helicopter" Information Sciences 177 1999–2022
[8] Luo C.C, R.F Liu, C.D Yang and Y.H Chang. (2003) "Helicopter H∞ control design with robust flying quality". Aerospace Science and Technology 7 159–169.
[9] Roushandel. A, A. Khosravi, A. Alfi, A Robust LMI-Based Stabilizing Control Method for Bilateral Teleoperation Systems,International Review of Automatic Control, Vol. 6 N.2,March 2013
[10] McLean D and H Matsuda. (2018) "Helicopter station keeping: comparing LQR, fuzzy-logic and neural net controllers". Engineering Application of Artificial Intelligence 11411-418.
[11] Feedback Instruments Ltd.Twin Rotor MIMO System Advanced Technique Manual, Crowborough, U.K.
[12] Ramalakshmil. AP.S. , P.S. Manoharan (2012) Non-linear Modeling and PID Control of Twin Rotor MIMO System IEEE International Conference on Advanced Communication Control and Computing Technologies (ICACCCT)
[13] Hassan J and J, S. Shamma (2017) Design Of Real-Time Implementable Distributed Suboptimal Control: An LQR Perspective.IEEETransactions on Control of Network Systems
[14] Li Sun, (2018) Research on Multi Model LQR Control Method of Three DOF Helicopter, 10th International Conference on Intelligent Human-Machine Systems and Cybernetics.
[15] Thomas I, Femi T, Mija S.J, (2020), Trajectory Tracking of Unmanned Helicopter Using Optimized Integral LQR Controller, Fourth International Conference on Inventive Systems and Control (ICISC). Publisher: IEEE.