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doi:10.15199/48.2021.04.08

LQGi/LTR controller with integrators and feedforward controller applied to a Twin Rotor MIMO System

***Abstract**. This paper presents a comparison between LQGi and LQGi/LTR control of a Twin Rotor MIMO System (TRMS)(made by feedback instrument company). The Linear Quadratic Gaussian (LQG) means a linear quadratic regulator with KALMAN filter. LQG/LTR is an LQG controller with Loop Transfer Recovery. For both we introduced an integrator to deal with steady state errors and disturbances. A feed forward controller has been added to improve the tracking performances of the system. The results are in simulation and in real time*

***Streszczenie.** Przedstawiono porównanie sterowania LQG I LQG/LYR systemu MIMO z dwoama wirnikami TRMS. W sterowaniu LQG wykorzystano filtr Kalmana. Dodatkowo wprowadzono układ całkujący oraz sterownik typu feed forward. Sterowanie analizowano przez symulacje oraz eksperymentalnie (**Sterowniki LQGi/LTR z integratorem i układem feedforward do sterowania systemu MIMO z podwójnym wirnikiem**)*

Keywords: Linear Quadratic Gaussian, loop transfert recovery, Kalman filter, Twin Rotor MIMO System, Integrator.
Słowa kluczowe: układ z podwójnym wirnikiem, sterowanie LQG/LTR, filtr Kalmana.

Introduction

Unmanned helicopters are one of the most important Unmanned Aerial Vehicles (UAVs), so lot of researches are oriented to this field including control and vision. The “Twin Rotor MIMO System” is a laboratory set-up that simulates the dynamic of a helicopter; we will use it to validate our control experiments.

In the literature many works have been done on this prototype such as [1], where they used a cross PID control technique. An LQR controller using modal analysis has applied in [2], where he presents experimental results. In addition, a hierarchical control architecture, which combines a baseline feedback controller with an Iterative Learning Control algorithm, was developed and applied on this system in [3].

A comparison study between the LQG control and LQG/LTR [4] is presented in order to acquire the impact of the loop transfer recovery; this method was also applied in discrete time control and on a system similar to our one in [5]. This technique was also applied on several system for several purpose, [6] used it to control a three-phase voltage source converter (VSC) system acting as a rectifier. [7] Also used it to control a multivariable active suspension system, and he compared it with LQR and with poles placement control in regulation and the results was in simulation; and more other works[8], [9], [10].

The current work is organized as follow: firstly, the control law was presented as notes and we presented also the model of the plant; secondly we exhibited the results in simulation and experimentations with interpretation of these results; and we finished with a conclusion.

Control law

The Gaussian linear command called LQG is a method that allows calculating the gain of a control by state feedback in a particular concern to reduce the white noises. It is a combination of the two optimal commands the LQR (Linear Quadratic regulator) and LQE (linear quadratic estimator, Kalman estimator) that can be calculate independently according to the principle of separation [13].

Linear quadratic regulators

Calculating the LQR is based on minimizing the coast function below

(1) $$J\ \ u(.)\ =\int_0^{\infty}\left[x^TQx+u^TRu\right]dt,....Q\geq 0,R\succ 0$$

where Q is (n*n) matrix, it represents how bad of penalty it is if “x” is closed to the target. R is (m*m) matrix, it is the picture of the energy to spend to get these performances

This Q and R lead us to best K that minimizes J

The feedback control law that stabilize our system and minimize the cost function above is

(2.2) $$u=-K.x$$

The calculating of the gain K is well detailed in [14] using the Riccatie Equation.

Kalman filter

Kalman filter is basically the analog of the LQR for observation; it is an optimal full state estimator; the input of the Kalman filter is the control signal and the output of the system, and its output of the kalman filter is the estimated state vector $\hat{x}$.

We give it the type of disturbances and type of measurement noise caused by the sensors.

This kalman filter minimizes the cost function below:

(2.3) $$J=E((X-\hat{X})^T(X-\hat{X}))$$

So we want to find a Kf that minimize the error between X and $\hat{X}$.

Below the equation that we use to calculate the estimated state:

(2.4) $$\hat{x}=A\hat{x}+Bu+K_f(y-C\hat{x})$$

Where: Kf is the KALMAN filter gain.

We use the same linear algebra to solve for this Kf (Riccatie equation)[14].

Remember that the controller we designed before need all the states and we used this kalman filter to estimate the non-measurable states, so the estimated states and the control law will be gather in the following equation:

(2.5) $$\dot{\hat{x}}=(A-BK-K_fC)\hat{x}+K_fy$$
$$u=-K\hat{x}$$

Loop transfer recovery LTR

Both Kalman filter and LQR problems have robust performance and robust stability characteristics- they has excellent stability margins, but the blending of these two techniques does not have any guarantee to keep their robustness.

To solve this problem, we need to continue design with loop transfer recovery (LTR) approach to have the same performances as LQR or Kalman filter problem.[15]

We cannot apply this technique on every system, it is allowed only for systems that verify the following hypothesis: a proper system (D=0), C(SI- A)-1B is a minimum phase system to get a full-state loop transfer properties recovered[16]. Also when we apply it on a system, the dynamics of this system are inversed and that's why LQG/LTR procedure is not valid for non-square plant[17].

There are two type of recovering: plant input recovering – in the LQR controller –or plant output recovering- in the LQE estimator- [18]. In our case, we will do recovering in output of the plant, because we have confidence in the actuator robustness. Therefore, we must design the Kalman filter at first and then we move to the LQR regulator.

Some adjustments on Q matrix are being as follow:

$$Q = Q_0 + qC^T C \qquad\qquad R = R_0 \tag{2.6}$$

And we increase the parameter q until the loop transfer Kf(s)G(s) of the LQG corrector converged sufficiently closed to C(sI-A)-1Kf to recover all the robustness margin that we lost when combining the Kalman filter with the LQR regulator.

Plant model

The modeling of the plant used here follows the same as in [11], using the Newtonian method. The TRMS mechanical unit consists of two rotors placed on a beam together with a counterbalance. The whole unit is attached to the tower allowing for safe helicopter experiments. Fig.1

Apart from the mechanical units, the electrical unit (placed under the tower) plays an important role for TRMS control. It allows for measured signals transfer to the PC and control signal application via an I/O card. The mechanical and electrical units provide a complete control system setup.[12]

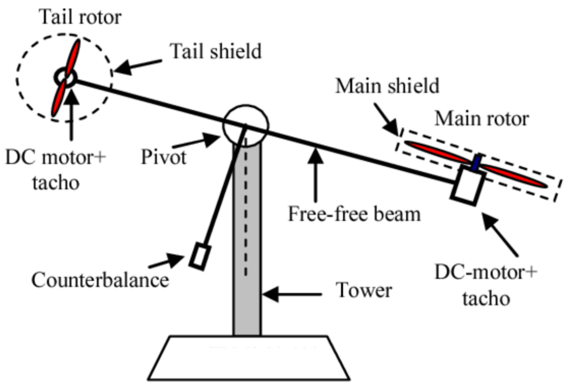

Fig.1. Twin Rotor MIMO System (TRMS)

Here is the form of the model

$$\dot{x} = A\ x\ + B\ x\ u \tag{2.7}$$

As in the first paragraph, in [11] the steps of modeling are well presented and well detailed, so we will ignore these details and we will present below the final equation that contains the non-linear model

$$\begin{cases}
\frac{d}{dt}\psi = \dot{\psi} \\
\frac{d}{dt}\dot{\psi} = \frac{1}{I_1}\left[a_1\tau_1^2 + b_1\tau_1 - M_G \sin(\psi) - B_{1\psi}\dot{\psi} - B_{2\psi} sign(\psi) - K_{gy}\cos(\psi)\dot{\phi}(a_1\tau_1^2 + b_1\tau_1)\right] \\
\frac{d}{dt}\phi = \dot{\phi} \\
\frac{d}{dt}\dot{\phi} = \frac{1}{I_2}\left[a_2\tau_2^2 + b_2\tau_2 - B_{1\phi}\dot{\phi} - B_{2\phi} sign(\phi) - K_c 1.75(a_1\tau_1^2 + b_1\tau_1)\right] \\
\frac{d}{dt}\tau_1 = -\frac{T_{10}}{T_{11}}\tau_1 + \frac{k_1}{T_{11}}u_1 \\
\frac{d}{dt}\tau_2 = -\frac{T_{20}}{T_{21}}\tau_2 + \frac{k_2}{T_{21}}u_2
\end{cases} \tag{2.8}$$

Bellow the state and the output vectors:

$$\boldsymbol{x} = [\boldsymbol{\psi}\dot{\boldsymbol{\psi}}\boldsymbol{\varphi}\dot{\boldsymbol{\varphi}}\boldsymbol{\tau}\mathbf{1}\boldsymbol{\tau}\mathbf{2}]^{\boldsymbol{T}} \qquad \boldsymbol{y} = [\boldsymbol{\psi}\boldsymbol{\varphi}]^{\boldsymbol{T}}$$

where:

$\boldsymbol{\psi}$: *Pitch* (*elevation*)

$\boldsymbol{\tau}\mathbf{1}$: the *principal* torque.

$\boldsymbol{\varphi}$: *Yaw*(*azimuth*).

$\boldsymbol{\tau}\mathbf{2}$: the tail torque.

We want to change the form of our model from the one presented above in equation (2.8) to the linear form bellow:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{2.9}$$

where

States : $x \in \mathbb{R}^6$ outputs : $y \in \mathbb{R}^2$ and inputs : $u \in \mathbb{R}^2$

We can linearize our system using the Taylor approximation, we proceed to cancel all the derivatives of our state vector $\dot{x} = 0$ with $u1 = u2 = 0$, we thus obtain the point of equilibrium on which we have opted :

$$\begin{cases} x_{10} = 0 \\ x_{20} = 0 \\ x_{30} = 0 \\ x_{40} = 0 \\ x_{50} = 0 \\ x_{60} = 0 \end{cases}$$

The matrixes A,B,C can be obtained by applying 'Jacobian matrix method'-using matlab

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{M_G}{I_1} & -\frac{B_{1\psi}}{I_1} & 0 & 0 & \frac{b_1}{I_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{B_{1\phi}}{I_2} & -1.75K_c\frac{b_1}{I_2} & \frac{b_2}{I_2} \\ 0 & 0 & 0 & 0 & -\frac{T_{10}}{T_{11}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{T_{20}}{T_{21}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{k_1}{T_{11}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{k_2}{T_{21}} \end{bmatrix}^T \tag{2.10}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

A table in [12] contains all the TRMS model parameters such B1,B2,K2, K2,T10…

Simulation results

The nonlinear model in (2.8) is used to simulate the controller to validate the effectiveness of this controller; the schema of implementation presented in figure 2. First, we will give the model a step signal to test its performances in regulation. A pre-filter has been putted after the reference signal source to get a smooth input and to avoid the sudden changes. Secondly, we will test its performances in trajectory tracking by giving the input a sinusoidal signal.

These tests were given with the LQG regulator then with the LQG/LTR one to see the difference between them.

The structure of the control design

The schema bellow presents the structure of controller implementation:

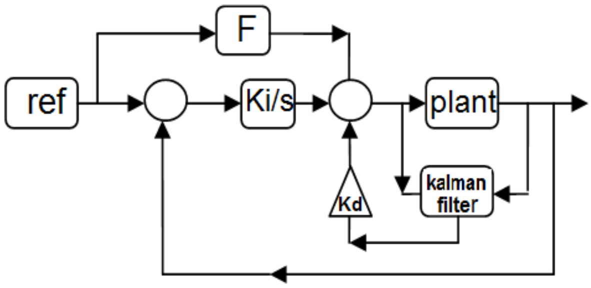

Fig.2. The structure of the controller LQG with the integrator and the feedforward filter

- F is the feedforward controller-model inverter
- Kd is state space feedback
- Ki is the output feedback
- Ref is the reference signal.
- U,Y input and output signals.
-

Regulation

The model of plant is fed by a step signal with amplitude of 0.4rad for the pitch and 0.8rad for the yaw to test its performances in regulation with disturbances rejection.

For every graph bellow, the blue continues line represents the reference and the red discontinues line represents the response, and the control signal of the system is presented for all experiences to make the comparison more efficient.

LQG:

The pitch and the yaw responses for the linear quadratic Gaussian control law.

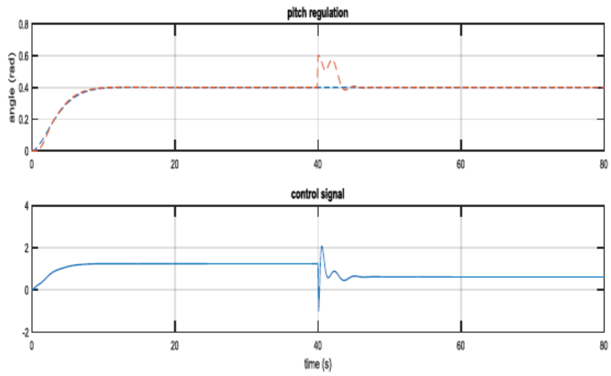

Fig. 3 LQG response for pitch angle regulation

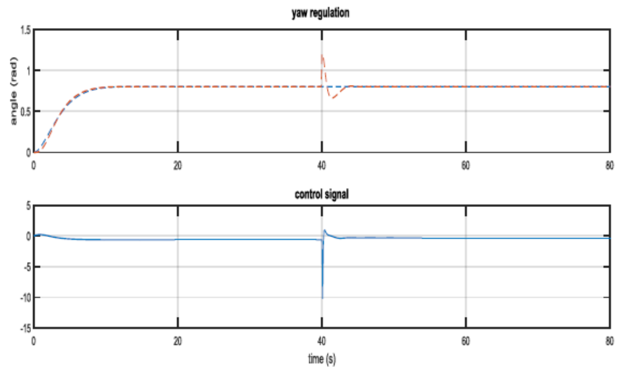

Fig. 4 LQG response for yaw angle regulation

LQG/LTR:

The pitch and yaw responses for the linear quadratic Gaussian control with law loop transfer recovery.

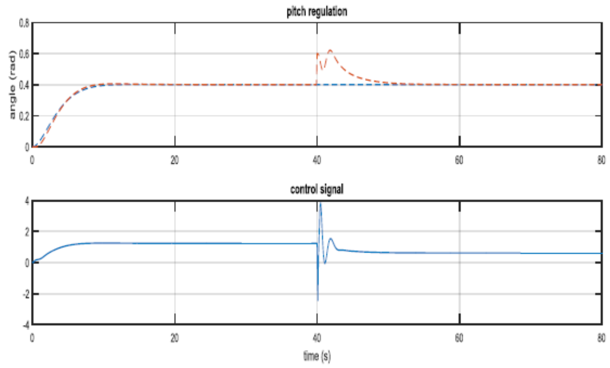

Fig. 5 LQG/LTR response for pitch angle regulation

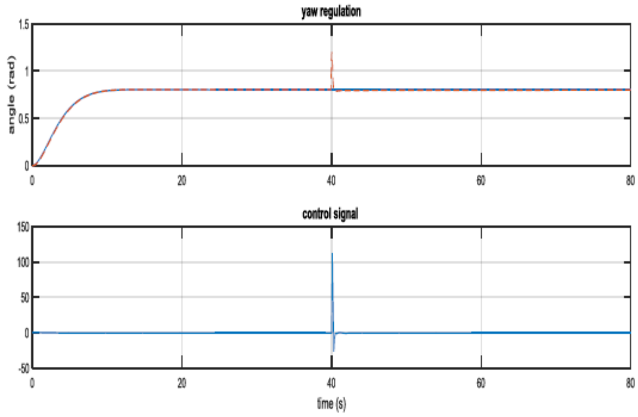

Fig. 6 LQG/LTR response for yaw angle regulation

For pitch angle regulation, the transient and permanent regimes are excellent for both (LQG and LQG/LTR): they reject a disturbance of 50% of the reference, and with a smooth and optimal control signal. For both there is no delay between the input and the output. The robustness of the LTR technique against disturbances was present for yaw and it made the difference; the LQG took 4 seconds to reject totally the perturbation, but the LTR took only 1 second, so the difference between the two methods appeared clearly.

Trajectory tracking

In this step we give to the input a sinusoidal signal centered in 0.4 rad with amplitude equal to 0.2 rad and frequency equal to 1rad/sec for the pitch angle, and amplitude of 0.4 with a frequency equal to 1rad/sec for yaw. These two types of inputs to prove that these controllers are designed for multi-objective control: regulation and tracking

LQG:

The response with the LQG controller

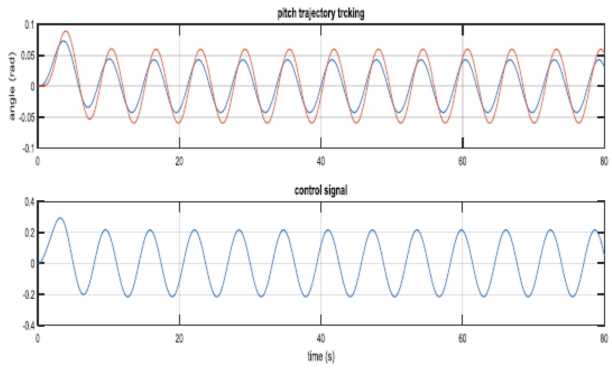

Fig. 7 LQG response for pitch angle tracking

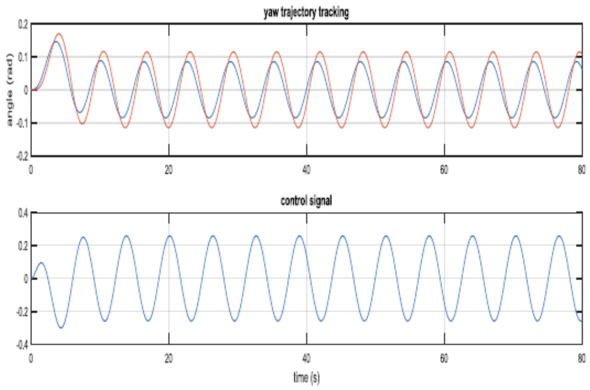

Fig. 8 LQG response for yaw angle tracking

LQG/LTR:

Test of TRMS tracking with LQG/ LTR regulation

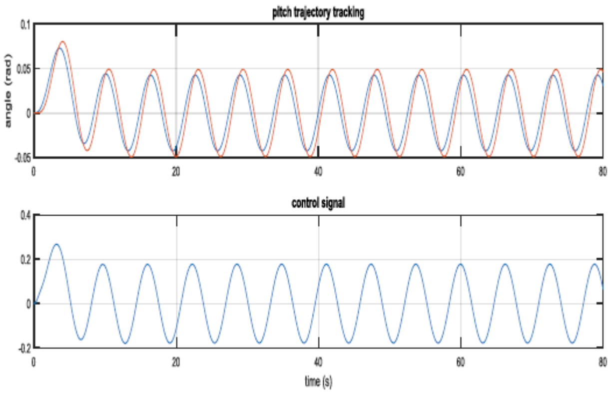

Fig. 9 LQG/LTR response for pitch angle tracking

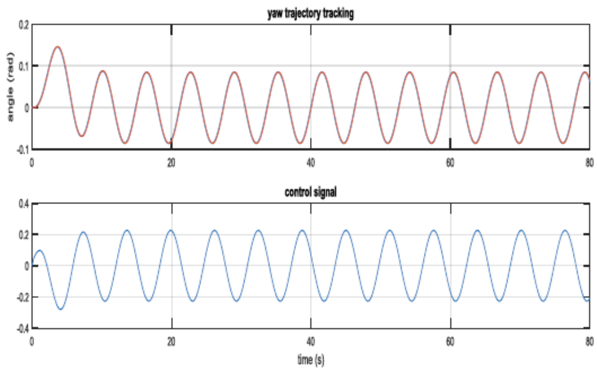

Fig. 10 LQG/LTR response for yaw angle tracking

For trajectory tracking, both strategies did well, and the control signal was excellent. The response with the LQG/LTR controller was very closer to the reference than the response with the LQG for the two angles; this appears clearly for Yaw angle.

Experimental results

As in simulation, we will apply step signal with the same amplitude, but the disturbance here in experimentation will be real one. We are going to hit the cage of the propeller with hand to get a hazardous perturbation. Also we kept the same amplitude as in simulation for the trajectory tracking and we reduce the frequency. A comparison grid will be presented in the end of this part to prove the difference between these two controllers.

Regulation

The system was fed by a step signal with amplitude of 0.4rad for the pitch and 0.8rad for the yaw to test its performances in regulation with disturbances rejection.

The disturbances here were given by hand hitting.

For every graph bellow, the blue line represents the reference and the red line represents the response, under it, we find the control signal.

LQG :

The response with the LQG controller applied on real system for regulation:

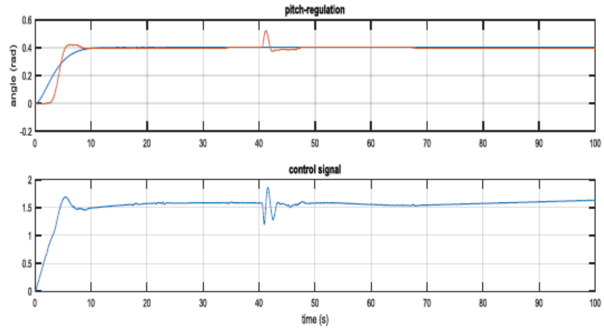

Fig. 11 LQG response for pitch angle regulation

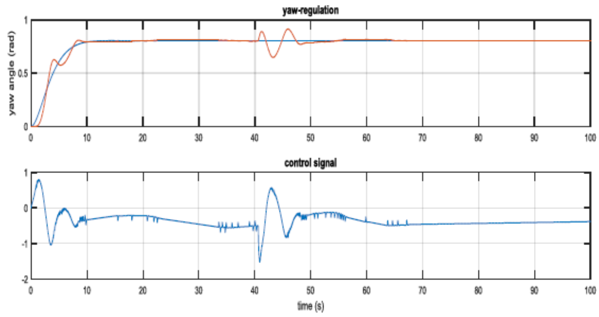

Fig. 12 LQG response for yaw angle regulation

LQG/LTR :

The response with the LQG controller with LTR recovery applied on real system for regulation:

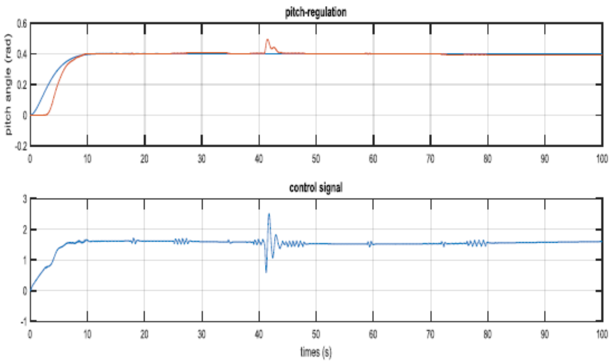

Fig. 13 LQG/LTR response for pitch angle

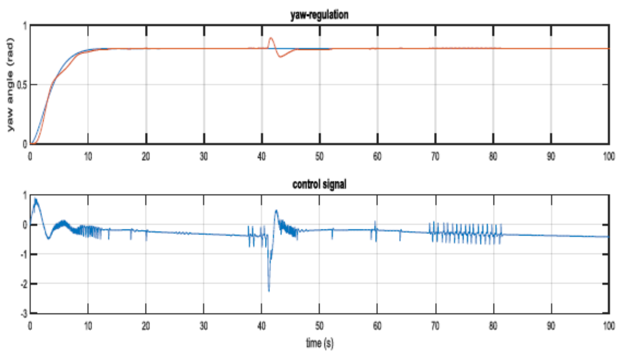

Fig. 14 LQG/LTR response for yaw angle regulation

The quality of the control signal in LQG contains less oscillation, but generally theses two control laws are the same in this point.

We remark that the LTR improve the response in the transient regime.

The transient regime for the LQG contain overshoot, and the response oscillate many time around the desired angle before reaching it, in the other hand the overshoot is zero for LTR and the response rise quickly and exactly to the reference without oscillation.

For all disturbances rejection, we have seen that the LTR reject the perturbation in minimum time and with less oscillation in contrast to the LQG, and especially for the yaw.

Trajectory tracking

The input is fed by a sinusoidal signal centered in 0.4 rad with amplitude equal to 0.2 rad and frequency equal to 0.25rad/sec for elevation and azimuth.

***LQG*:**

The response with the LQG controller applied on real system for trajectory tracking:

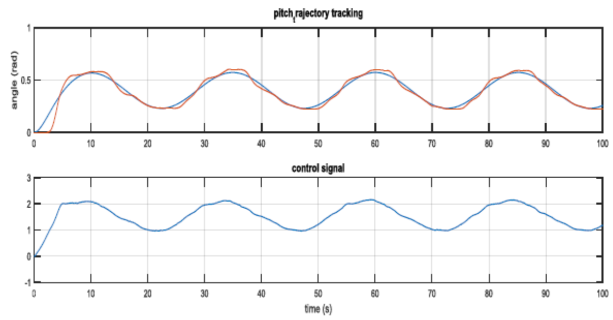

Fig. 15 LQG response for pitch angle tracking

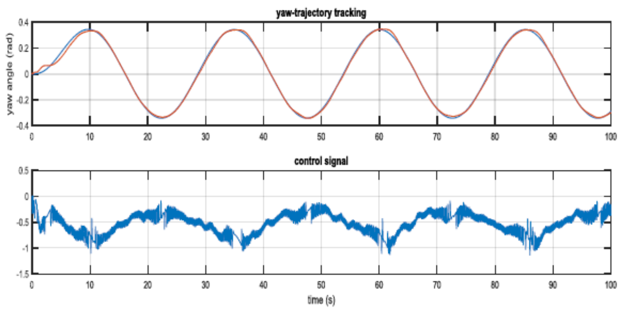

Fig. 16 LQG/LTR response for yaw angle tracking with control signal

Both strategies are very performing regarding to control signal -filtered and less chattering with low amplitude-, but the control signal of the LQG is more smooth then that of the LQG/LTR.

In the other hand, the trajectory tracking of the LQG response contains more oscillation around the desired signal than the LTR response, and specially for the yaw as always, it appears clearly, it is following the reference perfectly; this make the LQG/LTR final more efficient in this point.

Experimental results were as performing as simulation results, this prove the validity of system's model used for this system.

The azimuth responses are always better than the elevations responses in every experience because the pitch is always resisting its weight; its motion is in the vertical plane where the gravity is present, but the yaw propeller motion is in the horizontal plane where the effect of the gravity does not affect this angle.

We remark that we lose the optimality of the control signal when recovering the robustness of the loop, so we are in front of a tradeoff between the robustness and performances - optimality of the control signal.[19]

Comparison grid:

Below two comparative grids present: the variance of the control signal (var) and the mean absolute error (MAE) for the two objectives: regulation and tracking.

The results presented are only for experimental results

Table 1 variance of the control signal and the MAE for regulation

		LQG	LQG/LTR
Variance of the control signal	Pitch	Var=0.0407	Var=0.0507
	Yaw	Var=0.0549	Var=0.0411
M-A of error	Pitch	mae=0.0098	mae=0.0101
	Yaw	mae=0.0148	mae=0.0082

Table 2 variance of the control signal and the MAE for tracking

		LQG	LQG/LTR
Variance of the control signal	Pitch	Var=0.1938	Var=0.2107
	Yaw	var=0.0483	Var=0.0390
M-A of error	Pitch	mae=0.0214	mae=0.0287
	Yaw	mae=0.0186	mae=0.0094

The error is very small for all experiences; it is in the order of 10-2 and 10-3.

In this part of comparison between these two methods (LQG and LQG/LTR) we can remark that for pitch the LQG method gives minimum error but there isn't a big difference, and for the yaw the LQG/LTR gives less error with a difference of 10^{-1}.

Conclusion

After the modeling of the TRMS and the linearization of this non-linear model, an LQR controller has been used with a Kalman filter –LQG control- to improve the performances of this non-linear MIMO system. An integrator is introduced to deal with disturbances; and a feedforward controller was added to get good performances in tracking.

The main objective of this work is see the effect of the loop transfer recovering with the LQG controller, so a comparison is presented between the LQG with loop transfer recovering and without it, the experimental results as performing as the simulation resukts; this validates our model used in simulation and increase the confidence on it. The two techniques were excellent in permanent regime with optimal control signal. The tradeoff well known in literature between performances and robustness was very clear in this work, the control signal of the robust LQG/LTR was more dense comparing to the optimal LQG control signal, but generally the difference was not very big and both of them gave a smooth and optimal control signal.

Moreover, both methods were able to stabilize the system with these excellent results though the system was far from linearization point. The LQG/LTR was more performing than the LQG regarding to robustness against disturbances, trajectory tracking and the behavior of the system in the transient regime.

The next step of this proposal is: trying to deal with the delay seen in the transient regime for pitch and the density and charting presented in the control signal for trajectory tracking by optimizing the Q and R matrixes and the adjustment parameter "q".

Acknowledgments

The authors would like to thank REZOUG Nabil, BOUAYAD.D Amar and A. Halima for their help.

The author also would declare that the experimentations were done in the LAAS Laboratory in ENPO.

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