1. Waldemar WOJCIK[1], 2. Zinovii NYTREBYCH[2], 3. Oksana MALANCHUK[3], 4. Maria VISTAK[3], 5. Liubov KRAVCHENKO[4], 6. Bogdan PINAEV[5], 7. Saule RAKHMETULLINA[6], 8. Ainur TOIGOZHINOVA[7]

Lublin University of Technology, Poland (1), Lviv Polytechnic National University, Ukraine (2), Danylo Halytsky Lviv National Medical University, Ukraine (3), Poltava V.G. Korolenko National Pedagogical University, Ukraine (4), Vinnytsia National Technical University, Ukraine (5), East Kazakhstan State Technical University named after D.Serikbayev, Kazakhstan (6) Academy of Logistics and Transport, Kazakhstan (7)
ORCID: 1. 0000-0002-0843-8053; 2. 0000-0002-9599-8517; 3. 0000-0001-7518-7824; 4. 0000-0001-5192-4017; 5. 0000-0001-9065-8524; 6. 0000-0001-9592-0640; 7. 0000-0002-3142-0249; 8. 0000-0002-0305-2776

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Modelling of the processes in electrical systems by two-point problem for nonhomogeneous telegraph equation

Abstract. The two-point problem for the nonhomogeneous telegraph equation is a mathematical model to describe propagation of electromagnetic wavesunder the action of external force at given behavior of the process at two time moments. The differential-symbol method of constructing an exact analytical solution of the problem is proposed. The class of quasipolynomials as a class of existence and uniqueness of the solution of the problem is indicated. The examples to research propagation of waves with two given states are proposed. The presented results can be effectively used in the design and studying of parameters of the electrical engineering systems.

Streszczenie. *Problem dwupunktowy dla niejednorodnego równania telegraficznego jest matematycznym modelem opisu propagacji fal elektromagnetycznych pod działaniem siły zewnętrznej przy danym zachowaniu się procesu w dwóch momentach czasowych. Zaproponowano metodę różniczkowo-symboliczną konstruowania dokładnego analitycznego rozwiązania tego problemu. Wskazano klasę quasi-wielomianów jako klasę istnienia i jednoznaczności rozwiązania problemu. Zaproponowano przykłady do badania propagacji fal o dwóch zadanych stanach. Przedstawione wyniki mogą być efektywnie wykorzystane w projektowaniu i badaniu parametrów systemów elektrotechnicznych. (**Modelowanie procesów w systemach elektrycznych za pomocą problemu dwupunktowego dla niejednorodnego równania**).*

Keywords: Electromagnetic waves, Differential-symbol method, Two-point problem, Electrical system.
Słowa kluczowe: fale elektromagnetyczne, metoda różniczkowo-symboliczna, problem dwupunktowy, system elektryczny.

Introduction

Creating models of technical systems is an urgent task of the design and study of these systems. It allows to predict the physical and mechanical characteristics and quality indicators of materials and make adjustments to the algorithm of the system at the design stage of the device and optical sensors [1-7].

Mathematical modelling is one of the important directs of study of systems different nature [8]. Mathematical models at the component level of design of various physical processes are often described by partial differential equations [9, 10], therefore the study of such equations is an actual task. In particular, the conditions of existence and uniqueness of partial differential equation solutions [11] which satisfy initial and multipoint conditions by time variable are studied in papers [12, 13].

The models of hyperbolic partial differential equations of the structures vibrations are the basis for fundamental equations of atomic physics. The telegraph equation is an important equation for modeling various problems arising in engineering and science fields [14]. Models which contain telegraph equations describe various processes, such as the distribution of electric current in a conductor [15], the propagation of electromagnetic waves [19], and allow us to take into account the real existing resistances of medium and determine the nature of the damping of oscillations which are caused by these resistances.

The telegraph equation describes wave propagation [18], signal analysis [22], random walk theory [19] etc. Considerable attention is paid to the development of accurate and numerical methods for finding solutions of one-dimensional [20], two-dimensional [21,22] and three-dimensional [23] telegraph equations.

Often not only equations, but the Cauchy problem for them [24], that is equations with given values of the process and the speed of process change at the initial time moment are models of describing process. The problems for telegraph equation under given behaviour of process at two time moments are studied in the works [25,26]. For finding problem solutions for telegraph equation there are used many methods, in particular Reduced Differential Transform Method in [23, 35], Chebyshev Tau method in [26], variation iteration and homotopy perturbation methods in [8, 36].

In the present work the two-point problem for three-dimensional nonhomogeneous telegraph equation with given values of unknown function at two time moments is studied. The application of the differential-symbol method [28, 29] makes it possible to find the conditions of the existence and uniqueness of the solution of a two-point in time problem, and, most importantly, to construct an exact analytical solution. We note that the problems with local two-point in time conditions were investigated in work [30] using differential-symbol method.

In paper [31], there consider nonhomogeneous telegraph equation of the form

$$\Delta u-\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}-b\frac{\partial u}{\partial t}-cu=-f\left(x,y,z,t\right),$$

in which an unknown function u determines the distribution of electric current, $v$is the speed of signal propagation. In the presented work we consider the case when the insulation leakage is minimal, that is the coefficient near the unknown function u is zero. This equation describes the processes of sound and electromagnetic waves propagation in the medium. The density and pressure of the gas, the potential velocity, the components of the electric and magnetic fields and the corresponding potentials satisfy this equation [32-34].

The following tasks have been completed:

- Describing two-point problem
- Finding the conditions of existence of unique solution of the two-point problem for telegraph equation
- Analytical form of the problem solution.
- Realization of method for some functions.

Two-point problem for telegraph equation

We study electromagnetic waves which is described by an unknown function $U\left(t,\overline{x}\right)$. This function is a solution of the nonhomogeneous equation

(1) $$\left[\frac{\partial^2}{\partial t^2}+a\frac{\partial}{\partial t}+b\Delta_3\right]u\left(t,\overline{x}\right)=f\left(t,\overline{x}\right),$$

with given values of process at two time moments

(2) $u\left(0,\overline{x}\right)=0$, $u\left(\tau,\overline{x}\right)=0$.

In equation (1), constants a and b are determined by the physical characteristics of the conductor, function $f\left(t,\overline{x}\right)$ is the action of the external force onto propagation of electromagnetic waves.

Problem (1), (2) does not always have a unique solution in the class of quasipolynomial. For example, the problem

$$\frac{\partial^2 u}{\partial t^2}-\Delta_3 u=f\left(t,\overline{x}\right),\ u\left(0,\overline{x}\right)=0,\ u\left(2\pi,\overline{x}\right)=0,$$

which is problem (1), (2) for $a=0,\ b=-1,\ \tau=2\pi$, can have more than one solution because the function $u\left(t,\overline{x}\right)=\sin[x+t]-\sin[x-t]$ is the solution of the corresponding homogeneous problem. Therefore, problem (1), (2) is incorrectly solvable in the class of quasipolynomials [35-37].

The conditions of unique solvability of the two-point problem for telegraph equation

For nonempty set $L\subseteq\mathbb{C}^3$ let's consider the class of quasipolynomials $K_{\mathbb{C},L}$, that is the class of functions of the form

(3) $$g\left(t,\overline{x}\right)=\sum_{k=1}^{m}\sum_{j=1}^{N}Q_{kj}\left(t,\overline{x}\right)e^{\beta_j t+\alpha_{k1}x+\alpha_{k2}y+\alpha_{k3}z},\quad m,N\in\mathbb{N},$$

where $\beta_1,\dots,\beta_N$ are different complex numbers, the vectors $\alpha_1=\left(\alpha_{11},\alpha_{12},\alpha_{13}\right),\dots,\alpha_m=\left(\alpha_{m1},\alpha_{m2},\alpha_{m3}\right)$ are different and belong to the set L, $Q_{11}\left(t,\overline{x}\right),\dots,Q_{mN}\left(t,\overline{x}\right)$ are polynomials with complex coefficients.

By replacing the variables t,x,y,z by $\frac{\partial}{\partial\lambda}$, $\frac{\partial}{\partial\nu_1}$, $\frac{\partial}{\partial\nu_2}$ and $\frac{\partial}{\partial\nu_3}$ respectively, for each quasipolynomail (3) we introduce the quasipolynomial differential expression $g\left(\frac{\partial}{\partial\lambda},\frac{\partial}{\partial\nu_1},\frac{\partial}{\partial\nu_2},\frac{\partial}{\partial\nu_3}\right)\equiv g\left(\frac{\partial}{\partial\lambda},\frac{\partial}{\partial\nu}\right)$. Its action onto an entire function $G=G(\lambda,\nu_1,\nu_2,\nu_3)\equiv G(\lambda,\nu)$ is determined by equality

(4) $$\begin{aligned}&g\left(\frac{\partial}{\partial\lambda},\frac{\partial}{\partial\nu}\right)G(\lambda,\nu)=\\&=\sum_{k=1}^{m}\sum_{j=1}^{N}Q_{kj}\left(\frac{\partial}{\partial\lambda},\frac{\partial}{\partial\nu}\right)G(\lambda+\beta_j,\nu_1+\alpha_{k1},\nu_2+\alpha_{k2},\nu_3+\alpha_{k3}).\end{aligned}$$

For problem (1), (2), let's consider the function of the form

(5) $$\eta\left(\nu\right)=e^{-\frac{a}{2}\tau}\frac{\sinh\left[\tau R\left(\nu\right)\right]}{R\left(\nu\right)},$$

where $R\left(\nu\right)=\frac{1}{2}\sqrt{a^2-4b\|\nu\|^2}$, $\|\nu\|^2=\nu_1^2+\nu_2^2+\nu_3^2$.

Let the set H be set of zeroes of the function (5). The problem (1), (2) has unique solution in the class $K_{\mathbb{C},L}$ [32].

We study the problem (1), (2) in the case if the function $f\left(t,\overline{x}\right)$ has quasipolynomial form. Therefore, the unknown function $u\left(t,\overline{x}\right)$ also has quasipolynomial form.

Let's write the ordinary differential equation

(6) $$\frac{d^2T}{dt^2}+a\frac{dT}{dt}+b\|\nu\|^2T=0.$$

The functions

(7) $$\begin{aligned}T_1\left(t,\nu\right)&=e^{-\frac{a}{2}(t+\tau)}\frac{\sinh\left[\left(\tau-t\right)R\left(\nu\right)\right]}{\eta\left(\nu\right)R\left(\nu\right)},\\T_2\left(t,\nu\right)&=e^{-\frac{a}{2}t}\frac{\sinh\left[tR\left(\nu\right)\right]}{\eta\left(\nu\right)R\left(\nu\right)}\end{aligned}$$

are the solutions of equation (6) and satisfy following conditions:

(8) $$T_1\left(0,\nu\right)=1,\ T_1\left(\tau,\nu\right)=0,\ T_2\left(0,\nu\right)=0,\ T_2\left(\tau,\nu\right)=1$$

Let introduce the function

(9) $$F\left(t,\lambda,\nu\right)=\frac{e^{\lambda t}-T_1\left(t,\nu\right)-e^{\lambda\tau}T_2\left(t,\nu\right)}{\lambda^2+a\lambda+b\|\nu\|^2}.$$

Taking into account functions (7), formula (9) can be written in the form

(10) $$\begin{aligned}&F\left(t,\lambda,\nu\right)=\\&=\frac{e^{\lambda t}\sinh\left[\tau R(\nu)\right]-e^{\lambda-\frac{a}{2}(t-\tau)}\sinh\left[tR(\nu)\right]-e^{-\frac{a}{2}t}\sinh\left[\left(\tau-t\right)R(\nu)\right]}{\left(\lambda^2+a\lambda+b\|\nu\|^2\right)\sinh\left[\tau R(\nu)\right]}\end{aligned}$$

The solution of problem (1), (2) is the function

(11) $$u(t,\overline{x})=f\left(\frac{\partial}{\partial\lambda},\frac{\partial}{\partial\nu}\right)\left\{F(t,\lambda,\nu)\,e^{\nu\cdot\overline{x}}\right\}\Big|_{\lambda=0,\nu=O},$$

where $O=(0,0,0)$, $\nu\cdot\overline{x}=\nu_1x+\nu_2y+\nu_3z$.

The problem (1), (2) has only one solution in the class $K_{\mathbb{C},L}$. It can be proved by contradiction, analogously as in [32].

The examples of the two-point problem for some form of the external force

We show examples of solving the problems with specific parameters of the telegraph equation and for some forms of external force.

Example 1. Let's consider the following two-point problem:

(12) $$\frac{\partial^2 u}{\partial t^2}+4\frac{\partial u}{\partial t}-\Delta_3 u=c,u\left(0,\overline{x}\right)=0,u\left(1,\overline{x}\right)=0.$$

Problem (12) is of the same class as problem (1), (2), for which $f(t,\overline{x})=c,\ c=const>0$, $a=4,\ b=-1$, $\tau=1$.

Since $R(O)=2,\eta(O)=\frac{e^{-2}\sinh 2}{2}\neq 0$, then $O\notin L$. Taking into account that

$$T_1(t,O)=\frac{e^{-2t}\sinh[2-2t]}{\sinh 2},\ T_2(t,O)=\frac{e^{-2(t-1)}\sinh[2t]}{\sinh 2},$$

we find the solution of problem (12) by formula (11):

$$\begin{aligned}u\left(t,\overline{x}\right)&=cF(t,0,O)=\\&=c\lim_{\lambda\to0}\frac{e^{\lambda t}-T_1\left(t,O\right)-e^{\lambda}T_2\left(t,O\right)}{\lambda^2+4\lambda}\\&=c\lim_{\lambda\to0}\frac{e^{\lambda t}\sinh[2]-e^{-2t}\sinh[2-2t]-e^{\lambda-2t+2}\sinh[2t]}{\left(\lambda^2+4\lambda\right)\sinh[2]}\\&=c\lim_{\lambda\to0}\frac{e^{\lambda t+4}-e^{\lambda t}-e^{-4t+4}+1-e^{\lambda+4}+e^{\lambda-4t+4}}{\left(\lambda^2+4\lambda\right)\left(e^4-1\right)}=c\left[\frac{0}{0}\right]\end{aligned}$$

$$=c\lim_{\lambda\to0}\frac{te^{\lambda t+4}-te^{\lambda t}-e^{\lambda+4}+e^{\lambda-4t+4}}{\left(2\lambda+4\right)\left(e^4-1\right)}=c\frac{t(e^4-1)+e^4\left(e^{-4t}-1\right)}{2\left(e^4-1\right)}$$

The solution of the problem (12)

$$u\left(t,\bar{x}\right)=c\frac{t(e^{4}-1)+e^{4}\left(e^{-4t}-1\right)}{2\left(e^{4}-1\right)}$$

depends only on time and does not depend on spatial variables. The function

$$v\left(t\right)=c\frac{1-e^{4}+4e^{4-4t}}{4\left(1-e^{4}\right)}$$

is the speed of propagation of electromagnetic waves. The behavior of the functions $u\left(t,\bar{x}\right)$ and $v\left(t\right)$ for $c = 2$ are represented in Fig.1 and Fig.2 respectively.

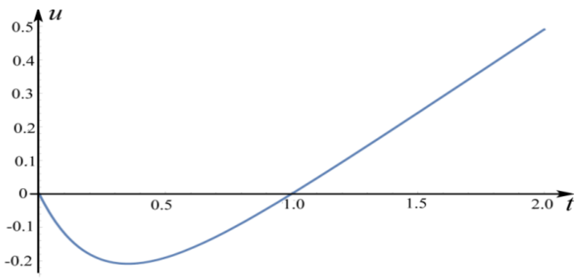

Fig.1. The behavior of the function $u\left(t,\bar{x}\right)$

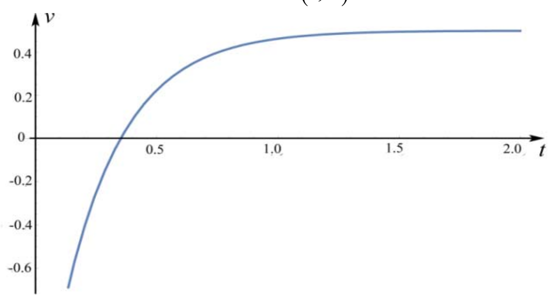

Fig.2. The function $v\left(t\right)$

Example 2. Let us research the problem

(13) $$\frac{\partial^{2}u}{\partial t^{2}}+4\frac{\partial u}{\partial t}-\Delta_{3}u=\cos(2t+x-2y),u\left(0,\bar{x}\right)=0,u\left(1,\bar{x}\right)=0$$

which is problem (1), (2) for $f\left(t,\bar{x}\right)=\cos(2t+x-2y)$, $a = 4$, $b = -1$, $\tau = 1$.

We find the solution of the problem (13) by formula (11):

$$u(t,\bar{x})=\cos\left(2\frac{\partial}{\partial\lambda}+\frac{\partial}{\partial\nu_{1}}-2\frac{\partial}{\partial\nu_{2}}\right)\left\{F(t,\lambda,\nu)e^{\nu\cdot\bar{x}}\right\}\Big|_{\lambda=0,\nu=O}=$$

$$=\frac{F(t,2i,i,-2i,0)e^{i(x-2y)}+F(t,-2i,-i,2i,0)e^{-i(x-2y)}}{2}$$

$$=\frac{1}{65}\left(\cos[2t+x-2y]+8\sin\left[2t+x-2y\right]\right.$$

$$-e^{-2t+2}\frac{\sin\left[t\right]}{\sin\left[1\right]}\left(\cos\left[x-2y+2\right]+8\sin\left[x-2y+2\right]\right)$$

$$\left.+e^{-2t}\left(\cos\left[x-2y\right]+8\sin\left[x-2y\right]\right)\left(ctg[1]\sin[t]-\cos\left[t\right]\right)\right)$$

Changing $x-2y$ in the solution by w, we get

$$u\left(t,w\right)=\frac{1}{65}\left(\cos[2t+w]+8\sin\left[2t+w\right]\right.$$

$$+e^{-2t}\left(\cos\left[w\right]+8\sin\left[w\right]\right)\left(ctg[1]\sin[t]-\cos\left[t\right]\right)$$

$$\left.-e^{-2t+2}\frac{\sin\left[t\right]}{\sin\left[1\right]}\left(\cos\left[w+2\right]+8\sin\left[w+2\right]\right)\right).$$

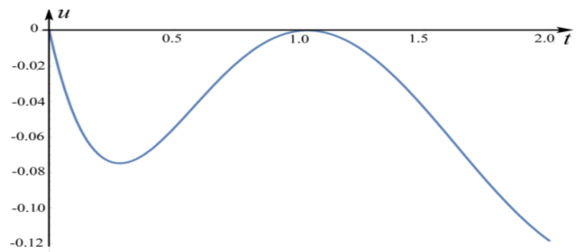

Fig.3. The behavior of the function $u\left(t,\bar{x}\right)$ at the point $\bar{x}=O$

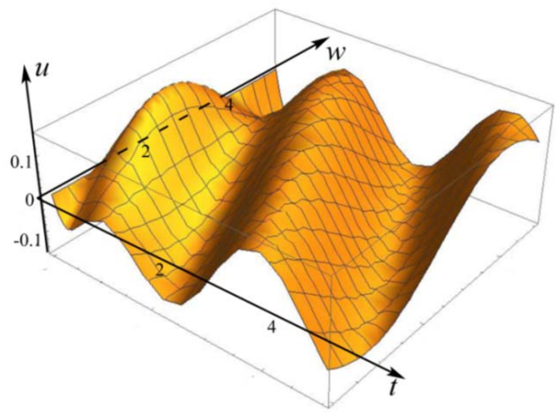

Fig.4. The behavior of function $u\left(t,w\right)$

We note that the solution of problem (13) (as well as the right-hand side of the equation $f\left(t,w\right)=\cos(2t+w)$) is 2π –periodic function by w.

Conclusions

The process of propagation of electromagnetic waves in an unbounded domain with known values of the process at two moments of time is modelled by a two-point problem in time Conditions of existences a unique solution are found and the formula for constructing this solution was found. Proposed method proved to be useful, what was illustrated by the examples.

Authors: *Waldemar Wojcik, Lublin University of Technology, Lublin, Poland, e-mail: waldemar.wojcik@pollub.pl; Zinovii Nytrebych, Lviv Polytechnic National University, Lviv, Ukraine, e-mail: znytrebych@gmail.com; Oksana Malanchuk, Danylo Halytsky Lviv National Medical University, Lviv, Ukraine, e-mail: oksana.malan@gmail.com; Maria Vistak, Danylo Halytsky Lviv National Medical University, Lviv, Ukraine, e-mail: vistak_maria@ukr.net; Liubov M. Kravchenko, Poltava V.G. Korolenko National Pedagogical University, e-mail: 63327@ukr.net; Bogdan Pinaev, Vinnytsia National Technical Univerty, Ukraine, e-mail: pinaev.bogdam@gmail.com; Saule Rakhmetullina, East Kazakhstan State Technical University named after D.Serikbayev, Ust-Kamenogorsk, Kazakhstan, e-mail: rakhmetulinas@mail.ru; Ainur Toigozhinova Academy of Logistics and Transport, Kazakhstan e-mail: aynur_t@mail.ru.*

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