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Internal Model Control for Underactuated Systems based on Novel Virtual Inputs Method

***Abstract**. An internal model control strategy is proposed in this paper for underactuated linear systems. Their associated models are non-square. When applying internal model control strategy, a specific inversion of a square model is needed to design the controller. For that reason, squaring the model of underactuated system by adding virtual control inputs is proposed in this paper. The obtained internal model structure is then modified in order to eliminate the excess inputs. Simulation results on a three inputs/four outputs system is considered to prove the effectiveness and reliability of the proposed method.*

***Streszczenie.** W artykule zaproponowano strategię kontroli modelu wewnętrznego dla niedostatecznie dostosowanych systemów liniowych. Powiązane z nimi modele nie są kwadratowe. Przy stosowaniu strategii kontroli modelu wewnętrznego do zaprojektowania regulatora potrzebna jest specyficzna inwersja modelu kwadratowego. Z tego powodu w niniejszym artykule zaproponowano podniesienie do kwadratu modelu niedostatecznie uruchomionego systemu poprzez dodanie wirtualnych wejść sterujących. Uzyskana struktura modelu wewnętrznego jest następnie modyfikowana w celu wyeliminowania nadmiernych nakładów. Uważa się, że wyniki symulacji w systemie trzech wejść / czterech wyjść potwierdzają skuteczność i niezawodność proponowanej metody.(**Kontrola modelu wewnętrznego dla systemów niedostatecznie aktywowanych w oparciu o nową metodę wirtualnych wejść**)*

Keywords: Internal model control, virtual control, underactuated linear systems, NERGA, ETF.
Słowa kluczowe: Wewnętrzna kontrola modelu, kontrola wirtualna, niedostatecznie uruchamiane systemy liniowe, NERGA, ETF.

Introduction

The control of underactuated systems is an open and interesting automation field. Underactuated systems have fewer inputs than degrees of freedom [1]. Therefore, some degrees of freedom are not excited directly by the actuators. This class of systems includes many applications in robotics, aerospace, naval vessels and submarines. During the past few years, this research area has attracted much attention. Generalized predictive control is proposed in [2], for the stabilization of the inertia wheel inverted pendulum. Position tracking for nonlinear underactuated vehicles is addressed in [3] via a nonlinear Lyapunov controller. Robust control design is proposed for a class of underactuated uncertain nonlinear systems in [4]. In [5], the control of underactuated systems with viability constraints is considered. A class of nonholonomic control solutions is redesigned by means of switching control, so that system trajectories are viable and converge to a goal set.

In this paper, Internal Model Control (IMC) design is investigated for linear underactuated mechanical systems. The IMC strategy is a powerful control approach thanks to its robustness, simplicity and good control performance [6, 7].

The IMC controller corresponds to a specific inversion of an appropriate identified model, since direct inversion is rarely achievable [7, 8]. For underactuated systems, several approaches are considered for the synthesis of a specific inversion of the identified model. In this paper, two approaches will be briefly introduced, the non-square effective relative gain (NERGA) and the equivalent transfer function method (ETF). The NERGA strategy consists on the measurement of interactions between the loops of the system in order to square the system and make it invertible [9, 10]. While, the ETF method consists on tuning the pseudo-inverse of the process to design the internal model controller [11, 12].

In this paper, we focus on the virtual outputs method. The idea behind this approach is simple. Through the introduction of appropriate number of virtual inputs to square the underactuated model. The IMC controller can then be designed and the excess control inputs are eliminated when applied to the system. This method is compared to NERGA and ETF approaches.

Internal model control

The Internal model control is an advanced control strategy introduced by Garcia and Morari in 1982. It has been developed in many forms such as continuous-time, discrete-time, SISO, MIMO stable systems. In the current section, the IMC principle will be presented in the case of square system and a specific design of the MIMO IMC controller will be discussed [13,14]. The MIMO IMC structure is depicted in Fig. 1. G(s) is the multivariable process; M(s) is the model; the MIMO IMC controller C(s) is chosen as the model inverse ; y(s) and ym(s) are the process and model output vectors; u(s) represent the control vector; v(s) is the external disturbance vector; r(s) represents the reference vector; d(s) describes the process-model mismatch and e(s) is the error vector.

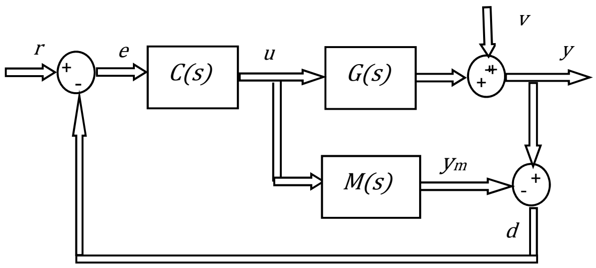

Fig. 1. IMC structure for MIMO systems

The process G(s) of dimension n can be effectively represented by the first order plus time delay (FOPTD) or the second order plus time delay (SOPTD), it is described by the following equation [11]:

(1) $$G(s)=\begin{pmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{pmatrix}$$

$g_{ij}(s)$ is given by the following equation:

(2) $$g_{FOPTD}(s)=\frac{ke^{-\theta s}}{\tau s+1},\quad g_{SOPTD}(s)=\frac{ke^{-\theta s}}{as^2+bs+1}$$

where $k=\left[k_{ij}\right]$, $\theta=\left[\theta_{ij}\right]$ and $\tau=\left[\tau_{ij}\right]$, are respectively steady gain, time delay and time constants.

The closed loop IMC mapping for square systems of dimension n is represented as follows:

(3) $$y(s)=\frac{C(s)G(s)}{I_m+C(s)\big(G(s)-M(s)\big)}r+\frac{I_m-C(s)M(s)}{I_m+C(s)(G(s)-M(s))}v$$

The Internal Model Control, is stable if and only if all elements of the matrix on the right hand of Eq. (3) are stable in open loop. The realization of an IMC controller as the model inverse is essential to ensure perfect set-point tracking. Unfortunately, model inversion for physical systems gives a structure generally unrealizable. For that reason, a specific inversion method is proposed. The IMC controller design proposed for MIMO square systems is illustrated by Fig. 2, [7, 8]

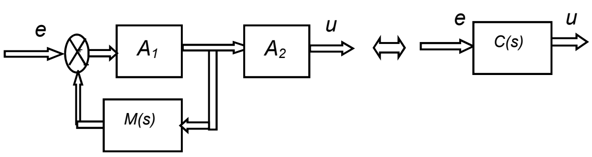

Fig. 2. MIMO internal model controller

The controller transfer function C(s) is described by the following equation:

(4) $$C(s)=A_2(I_n+A_1M(s))^{-1}A_1$$

Where $A_1=\lambda I_{n\times n}$ is a gain matrix, λ is a positive coefficient chosen sufficiently high in order to ensure a better approximation of the model inverse and the matrix $A_2=(A_1M(0))^{-1}\times(I_{n\times n}+A_1M(0))$

Internal model control for underactuated systems

In the present paper, we will dedicate the effort to control underactuated systems having fewer control inputs (n) than degrees of freedom (m), (n<m). The control task of such systems is harder than fully actuated systems. The non-square effective relative gain (NERGA) and the equivalent transfer function method are proposed in literature and are briefly described next.

Model inversion based on NERGA method :

Non-Square Effective relative gain array (NERGA) method solves the problem of singularity of the process transfer function matrix. This method makes the process square by eliminating outputs or inputs that do not affect the dynamics of the multivariable system. In the literature the NERGA method is described as follows [6, 10, 15]:

Step 1: Calculate the effective gain matrix

(5) $$E=G(0)\otimes\Omega$$

Where G(0) is the steady state gain matrix, $\otimes$ is the Hadamard product and Ω is the bandwidth frequency matrix.

Step 2: Calculate the NERGA

(6) $$\Phi=E\otimes\left(E^{+}\right)^{T}$$

$E^{+}=E^{H}\left[EE^{H}\right]^{-1}$, E^{+} is the generalized inverse of E.

Step 3: calculate the sum of each NERGA row and column

By using information from matrix NERGA, it is possible to uniquely determine the sums of elements in each row and each column described as follows:

(7) $$RS=\left[\sum_{j=1}^{n}\phi_{1j}\quad\sum_{j=1}^{n}\phi_{2j}\quad\ldots\quad\sum_{j=1}^{n}\phi_{nj}\right]^{T}$$

(8) $$CS=\left[\sum_{i=1}^{n}\phi_{i1}\quad\sum_{j=1}^{n}\phi_{i2}\quad\ldots\quad\sum_{j=1}^{n}\phi_{im}\right]^{T}$$

Step 4: eliminate the output that corresponds to the smallest sum row

If RS is very close to 0, so the output loop has poor impact on the input loop. This output loop can be removed.

Model inversion based on ETF method :

The equivalent transfer function (ETF) method has been developed to approximate the inverse of the system transfer function matrix. The inverse process equation is expressed as follows [10, 16].

(9) $$\hat{G}(s)=\left[G(s)\right]^{-T}=\begin{pmatrix}\frac{1}{\hat{g}_{11}(s)} & \frac{1}{\hat{g}_{12}(s)} & \cdots & \frac{1}{\hat{g}_{1m}(s)}\\ \frac{1}{\hat{g}_{21}(s)} & \frac{1}{\hat{g}_{22}(s)} & \cdots & \frac{1}{\hat{g}_{2m}(s)}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{1}{\hat{g}_{n1}(s)} & \frac{1}{\hat{g}_{n2}(s)} & \cdots & \frac{1}{\hat{g}_{nm}(s)}\end{pmatrix}$$

The ETFs parameters for FOPTD and SOPTD have the following form [17, 18, 19].

(10) $$\hat{g}_{FOPTD}(s)=\frac{\hat{k}e^{-\hat{\theta}s}}{\hat{\tau}s+1},\quad \hat{g}_{SOPTD}(s)=\frac{\hat{k}e^{-\hat{\theta}s}}{as^2+bs+1}$$

where $\hat{k}=\left[\hat{k}_{ij}\right]$, $\hat{\theta}=\left[\hat{\theta}_{ij}\right]$ and $\hat{\tau}=\left[\hat{\tau}_{ij}\right]$ are steady gain, time delay and time constants.

(11) $$\hat{K}=K^{N}\oplus\Lambda^{N}$$

$\hat{K}$ is the steady state gain matrix, Λ^{N} is The NRGA and $\oplus$ is the Hadamard division.

(12) $$\hat{\theta}=\Gamma^{N}\otimes\theta$$

where Γ^{N} is the NRARTA.

(13) $$\hat{\tau}=\Gamma^{N}\otimes\tau$$

This method has been applied for the design of internal model controller for non-square multivariable system; the controller is calculated as follows [20, 21].

(14) $$C(s)=\hat{G}(s)G_{m+}(s)G_{f}(s)$$

where: $G_f(s)$ is a low pass filter allowing the realization of the internal model controller, $G_{m+}(s)$ is the process model non-minimum phase, it contains the time delay part and the right half-plane zeros [22].

Internal model control based on virtual inputs method :

As discussed previously, the IMC controller is chosen as the model inverse which requires a square transfer matrix [20]. However, the transfer matrix of the model M(s) isn't square so non invertible. The process model M(s) must be square to evaluate the controller parameters. To circumvent this problem, virtual inputs of dimension (m-n) should be added to square the model transfer function matrix. The IMC controller can then be designed. The excess control inputs are eliminated when applied to the system. The virtual inputs method is illustrated through Fig. 3.

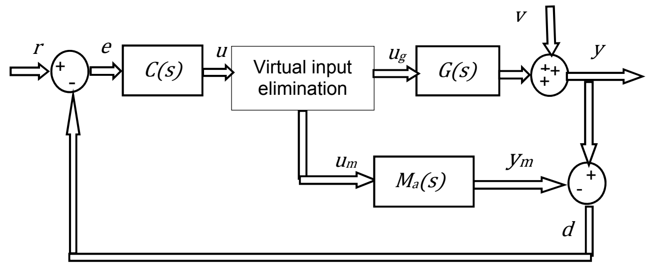

Fig. 3. Internal model control structure for under actuated systems

where : $G(s)$ is the process transfer matrix of dimension (m, n), $M_a(s)$ is the augmented square transfer matrix of the model transfer matrix $M(s)$ of dimension (m, n) and the controller $C(s)$ of dimension (m, m) obtained as the approximate inverse of the square augmented model $M_a(s)$

Let's consider the non-square process *G(s)* and model transfer matrices *M(s)* are given as follows:

(15) $$G(s)=\begin{pmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1}(s) & g_{m2}(s) & \cdots & g_{mn}(s) \end{pmatrix}$$

(16) $$M(s)=\begin{pmatrix} M_{11}(s) & M_{12}(s) & \dots & M_{1n}(s) \\ M_{21}(s) & M_{22}(s) & \dots & M_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1}(s) & M_{m2}(s) & \dots & M_{mn}(s) \end{pmatrix}$$

The IMC based on virtual inputs method is described as follows:

Step 1: Square the model of the underactuated system

The model described by Eq. (16) is squared by adding the missing transfer functions bloc of dimension (m, (m-n)), the augmented model $M_a(s)$ is then described as follows:

(17) $$M_a(s)=\left(\underbrace{\begin{matrix} M_{11}(s) & M_{12}(s) & \dots & M_{1n}(s) \\ M_{21}(s) & M_{22}(s) & \dots & M_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1}(s) & M_{m2}(s) & \dots & M_{mn}(s) \end{matrix}}_{M(s)} \left| \underbrace{\begin{matrix} M_{1n+1}(s) & M_{12}(s) & \dots & M_{1m}(s) \\ M_{2n+1}(s) & M_{22}(s) & \dots & M_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ M_{mn+1}(s) & M_{m2}(s) & \dots & M_{mm}(s) \end{matrix}}_{\text{transfer matrix of dimension } ((m-n)\times m)} \right.\right)$$

Step 2: Calculate the IMC controller C(s)

The IMC controller C(s) is obtained as discussed in the previous section. It corresponds to the approximate inverse of the square model $M_a(s)$.

The control input vector $u \in \Re^{m\times 1}$ is composed of $u_g \in \Re^{n\times 1}$ the real control inputs acting on the process and (m-n) virtual inputs:

(18) $$u = \left[\underbrace{u_1 \dots u_n}_{u_g} \left| \underbrace{u_{n+1} \dots u_m}_{\text{virtual inputs}} \right.\right]$$

Step 3: Eliminate the excess control inputs

The virtual inputs should be eliminated before being applied to the square model $M_a(s)$, in order to reduce the process-model mismatch. The control input vector um is described by the following equation:

(19) $$u_m = \left[\underbrace{u_1 \dots u_n}_{u_g} \left| \underbrace{u_{n+1} = 0 \dots u_m = 0}_{\text{eliminated virtual inputs}} \right.\right]^T$$

Consider the matrices A and B defined as follows:

(20) $$A=\left[I_{n\times n} \left| \underbrace{\begin{matrix} 0 & \dots & 0 \\ \vdots & \dots & \vdots \\ 0 & \cdots & 0 \end{matrix}}_{\text{nul matrix of dimension } ((m-n),\, n)} \right.\right] \quad B=\begin{bmatrix} A \\ \left.\begin{matrix} 0 & \dots & 0 \\ \vdots & \cdots & \vdots \\ 0 & \cdots & 0 \end{matrix}\right\}((m-n),m) \end{bmatrix}$$

The control inputs applied to the process and the model can be described then by the following equations:

(21) $$u_g = A\times u \quad , \quad u_m = B\times u$$

Simulation results

To demonstrate the effectiveness of the proposed controller design method for underactuated systems, we consider the following process, with 3 inputs (u_1, u_2, u_3) and 4 outputs (y_1, y_2, y_3, y_4) described as follows:

(22) $$G(s)=\begin{bmatrix} \dfrac{e^{-8s}}{6s^2+17s+1} & \dfrac{-9e^{-5s}}{s^2+4s+1} & \dfrac{13e^{-3s}}{3s^2+35s+1} \\ \dfrac{-5e^{-13.5s}}{2s^2+19s+1} & \dfrac{8e^{-2s}}{s^2+33s+1} & \dfrac{7e^{-5s}}{s^2+3s+1} \\ \dfrac{-16e^{-3s}}{s^2+5s+1} & \dfrac{3e^{-7s}}{s^2+14s+1} & \dfrac{e^{-11s}}{3s^2+25s+1} \\ \dfrac{5e^{-4s}}{10s+1} & \dfrac{6e^{-5s}}{100s+1} & \dfrac{5e^{-4s}}{10s+1} \end{bmatrix}$$

The eigen values of *G(s)* $(\lambda_1 ... \lambda_6)$ have a negative real part, which confirms the process stability and justifies the IMC application.

(23) $$\begin{aligned} &\lambda_1 = -32.96, \lambda_2 = -13.92, \lambda_3 = -11.63 \\ &\lambda_4 = -9.44, \lambda_5 = -8.29, \lambda_6 = -4.79 \end{aligned}$$

Simulation parameters for the NERGA inversion method:

The NERGA algorithm is applied to the proposed system. The steady state gain matrix *G(0),* band width Ω, effective energy transmission ratio *EG*, NERGA Ψ, sum of the elements in each row *RS* and sum of the elements in each columns *CS* are calculated as follows:

(24) $$G(0)=\begin{bmatrix} 1 & -9 & 13 \\ -5 & 8 & 7 \\ -16 & 3 & 1 \\ 5 & 6 & 5 \end{bmatrix}, \Omega=\begin{bmatrix} 5.99e^{-2} & 2.65e^{-1} & 2.85e^{-2} \\ 5.27e^{-2} & 3.02e^{-2} & 3.73e^{-1} \\ 2.07e^{-1} & -7.16e^{-2} & 4e^{-2} \\ 9.97e^{-2} & 9.97e^{-3} & 9.97e^{-2} \end{bmatrix}$$

$$EG=\begin{bmatrix} -2.52e^{-2} & -1.25e^{-3} & -2.93e^{-1} & 4.90e^{-2} \\ -4.12e^{-1} & 5.48e^{-2} & -8.71e^{-3} & 2.04e^{-2} \\ 3.70e^{-2} & 3.63e^{-1} & -1.71e^{-2} & 7.34e^{-2} \end{bmatrix}$$

(25) $$\Psi=\begin{bmatrix} -1.51e^{-3} & 9.87e^{-1} & 1.37e^{-2} \\ 3.31e^{-4} & 1.32e^{-2} & 9.50e^{-1} \\ 2.07e^{-1} & -1.87e^{-3} & -6.88e^{-4} \\ 2.44e^{-2} & 1.22e^{-3} & 3.66e^{-2} \end{bmatrix}$$

(26) $$RS=\begin{bmatrix}0.99 & 0.96 & 0.97 & 0.06\end{bmatrix}^T, \quad CS=\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$$

From the results shown previously, we notice that the fourth term in RS is the closest term to zero among the other three terms. Thus, the fourth output loop of the system can neglect and consequently removed.

The gain matrix considered for NERGA is $A_1 = 0.01\times I_3$

Simulation parameters for the ETF inversion method :

By using the NRGA, NRNGA, NRARTA concepts, the ETF model parameters for process *G(s)* are deduced through the equations below:

(27) $$\hat{K} = \begin{bmatrix} -2.65e^{2} & -2.40e^{1} & 2.07e^{1} \\ -2e^{2} & 2.20e^{1} & 2.97e^{1} \\ -1.89e^{1} & -3.80e^{2} & 1e^{3} \\ 3.66e^{1} & 2.21e^{1} & 3.68e^{1} \end{bmatrix}, \hat{\tau} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2.64 & 2.79 & 9.82 \end{bmatrix}$$

(28) $$\hat{\theta} = \begin{bmatrix} 5.36e^{0} & 1.23e^{1} & 3.73e^{-1} \\ 9.44e^{-2} & 3.90e^{-1} & 1.67e^{1} \\ 3.44e^{0} & 2.52e^{0} & 2.41e^{-1} \\ 1.05e^{0} & 2.79e^{-2} & 3.92e^{0} \end{bmatrix}$$

The ETF matrix is expressed by Eq. (28).

(29) $$\hat{G}(s) = \begin{bmatrix} \frac{(6s^2+17s+1)e^{5.36s}}{-265.1} & \frac{(s^2+4s+1)e^{12.35s}}{-24.1} & \frac{(3s^2+35s+1)e^{0.37s}}{20.7} \\ \frac{(2s^2+19s+1)e^{0.09s}}{-201} & \frac{(s^2+33s+1)e^{0.39s}}{22.1} & \frac{(s^2+3s+1)e^{16.75s}}{29.7} \\ \frac{(s^2+5s+1)e^{3.44s}}{-19} & \frac{(s^2+14s+1)e^{2.52s}}{-380.2} & \frac{(3s^2+25s+1)e^{0.24s}}{1008.6} \\ \frac{(2.64s+1)e^{1.05s}}{36.6} & \frac{(2.79s+1)e^{0.02s}}{22.1} & \frac{(9.82s+1)e^{3.92s}}{36.9} \end{bmatrix}$$

Finally, we can obtain an IMC controller as described below:

(30) $$C_{IMC}(s) = \begin{bmatrix} \frac{(6s^2+17s+1)e^{-6.98s}}{-265.1(13s+1)^2} & \frac{(2s^2+19s+1)e^{-16.65s}}{-201(12s+1)^2} & \frac{(s^2+5s+1)}{-19(7s+1)^2} & \frac{(2.64s+1)e^{-2.87s}}{36.6(5s+1)^2} \\ \frac{(s^2+4s+1)}{-24.1(13s+1)^2} & \frac{(s^2+33s+1)e^{-16.36s}}{22.1(12s+1)^2} & \frac{(s^2+14s+1)e^{-0.91s}}{-380.2(7s+1)^2} & \frac{(2.79s+1)e^{-3.9s}}{22.1(5s+1)^2} \\ \frac{(3s^2+35s+1)e^{-11.97s}}{20.7(13s+1)^2} & \frac{(s^2+3s+1)}{29.7(12s+1)^2} & \frac{(3s^2+25s+1)e^{-3.19s}}{1008.6(7s+1)^2} & \frac{(9.82s+1)}{36.9(5s+1)^2} \end{bmatrix}$$

Hence, the feedback filter is obtained.

(31) $$G_{m+}(s) = diag\left\{e^{-12.35s}, e^{-16.75s}, e^{-3.44s}, e^{-3.92s}\right\}$$

(32) $$G_f(s) = diag\left\{\frac{1}{(13s+1)^2}, \frac{1}{(12s+1)^2}, \frac{1}{(7+1)^2}, \frac{1}{(5s+1)^2}\right\}$$

Simulation parameters for the novel virtual inputs inversion method :

With the assumption of imperfect modeling, the augmented transfer matrix $M_a(s)$ for the proposed inversion method is expressed as follows:

(33) $$M_a(s) = \begin{bmatrix} \frac{0.5e^{-2s}}{5s^2+15s+1} & \frac{-8e^{-2s}}{s^2+3s+1} & \frac{10e^{-3s}}{3s^2+30s+1} & \frac{2}{10s+1} \\ \frac{-2e^{-10s}}{2s^2+15s+1} & \frac{5e^{-2s}}{s^2+30s+1} & \frac{4e^{-2s}}{s^2+5s+1} & \frac{1}{5s+1} \\ \frac{-15e^{-3s}}{s^2+3s+1} & \frac{4e^{-5s}}{s^2+10s+1} & \frac{2e^{-10s}}{3s^2+20s+1} & \frac{2}{4s+1} \\ \frac{4e^{-4s}}{9s+1} & \frac{e^{-s}}{90s+1} & \frac{3e^{-4s}}{9s+1} & \frac{1}{5s+1} \end{bmatrix}$$

The first three columns of Ma corresponds to the transfer matrix M(s): $M(s) = (M_{ij})_{\substack{i=1\ldots4 \\ j=1\ldots3}}$, the last column of M_a matches the added transfer vector of dimension (4, (4-3)).

The $M_a(s)$ is stable. Its correspondent eigen values have negative real parts:

(34) $$\lambda_1 = -29.96, \lambda_2 = -9.96, \lambda_3 = -9.89, \lambda_4 = -7.43, \\ \lambda_5 = -6.61, \lambda_6 = -4.79$$

The gain matrix A_1 considered for the virtual inputs method and that ensures the regulator and the IMC structure stability is : $A_1 = 0.01 \times I_4$

<u>Nominal case</u>

The step responses of outputs $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$ in the nominal case (in the absence of disturbances) are displayed respectively in Fig. 4, Fig. 5, Fig. 6 and Fig. 7. It can be noticed that the internal model controller based on virtual inputs method, has a smaller overshoot as compared to other methods (ETF and NERGA), the system has good traceability and accuracy. In fact, for the ETF method, diminished control performance is noticed. While the application of NERGA method isn't efficient due to the neglected dynamics that may cause performance degradation. In our case, the output $y_4(t)$ is neglected. Besides, this method can't be efficiently applied when the sums of the elements of each column of the NERGA matrix are very close to each other's.

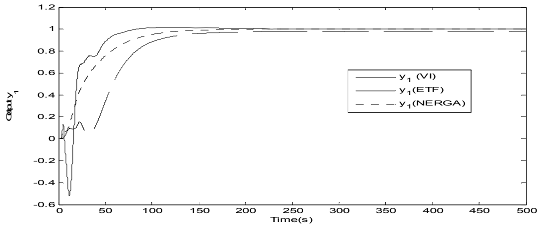

Fig. 4. System output $y_1(t)$ (nominal case)

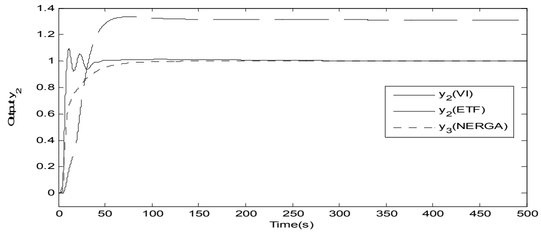

Fig. 5. System output $y_2(t)$ (nominal case)

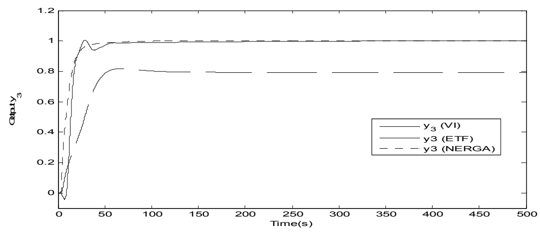

Fig. 6. System output $y_3(t)$ (nominal case)

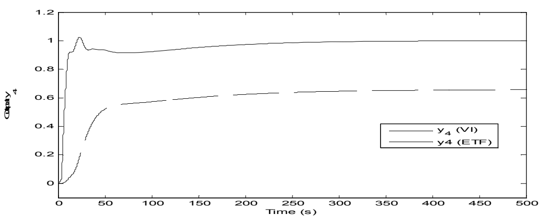

Fig. 7. System output y_4 (t) (nominal case)

<u>Robustness towards disturbances</u>

The robustness of the proposed method (IMC based on virtual inputs method) is proved through simulation results. The proposed disturbances are steps of amplitude 0.5 that occur at t = 150s. System outputs are displayed in Fig. 8, Fig. 9, Fig. 10 and Fig. 11. We can notice that the set-point tracking is established despite the persistent disturbances.

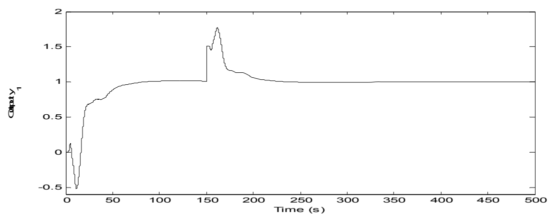

Fig. 8. System output $y_1(t)$ (in the presence of disturbances)

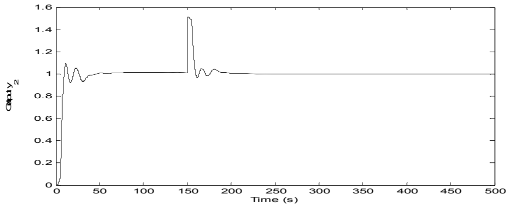

Fig. 9. System output $y_2(t)$ (in the presence of disturbances)

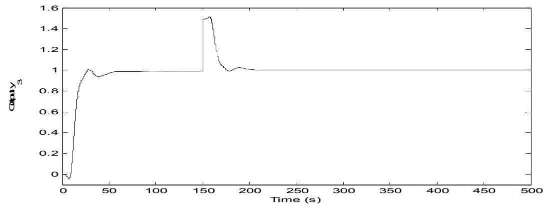

Fig. 10. System output $y_3(t)$ (in the presence of disturbances)

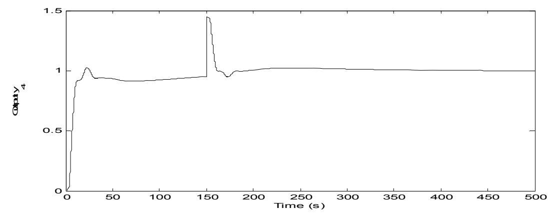

Fig. 11. System output $y_4(t)$ (in the presence of disturbances)

Conclusion

In this paper, a novel internal model control strategy for linear underactuated systems based on virtual inputs method was developed. The model of underactuated systems is non square. However, a square model is needed to design the internal model controller by a specific inversion method. The under-actuation issue was solved in this paper by squaring the system's model by adding the appropriate number of inputs. The excess control inputs are then eliminated when applied to the system. Numerical example exhibited the efficiency of the proposed control strategy based on virtual inputs method. The obtained controller is simple, easy to implement and presents good control and robustness performances as compared to the non-square effective relative gain array and equivalent transfer functions approaches considered in literature for the internal model control of underactuated systems.

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REFERENCES

[1] Manai N.H., Saidi I., Soudani D., Predictive control of an under-actuated System, The 3rd International Conference on Advanced Systems and Emergent Technologies, Tunisia (2019), 90-95.
[2] Touati N., Chemori A., Predictive Control for the Stabilization of a Fast Mechatronic System: from Simulation to Real-time Experiments, The International Federation of Automatic Control, 6th IFAC Symposium on Mechatronic Systems, Hangzhou, China, (2013), 237-242.
[3] Saidi I., Touati N., Nonlinear predictive control for trajectory tracking of underactuated mechanical systems, Przegląd Elektrotechniczny, 97 (2021), nr. 6, 30-33.
[4] Saidi I., N. Touati N., Sliding mode control to stabilization of nonlinear Underactuated mechanical systems, Przegląd Elektrotechniczny, 97 (2021), nr. 7.
[5] Din S. U., Khan Q., Rehman F. U., Akmeliawati R., Robust Control of Underactuated Systems: Higher Order Integral Sliding Mode Approach, Mathematical Problems in Engineering, (2016).
[6] Saidi I., N. Touati, Dhahri A., Soudani D., A comparative study on existing and new methods to design internal model controllers for non-square systems, Transactions of the Institute of Measurement and Control, 41 (2019), No.13, 3637-3650.
[7] Dhahri A., Saidi I., Soudani D., Internal model control for multivariable over-actuated systems with multiple time delay, 4th International Conference on control Engineering & Information Technology, Tunisia, (2016), 623-626.
[8] Touati N., Saidi I., Dhahri A., Soudani D., Internal multimodel control for nonlinear overactuated systems, Arabian Journal for Science and Engineering, (2018), 1-9.
[9] Dong R., Tan Y., Internal model control for dynamic systems with preceded backlash, Journal of Dynamic Systems, Measurement, and Control, 13 (2015), nr 2.
[10] Ling Q., Qibing J., Xuewei W., Based NERGA a novel decentralized PID for non-square system, Advanced materials research,180 (2011), 1143-1149
[11] Quan L., Zhang H., Yang Y., A novel internal model control method for non-square system, Applied Mechanics and Materials, 238(2012), 66-70
[12] Luan X., Chen Q., Liu F., Centralized PI control for high dimensional multivariable systems based on equivalent transfer function, ISA Transactions, (2014)
[13] Daniel KORENČIAK, Miroslav GUTTEN, Opportunities for integration of modern systems into control processes in intelligent buildings, Przegląd Elektrotechniczny, 88 (2012), nr 5, 266-269.
[14] Mateusz ŻYCHLEWICZ, Piotr DERUGO, Zastosowanie sterowania z modelem wewnętrznym opartego o sieci neuronowe dla napędu z elastycznym sprzęgłem, Przegląd Elektrotechniczny, 94 (2018), nr 2, 163-168.
[15] Garcia C. G., Morari M., Internal model control. 1.a unifying review and some results, Industrial Engineering Chemistry Process Design and Development, 2 (1982), 403–411.
[16] Jin Q., DU X., Jiang B., Novel centralized IMC-PID controller design for multivariable processes with multiple time delays, Industrial & engineering chemistry research, (2017)
[17] Dhahri A., Saidi I., Soudani D., A New internal model control method for MIMO over-actuated systems, International Journal of Advanced Computer Science and Applications, 7(2016), nr10, 304-309.
[18] Xiong Q., Cai W. J., He M. J., Equivalent transfer function method for PI/PID controller design of MIMO Process, Journal of Process Control, 17(2007), 665-673.
[19] Xiong Q., Jian C., Effective transfer function method for decentralized control system Design of Multi-Input Multi-Output Process, Journal of Process Control, 16 (2006), 773-784.
[20] Jin Q., Jiang B, Wang Q., Shan G., Decoupling internal model control for non-square processes based on equivalent transfer function, Transactions of the institute of Measurement and Control, 36(2014), nr 8, 1114-1131.
[21] Chen P. Y., Ou L. L., Modified internal model control and its application in non-square processes, Control and decision, 23(2008), nr5, 581-584.
[22] Bejaoui I., Saidi I., Xibilia M. G., Soudani D., Internal Model Control of Discrete Non-Minimum Phase Over-Actuated Systems with Multiple Time Delays and Uncertain Parameters, Journal of Engineering Science and Technology Review, vol. 12, no. 2, pp. 111-118, Juillet 2019.