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Analysis of Caputo-Fabrizio Operator Application for Synthesis of Fractional Order PID-controller

***Abstract**. Using the representation of Caputo and Fabrizio, the influence of substitution in the linear model of a two-mass system of integer derivatives on fractional order derivatives is shown in this paper. So, the change of parameters of PID-controller of fractional order in comparison with classical PID-controller is analyzed. The influence of the use of PID-controller of fractional order on the transient characteristics of the system is demonstrated in the results of this paper.*

***Streszczenie.** Korzystając z reprezentacji Caputo i Fabrizio, ukazano wpływ zastąpienia pochodnych całkowitego rzędu w liniowym, modelu układu dwóch mas, pochodnymi ułamkowego rzędu. Przeanalizowano zmianę parametrów regulatora PID ułamkowego rzędu względem klasycznego regulatora PID. Zademonstrowano wyniki przedstawiające wpływ wykorzystania regulatora PID ułamkowego rzędu na charakterystykę przejściową układu. (**Analiza wykorzystania operatora Caputo-Fabrizio w syntezie regulatora PID ułamkowego rzędu**)*

Keywords: Caputo-Fabrizio operator, fractional order PID-controller, two mass system, fractional order derivatives.
Słowa kluczowe: operator Caputo-Fabrizio, regulator PID ułamkowego rzędu, układ dwóch mas, pochodne ułamkowego rzędu.

Introduction

Today the industry is putting increasing demands on the quality and accuracy of regulation. The problem of synthesis of effective management is complicated as the presence of different types of nonlinearities and change system parameters during its operation. For these reasons, the use of traditional regulators, in particular, PID-controllers, does not always provide highly effective regulation and therefore there is a need to find new approaches.

Methods of nonlinear control theory, such as back-stepping [1], feedback linearization [2], passivity base control [3] due to their complexity are not widely used in industry

At the same time, many publications on the use of fractional order regulators in technical systems demonstrates their advantages over classical regulators.

Such, in particular, papers on two- and three-mass systems and control systems for flexible and mobile works [4-7]. The actual problem is and synthesis regulators $PI^{\lambda}D^{\mu}$ [8-11], which in turn is associated with having a more synthesized settings, and the ability to use various functions for approximating the Laplace transform of the derivative or integral fractional order.

Along with solving control problems, the use of fractional derivatives to describe processes in electric machines, in particular with permanent magnets, and systems with semiconductor converters is promising, as shown in [12-15].

Traditionally, until recently, in most cases, to describe the fractional derivative in the electromechanical converters models developing or the controllers synthesis used the representation of Caputo:

$$ {}^{C}D^{\alpha}g(t) = \frac{1}{\Gamma(1-\alpha)} \cdot \int_0^t (t-\tau)^{-\alpha} \cdot \dot{g}(\tau)\, d\tau, \quad (1)$$

and to describe the fractional integral - the representation of Riemann-Liouville:

$$ {}^{RL}I^{\alpha}g(t) = \frac{1}{\Gamma(\alpha)} \cdot \int_0^t (t-\tau)^{\alpha-1} \cdot g(\tau)\, d\tau. \quad (2)$$

In the case of a generalized integro-differential operator of fractional order, obtained form:

$$ {}_{\alpha}D_T^{\gamma}x(t) = \begin{cases} \int_0^t x(\tau) d\tau^{\gamma} & Re(\gamma) < 0 \\ x(t) & Re(\gamma) = 0 \\ \frac{d^{\gamma} x(t)}{dt^{\gamma}} & Re(\gamma) > 0 \end{cases} \quad (3)$$

where α, T - top and bottom limits.

When using the Laplace operator under zero initial conditions it turns out:

$$ L\{ {}_{\alpha}D_T^{\gamma}x(t); s\} = s^{\gamma}X(s). \quad (4)$$

To approximate the fractional derivative s^{γ} when $0 < \gamma < 1$ by integer operator, one of the most commonly is to use the Oustaloup approximation:

$$ s^{\gamma} \approx \omega_h^{\gamma} \prod_{k=1}^{N} \frac{s+\omega_k'}{s+\omega_k} \quad (5)$$

where $\omega_k' = \omega_c \left(\frac{\omega_c}{\omega_e}\right)^{\frac{2k-1-\gamma}{2N}}$, $\omega_k = \omega_c \left(\frac{\omega_c}{\omega_e}\right)^{\frac{2k-1+\gamma}{2N}}$, γ – the order of the fractional derivative, N – the order of approximation; (ω_e, ω_h) – frequency range.

This approximation does not always adequately reflect the fractional order derivative in the whole frequency range. Results that are more accurate obtained using the modified Oustaloup approximation:

$$ s^{\gamma} \approx \left(\frac{d \cdot \omega_{kn}}{b}\right)^{\gamma} \left(\frac{d \cdot s^2 + b \cdot \omega_n\, s}{d \cdot (1-\gamma) \cdot s^2 + b \cdot \omega_n \cdot s + d \cdot \gamma}\right) \times \times \prod_{k=-N}^{N} \frac{s+\omega_k'}{s+\omega_k}$$

where $0 < \gamma < 1$, γ – the order of the fractional derivative, N – the order of approximation; (ω_e, ω_h) – frequency range: $\omega_k' = \omega_e \left(\frac{\omega_n}{\omega_e}\right)^{\frac{2k-1+\gamma}{2N}}$, $\omega_k = \omega_e \left(\frac{\omega_n}{\omega_e}\right)^{\frac{2k-1+\gamma}{2N}}$.
In this case are recommended parameters in value b=10, d=9.

As noted in [16] inuse proposed approach causes some difficulties at the stage of synthesis control actions and the analysis of systems in the time domain.

In [17] Caputo and Fabrizio proposed a representation of a fractional derivative of order $0 < \alpha < 1$ in the form:

$$ {}^{CF}D^{\alpha}g(t) = \frac{(2-\alpha) \cdot M(\alpha)}{2 \cdot (1-\alpha)} \cdot \int_0^t e^{\frac{-\alpha \cdot (t-\tau)}{1-\alpha}} \cdot \dot{g}(\tau)\, d\tau \quad (6)$$

which is obtained from the Caputo representation by replacing the kernel $(t-\tau)^{-\alpha}$ on $e^{\frac{-\alpha \cdot (t-\tau)}{1-\alpha}}$, and $1/\Gamma(\alpha)$ on $1\big/\sqrt{2 \cdot \pi \cdot (1-\alpha^2)}$, and when $M(\alpha)$ normilized factor, which depends on α.

In [18] is shown, that $M(\alpha) = {}^{2}/_{(2-\alpha)}$ and then fractional order representation when order is in average $0 < \alpha < 1$ is described by equation:

(7) $$ {}^{CF}D^{\alpha}g(t) = \frac{1}{(1-\alpha)} \cdot \int_0^t e^{\frac{-\alpha\cdot(t-\tau)}{1-\alpha}} \cdot \dot{g}(\tau)\, d\tau. $$

The fractional integral can be represented as [19]:

$$ {}^{CF}I^{\alpha}g(t) = (1-\alpha)\cdot g(t) + \alpha \cdot \int_0^t g(\tau)\, d\tau $$

or [20]:

$$ {}^{CF}I^{\alpha}g(t) = \frac{1}{\alpha} \cdot \int_0^t e^{\frac{-(1-\alpha)\cdot(t-\tau)}{\alpha}} \cdot g(\tau)\, d\tau $$

Laplace transforms are often used to analyze systems with fractional derivatives [21,22]. In the case of the Caputo-Fabrizio operator using, we obtain (8):

$$ \mathcal{L}({}^{CF}D^{\alpha}g(t)) = \frac{1}{1-\alpha} \cdot \frac{1}{s+\frac{\alpha}{1-\alpha}}\left(s\cdot G(s) - g(0)\right) $$

$$ = \frac{1}{s\cdot(1-\alpha)+\alpha} \cdot \left(s \cdot G(s) - g(0)\right), $$

$$ \mathcal{L}({}^{CF}I^{\alpha}g(t)) = \frac{1}{\alpha} \cdot \frac{1}{s+\frac{1-\alpha}{\alpha}} \cdot G(s) = \frac{1}{\alpha\cdot s+1-\alpha} \cdot G(s) $$

or

(8) $$ \mathcal{L}({}^{CF}I^{\alpha}g(t)) = (1-\alpha)\cdot G(s) + \frac{\alpha}{s} \cdot G(s). $$

So, the Caputo-Fabrizio operator applying open possibility to represent integer derivative of the fractional derivative and integral as a result of Laplace transform. This avoids approximation of the fractional derivative s^{α} and integral $s^{-\alpha}$ and thus provide leeway to in modeling processes and synthesis control systems.

In [23, 24] it is noted about the limited application of the Caputo-Fabrizio and other operators with a non-singular kernel for the study of systems, in particular the need to fulfill the condition $f(0) = 0$, which follows from the (9):

(9) $$ {}^{CF}D_0^{\alpha}\left[{}^{CF}J_0^{\alpha}f(t)\right] = f(t) - exp\left(-\frac{\alpha}{1-\alpha}t\right)f(0). $$

This remark is not critical in solving the problem of control influence synthesis, because in classical control theory for the control systems synthesis traditionally use models in increments of variables and do not take into account the initial conditions.

Similarly, the initial conditions are assumed to be zero when using the Oustaloup approximation. And the possibility of transition to the model with integer derivatives allows to apply the classical methods of control effects synthesis without transforming to the frequency domain and applying the approximations required when using operators with a singular kernel to describe the derivative of the fractional order.

Aim and baseline of the research

In the traditional theory of automatic control, the PID-controller when using a parallel circuit is described by an equation (10) [25]:

(10) $$ u(s) = k_{p\_cl} \cdot \left(1 + \frac{1}{\tau_{i_{cl}} \cdot s} + \frac{\tau_{d_{cl}} \cdot s}{\frac{\tau_{d_{cl}}}{N} \cdot s + 1}\right) \cdot e(s) $$

where e - the error of the mismatch between the input and the output signals of the control circuit, $u(s)$ - the controller output signal; k_{p_cl}, τ_{I_cl}, τ_{d_cl} - controller parameters; $N = 2 \div 20$ and is often taken as equal 10.

For the fractional PID-controller, the equation (11) describing its operation:

(11) $$ u(t) = k_p \cdot e(t) + k_i \cdot I^{\mu}e(t) + k_d \cdot D^{\alpha}e(t), $$

and when using an operator with a non-singular kernel, in particular Caputo-Fabrizio, the following equations for the frictional order PID-controller are obtained as:

$$ u(s) = k_p \cdot e(s) + \left(k_i \cdot (1-\mu) + \frac{k_i \cdot \mu}{s}\right) \cdot e(s) + $$

$$ + \frac{\frac{k_d}{\alpha} \cdot s}{\frac{1-\alpha}{\alpha} \cdot s + 1} \cdot e(s) $$

or

$$ u(s) = k_p \cdot e(s) + \frac{k_i}{\mu \cdot s + (1-\mu)} \cdot e(s) + $$

$$ + \frac{\frac{k_d}{\alpha} \cdot s}{\frac{1-\alpha}{\alpha} \cdot s + 1} \cdot e(s) $$

Block diagrams of regulators are shown in fig. 1 and 2.

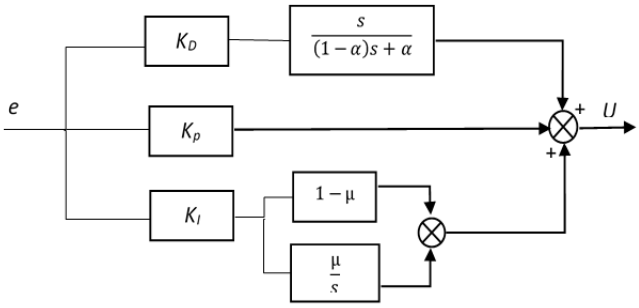


Fig.1. Block diagram of the fractional order regulator for which the integral component obtained on the basis of the Laplace image of the fractional order derivative using the Caputo-Fabrizio operator.

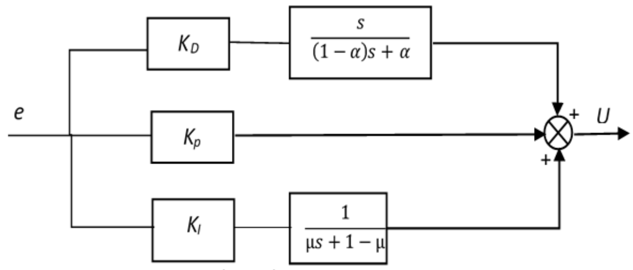


Fig.2. Block diagram of the fractional order PID-controller obtained using an integrated operator with a non-singular core [20].

For the structure, which is shown in Fig. 1, the equation can be written as follows:

$$ u(s) = \left(k_p + k_i \cdot (1-\mu)\right) x $$

$$ x\left(1 + \frac{1}{\frac{k_p+k_i\cdot(1-\mu)}{k_i\cdot\mu}} \cdot \frac{1}{s} + \frac{\frac{k_d}{\alpha\cdot\left(\left(k_p+k_i\cdot(1-\mu)\right)\right)} \cdot s}{\frac{1-\alpha}{\alpha} \cdot s + 1}\right) \cdot e(s) $$

Then, by analogy with the classic PID-controller, we obtain (12):

(12) $$ u(s) = k_p^* \cdot \left(1 + \frac{1}{\tau_i^* \cdot s} + \frac{\tau_d^* \cdot s}{\frac{\tau_d^*}{N} \cdot s + 1}\right) \cdot e(s), $$

where $k_p^* = k_p + k_i \cdot (1-\mu)$, $\tau_i^* = \frac{k_p+k_i\cdot(1-\mu)}{k_i\cdot\mu}$,

$$ \tau_d^* = \frac{k_d}{\alpha\cdot\left(k_p+k_i\cdot(1-\mu)\right)}, \quad \frac{1-\alpha}{\alpha} = \frac{\tau_d^*}{N}. $$

Thus, for the synthesis of the fractional controller it becomes possible to use the methods of classical PID - controller synthesis.

For example, in the case of PID control adjustment methods, in particular such as Ziegler-Nichols'; Chien, Hrones and Reswick; Tyreus-Luyben, to determine the parameters of the fractional controller, we obtain the following system of equations (13):

$$(13)\qquad \begin{cases} k_p + k_i \cdot (1-\mu) = \beta_1 \cdot k_u \\ \dfrac{k_p + k_i \cdot (1-\mu)}{k_i \cdot \mu} = \beta_2 \cdot T_u \\ \dfrac{k_d}{\alpha \cdot \left(k_p + k_i \cdot (1-\mu)\right)} = \beta_3 \cdot T_u \\ \dfrac{1-\alpha}{\alpha} = \dfrac{\beta_3}{N} \cdot T_u \end{cases}$$

where β_1, β_2, β_3 – parameters that depend on the selected method of the PID-controller adjusting, k_u and T_u – the factor of the proportional controller, at which the system occurs steady-state oscillations and the period of these oscillations, respectively.

From (13) we will received equation

$$\begin{cases} k_p = \beta_1 \cdot k_u \cdot \left(1 - \dfrac{1-\mu}{\beta_2 \cdot T_u \cdot \mu}\right) \\ k_i = \dfrac{\beta_1 \cdot k_u}{\beta_2 \cdot T_u \cdot \mu} \\ k_d = \dfrac{\beta_1 \cdot \beta_3 \cdot k_u \cdot T_u}{1 + \dfrac{\beta_3}{N} \cdot T_u} \\ \alpha = \dfrac{1}{1 + \dfrac{\beta_3}{N} \cdot T_u} \end{cases}$$

On the other hand, in the fractional order PID-controller at the same settings as the classic PID-controller, an additional signal is generated. Writing the equation for the PID-controller in the form (14):

$$(14)\quad u(s) = \left(k_p + k_i \cdot (1-\mu)\right) \cdot \left(1 + \frac{N \cdot \frac{1-\alpha}{\alpha} \cdot s}{\frac{1-\alpha}{\alpha} \cdot s + 1}\right) x$$

$$x\, e(s) + \frac{k_i \cdot \mu}{s} \cdot e(s)$$

We could use $T_f = \frac{1-\alpha}{\alpha}$, and also, after simplification – get equation (15)

$\boldsymbol{u(s)} = \left(k_p + k_i \cdot (1-\mu)\right) \cdot \frac{(N+1) \cdot T_f \cdot s + 1}{T_f \cdot s + 1} \cdot e(s) + \frac{k_i \cdot \mu}{s} \cdot e(s) = k_p \cdot \frac{(N+1) \cdot T_f \cdot s + 1}{T_f \cdot s + 1} \cdot e(s) + k_i \cdot (1-\mu) \cdot \frac{(N+1) \cdot T_f \cdot s + 1}{T_f \cdot s + 1} \cdot e(s) + \frac{k_i}{s} \cdot e(s) - \frac{k_i \cdot (1-\mu)}{s} \cdot e(s) = k_p \cdot \frac{(N+1) \cdot T_f \cdot s + 1}{T_f \cdot s + 1} \cdot e(s) + \frac{k_i}{s} \cdot e(s) + k_i \cdot (1-\mu) \cdot \left(\frac{(N+1) \cdot T_f \cdot s + 1}{T_f \cdot s + 1} - \frac{1}{s}\right) \cdot e(s) = \boldsymbol{u_{cl}(s)} + \Delta \boldsymbol{u(s)}$

$$(15)\qquad u(s) = u_{cl}(s) + \Delta u(s)$$

where

$u_{cl}(s) = k_p \cdot \frac{(N+1) \cdot T_f \cdot s + 1}{T_f \cdot s + 1} \cdot e(s) + \cdot \frac{k_i}{s} \cdot e(s)$ component, that corresponds to the classic PID-controller,

$\Delta u(s) = k_i \cdot (1-\mu) \cdot \left(\frac{(N+1) \cdot T_f \cdot s + 1}{T_f \cdot s + 1} - \frac{1}{s}\right) \cdot e(s)$ – the extra signal that corresponds to the fractional PID-controller.

This additional signal Δu(s) can be represented as:

$\Delta u(s) = k_i \cdot (1-\mu) \cdot \frac{(N+1) \cdot T_f \cdot s^2 + (1-T_f) \cdot s - 1}{(T_f \cdot s + 1) \cdot s} \cdot e(s) = k_i \cdot (1-\mu) \cdot \frac{(s+p_1) \cdot (s-p_2)}{(T_f \cdot s + 1) \cdot s} \cdot e(s),$

where

$$p_1 = \frac{1}{2} \cdot \frac{1 - T_f + \sqrt{4 \cdot N \cdot T_f + \left(1 + T_f\right)^2}}{(N+1) \cdot T_f},$$

$$p_2 = \frac{1}{2} \cdot \frac{T_f - 1 + \sqrt{4 \cdot N \cdot T_f + \left(1 + T_f\right)^2}}{(N+1) \cdot T_f}.$$

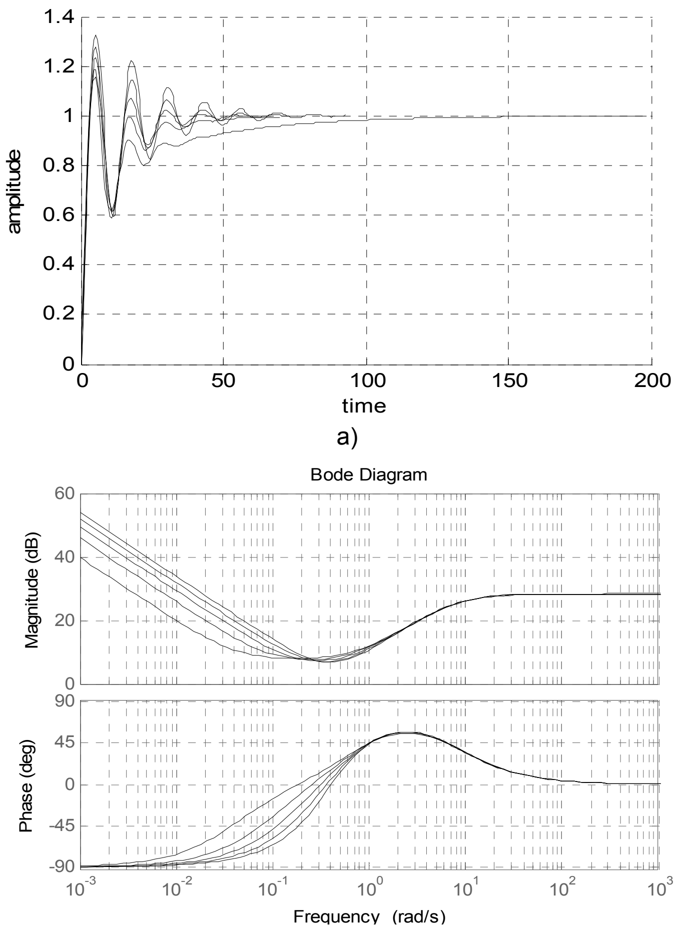
In a further signal $\Delta u(s)$ appear unstable component of zero.

Effect of change μ on the system transient characteristic with a fractional PID-controller in a close-loop control system of a dynamic object with a transfer function

$$W(s) = \frac{1}{12 \cdot s^2 + 0.3 \cdot s + 1}$$

at the controller settings $k_p = 2.1$, $k_i = 0.49$ and $T_f = 0.125$ are shown in Fig. 3a.

The Bode diagram and phase frequency characteristic of the fractional PID-controller when changing the parameter μ are given in fig. 3b.



b)

Fig. 3. Effect of parameter μ changing on transient characteristic of system and frequency characteristics in case of the fractional controller

In the case of the fractional order PID-controller, (block diagram in Fig. 2), we obtain transfer function:

$$W(s) = K_P + \frac{\frac{K_D}{\alpha} s}{T_{f1} s + 1} + \frac{\frac{K_I}{1-\mu} s}{T_{f2} s + 1}$$

Then

$$W(s)=\frac{\left[K_P(T_{f1}T_{f2}s^2+(T_{f1}+T_{f2})s+1)\right]}{(T_{f1}s+1)(T_{f2}s+1)}$$
$$+\frac{\frac{K_I}{1-\mu}(T_{f1}s+1)+\frac{K_D}{\alpha}s(T_{f2}s+1)}{(T_{f1}s+1)(T_{f2}s+1)}.$$

And we can rewrite

$$W(s)=\frac{s^2(K_PT_{f1}T_{f2}+\frac{K_D}{\alpha}T_{f2})+K_P+\frac{K_I}{1-\mu}}{(T_{f1}s+1)(T_{f2}s+1)}$$
$$+s\left(K_P(T_{f1}+T_{f2})+\frac{K_D}{\alpha}+\frac{K_I}{1-\mu}T_{f1}\right)$$

where $T_{f1}=\frac{1-\alpha}{\alpha}$; $T_{f2}=\frac{\mu}{1-\mu}$.

After we input

$$K_D^*=(K_PT_{f1}T_{f2}+\frac{K_D}{\alpha}T_{f2})$$
$$K_I^*=K_P+\frac{K_I}{1-\mu}$$
$$K_P^*=K_P(T_{f1}+T_{f2})+\frac{K_D}{\alpha}+\frac{K_I}{1-\mu}T_{f1},$$

the transfer function can be rewritten as (16):

(16) $W(s)=\frac{1}{(T_{f1}s+1)}(K_D^*\cdot s^2+K_P^*\cdot s+K_I^*)\cdot\frac{1}{(T_{f2}s+1)}$.

It's corresponding known digital control theory in differential form submission PID-controller and connected in series with it filters high and low frequencies. The above transfer function can also be represented as:

$$W(s)=\frac{s}{(T_{f1}s+1)}\left(K_D^*\cdot s+\frac{K_I^*}{s}+K_P^*\right)\cdot\frac{1}{(T_{f2}s+1)}$$

From this transfer function, we can select parts, which corresponds to the series connection of the real differentiator, PID-controller and low-pass filter (Fig. 5).

Such a structure (see Fig. 4) can be interpreted as a PID- controller by the derivative of the discrepancy with the intensity determiner.

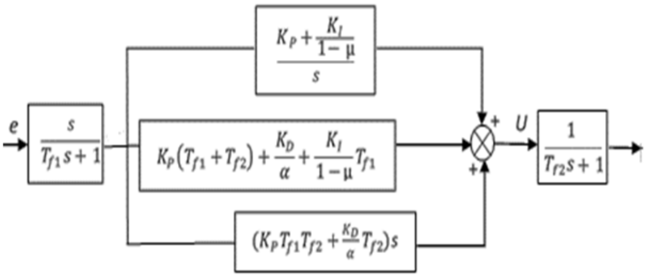


Fig. 4. Block diagram of the fractional order PID-regulator.

Taking into account that

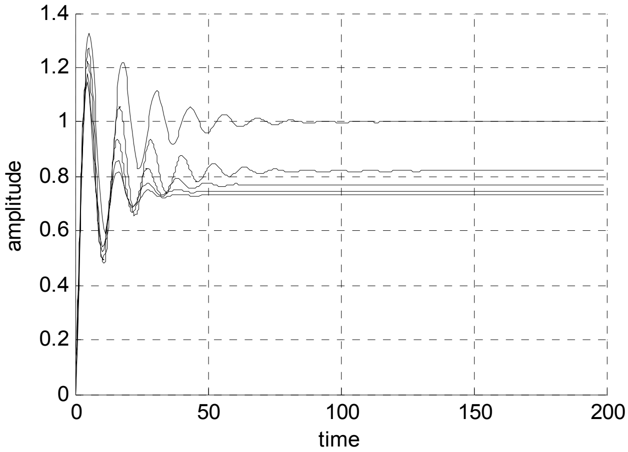
$$T_{f1}+T_{f2}=\frac{1-\alpha}{\alpha}+\frac{\mu}{1-\mu}=\frac{(1-\alpha)(1-\mu)+\alpha\mu}{\alpha(1-\mu)}$$

we obtain the dependences of the change of coefficients on the orders of fractional integration and differentiation:

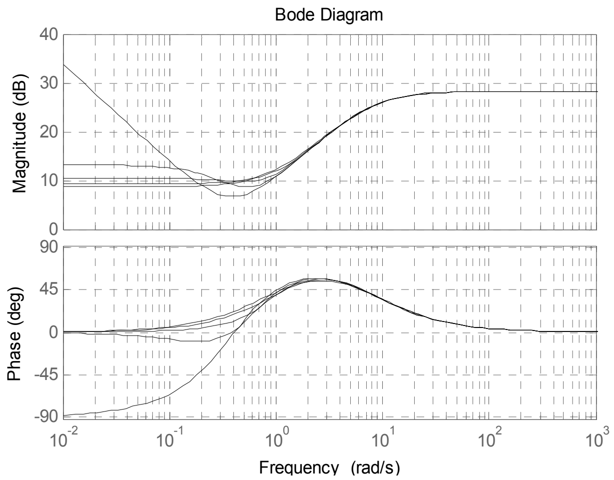
$$K_P^*=\frac{K_P\cdot((1-\alpha)\cdot(1-\mu)+\alpha\cdot\mu)+K_D\cdot(1-\mu)+K_I\cdot(1-\alpha)}{\alpha\cdot(1-\mu)}$$
$$K_D^*=\frac{K_P\cdot(1-\alpha)\cdot\mu+K_D\cdot\mu}{\alpha\cdot(1-\mu)},K_I^*=K_P+\frac{K_I}{1-\mu}.$$

Similarly, as for the structure of the fractional order PID-controller (is shown in Fig. 1), we investigate the effect of the change μ on the transient characteristic (Fig. 5a) and the frequency characteristics (Fig. 5b) of the fractional order PID-controller, the block diagram of which is shown in Fig. 4.

These results allow us to state that the characteristics of the controller significantly depend on the chosen operator to describe the integral of the fractional order.



a)



b)

Fig. 5. Effect of parameter μ changing on transient characteristic of system and frequency characteristics in case of the fractional controller

One of the advantages of the fractional order PID-controller, as shown in [26], is the greater number of debugging parameters compared to the classic PID-controller. In addition to the gain of the proportional, differential and integral components, there are two additional parameters, in particular – the order of the fractional integrator μ and the order of the fractional differentiator λ. This leads to the solution of the optimization problem in five-coordinate space. When using the operator Caputo-Fabrizio and the implementation of the fractional integral in accordance with [19], given that

$$\tau_d^*=\frac{k_d}{\alpha\cdot\left(k_p+k_i\cdot(1-\mu)\right)}=N\cdot\frac{1-\alpha}{\alpha}$$

and from which we obtained

$$k_p+k_i\cdot(1-\mu)=\frac{k_d}{N\cdot(1-\alpha)}$$

the equation for determining the output signal of the fractional order PID-controller will given as (17):

$$u(s)=\frac{k_d}{N\cdot(1-\alpha)}\cdot\left(1+\frac{N\cdot\frac{1-\alpha}{\alpha}\cdot s}{\frac{1-\alpha}{\alpha}\cdot s+1}\right)\cdot e(s)+\frac{k_i\cdot\mu}{s}\cdot e(s)$$

$$u(s) = \frac{k_d}{N \cdot (1-\alpha)} \cdot \frac{(N+1) \cdot \frac{1-\alpha}{\alpha} \cdot s + 1}{\frac{1-\alpha}{\alpha} \cdot s + 1} \cdot e(s) + \cdot \frac{k_i \cdot \mu}{s} \cdot e(s).$$

(17) $$u(s) = \frac{k_d \cdot (1+T_f)}{N \cdot T_f} \cdot \frac{(N+1) \cdot T_f \cdot s + 1}{T_f \cdot s + 1} \cdot e(s) + \cdot \frac{k_i^*}{s} \cdot e(s)$$

and the number of parameters for the settings is similar to the classic PID-controller.

At the same time the appropriate settings $k_{p\_cl} = \frac{k_d \cdot (1+T_f)}{N \cdot T_f}$ and $k_{i\_cl} = k_i^*$ of the classical regulator, it is possible to provide in system control signal which is realized by the adjusted fractional order regulator.

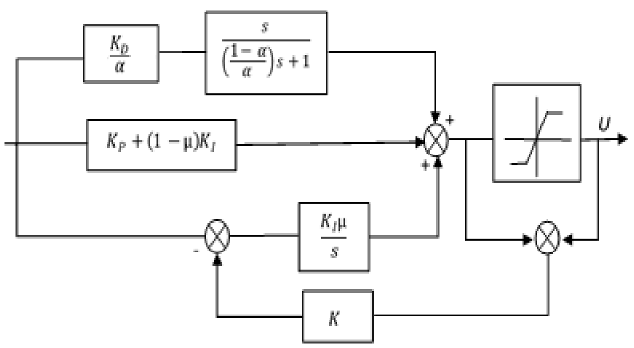
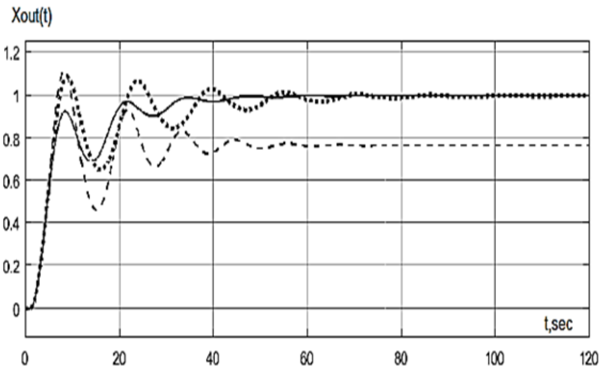
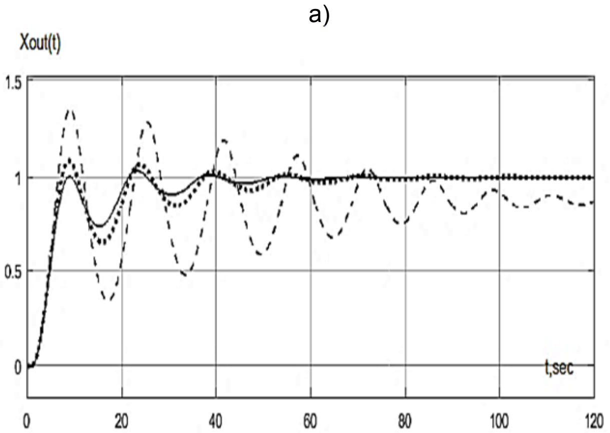


Fig. 6. Block diagram of the PID- controller with „anti wind-up" block.



a)



b)

Fig. 7. Transients of the different type of PID-controller (____ - fractional PID-regulator according to the scheme of fig. 1; _ _ _ - classic PID- controller; .... - fractional PID-controller according to the scheme of fig. 2) and for two cases (a - when μ=0.6, b – when μ=0.9)

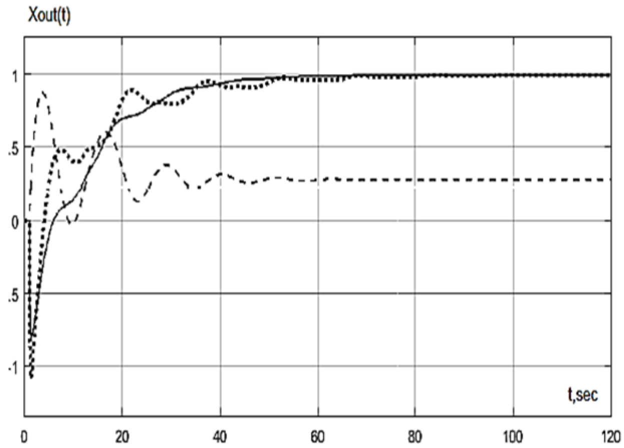
In control systems with a PID-controller, the output signal is traditionally limited to such level. At the same time, the task of eliminating integration becomes relevant. One of the traditional approaches to avoid integration (anti wind-up) is shown in Fig. 6.

It should be noted that this problem does not arise in the fractional order PID-controller (Fig. 2).

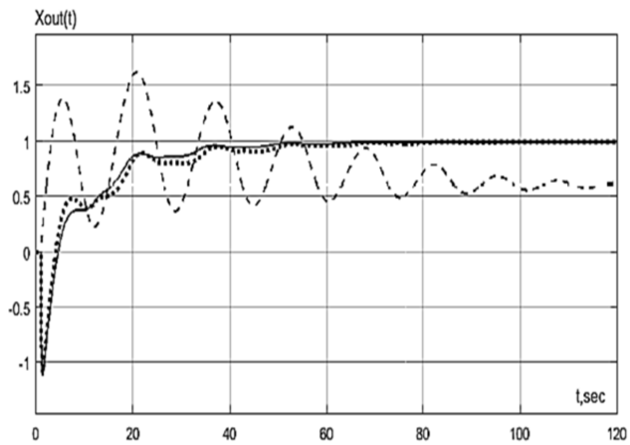
In fig. 7-8 transient characteristic for different cases of PID-regulator are shown.

There are the dependences of the change of the output signal of the system, and in Fig. 8 - changes in the integral component of the classical and fractional regulators, when setting the limit on the control signal ± 1.

Changing the parameter μ fractional order PID-controller of type 1 with the adjustment of the gain of the proportional, differential and integral components corresponding to the classical controller, provides additional opportunities to improve system performance. A similar change in the parameter μ fractional order PID-controller of type 2 provides a reduction in oscillation and, thus, reduce the control time.



a)



b)

Fig. 8. Transients of the different type of PID-controller and by block of sigmal limitation (____ - fractional PID- controller according to the scheme of fig. 1; _ _ _ - classic PID- controller; .... - fractional PID-regulator according to the scheme of fig. 2.) and for two cases (a - when μ=0.6, b – when μ=0.9)

The obtained dependences of the output coordinate changing demonstrate that in the control systems with output signal limitation of the PID-controller, the advantages of the fractional order controller are somewhat leveled.

At the same time in the system with the fractional order PID-regulator that is shown by block diagrams (Fig. 2) the quality of transients is slightly deteriorating.

Conclusions

The use of the Caputo-Fabrizio operator to representation fractional derivatives and integrals after Laplace transforming provides product with integer derivatives and integrators after application. It is, respectively, simplifies the synthesis of control effects and analysis of system behavior.

The use of fractional derivatives in linear models of a two-mass system is equivalent to changing individual parameters of the model and is not appropriate.

The use of fractional PID-controller with the adjustment of the coefficients corresponding to the classical controller when changing the parameter μ improves the characteristics of the system.

The classic PID controller is a partial case of the fractional PID-controller and, at the same time, at certain settings of parameters can reproduce work of the fractional PID-controller.

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