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doi:10.15199/48.2021.09.21

An Optimal Design of Non-Causal Recursive Digital Filters with Zero Phase Shift using Chebyshev approximation and Linear Programming

***Abstract**. This paper presents an optimal design for the special class of Non-Causal Recursive (NR) digital filters with zero phase shift. The design is based on the Chebyshev approximation problem. It can be transformed to an equivalent linear program under linear constraints of the zero phase. The given design yields more interesting pole-zero patterns that are not necessarily restricted to the classical design of Kormylo and Jain. The proposed optimal design allows an accurate zero phase shift and better magnitude characteristics in passband and stopband.*

Streszczenie.** W artykule przedstawiono optymalną procedurę projektowania dla specjalnej klasy nieprzyczynowych filtrów rekurencyjnych (NR) z zerowym przesunięciem fazowym. Projekt opiera się na problemie aproksymacji Czebyszewa. Można go przekształcić do równoważnego programu liniowego przy ograniczeniach liniowych fazy zerowej. Projekt daje bardziej interesujące wzory bieguna zerowego, które niekoniecznie ograniczają się do klasycznego projektu Kormylo i Jaina. Zaproponowana optymalna procedura umożliwia dokładne zerowe przesunięcie fazowe i lepszą charakterystykę amplitudy w paśmie przepuszczania i zatrzymywania.* (Optymalna procedura projektowania nieprzyczynowych rekurencyjnych filtrów cyfrowych z zerowym przesunięciem fazowym przy użyciu przybliżenia Czebyszewa i programowania liniowego***)

Keywords: non-causal recursive digital filters - linear programming - Chebyshev approximation – optimization – zero phase shift
Słowa kluczowe: filtry cyfrowe, projektowanie, aproksymacja Czebyszewa.

Introduction

Filters having high performances in both magnitude and phase are strongly recommended in special applications involving noise suppression, such as extracting the desired pure electrocardiogram (ECG) content from the measured ECG signal [1]. For the limited class of problems, where filtering is permissible off-line, a Non-causal Recursive (NR) digital filter can be realized by a tandem connection of an arbitrary transfer function $H_1(z)=H(z)$ and time reversed version of the same function $H_2(z^{-1})=H(z^{-1})$ [2-8] (Fig.1). This special class of NR digital filters can offer highly magnitude characteristics with theoretically zero phase shift and optimal computational burdens per output sample [4-6]. Throughout the last four decades, Kormylo and Jain's design [2] was the most widespread used in designing NR filters [4-13]. This classical design requires specifications in the frequency domain. It is based on identical transfer functions $H_1(z)=H_2(z)=H(z)$, it also uses the bilinear transform and well-known optimal designs of analog elliptic filters [14,15], such that the magnitude characteristic of the causal transfer function $H(z)$ has to match the square root of the desired frequency response of an NR filter. The design procedure is fast and simple, however, despite of using an optimal elliptic transfer function $H(z)$, the squared magnitude response $|H(e^{j\theta})|^2$ is not necessarily the optimal frequency response of the NR digital filter in both magnitude and phase (here, θ is the normalized frequency). Additionally, we encounter sometimes certain inefficiency in the use of identical transfer functions, because the Stop-Band zeros of the NR filter's transfer function $H_{NR}(z)=H(z)H(z^{-1})$ appear as double zeros along with elliptic poles on mirror images patterns inside and outside the unit circle. Willson and Orchard [11] have proposed an improvement of the reported design [2] by distributing all double zeros throughout the filter's Stop-Band (SB). The tandem connection $H_1(z)H_2(z^{-1})$ has zero phase shift and a flat magnitude characteristic in the Pass-Band (PB) along with an additional loss of 6 dB in the SB. However, the separate non-identical transfer functions $H_1(z)$ and $H_2(z)$ can not exhibit flat magnitude characteristics in the PB as the previous Kormylo and Jain's design [2]. Other downside to these designs [2, 11] is the fact that they are not based on an optimization methodology.

In this paper, an optimal design of an NR digital filter is proposed on the basis of the Chebyshev approximation problem. It can be transformed to an equivalent linear program under particular linear constraints that allow zero phase shift. The given approach enables more possible and interesting pole-zero placements, which are not necessarily restricted to the previous cases of elliptic poles in mirror-images pairs and double zeros laying on the unit circle. The design leads to an optimal solution in terms of magnitude and phase. The magnitude response can be better in both the PB and the SB over previous design of Kormylo and Jain [2]. Generally, we get flat magnitude characteristics in the tandem connection $H_1(z)H_2(z^{-1})$ as well as in the separate causal transfer functions $H_1(z)$ and $H_2(z)$. Additionally, according to several experiments, the zero phase response accuracy has generally improved with the proposed design.

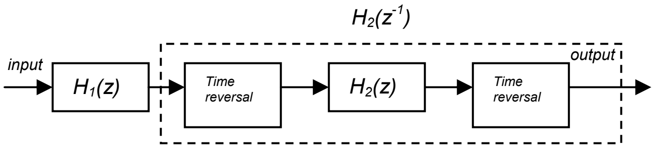

Fig.1. A Noncausal Recursive (NR) digital filter realization

First, formulations of the zero phase shift constraints and the Chebyshev approximation problem are presented. The constraints as functions of the normalized frequency θ are given in terms of PB and SB. A linear programming formulation is also explained. Afterwards, we describe the optimization procedure that solves iteratively the proposed linear programming. Next, two filter design examples are illustrated. Finally, conclusions are given at the end of this paper.

Formulation of the zero phase shift constraints

Consider the generalized NR filters realization [3] in Fig.1, where $H_1(z)$ *and* $H_2(z)$ be the transfer functions of the Nth order of causal IIR filters such that:

(1) $H_1(z)=\dfrac{\sum\limits_{k=0}^{N} b_k z^{-k}}{\sum\limits_{k=0}^{N} a_k z^{-k}}$ and $H_2(z)=\dfrac{\sum\limits_{k=0}^{N} \beta_k z^{-k}}{\sum\limits_{k=0}^{N} \alpha_k z^{-k}}$

a_k , b_k , α_k and β_k are real coefficients.

Thus, the equivalent transfer function $H_{NR}(z)= H_1(z)H_2(z^{-1})$ of 2Nth order of the NR filter can be expressed as:

(2) $$H_{NR}(z)=\frac{\sum\limits_{k=-N}^{N} c_k z^{-k}}{\sum\limits_{k=-N}^{N} d_k z^{-k}}=\frac{\hat{N}(z)}{\hat{D}(z)}$$

Where:

(3) $\hat{N}(z)=\sum\limits_{k=-N}^{N} c_k z^{-k}$ and $\hat{D}(z)=\sum\limits_{k=-N}^{N} d_k z^{-k}$

For $z=e^{j\theta}$, the frequency response of the NR filter is:

(4) $$H_{NR}\left(e^{j\theta}\right)=H_1(z)H_2\left(z^{-1}\right)\Big|_{z=e^{j\theta}}=\frac{\hat{N}(\theta)}{\hat{D}(\theta)}$$

Such that:

(5) $\hat{N}(\theta)=\sum\limits_{k=-N}^{N} c_k e^{-jk\theta}$ and $\hat{D}(\theta)=\sum\limits_{k=-N}^{N} d_k e^{-jk\theta}$

The equivalent frequency response $H_{NR}(e^{j\theta})$ of the NR filter can be expressed as:

(6) $$H_{NR}\left(e^{j\theta}\right)=\frac{c_0+\sum\limits_{k=1}^{N}\left(c_k+c_{-k}\right)\cos(k\theta)-j\sum\limits_{k=1}^{N}\left(c_k-c_{-k}\right)\sin(k\theta)}{d_0+\sum\limits_{k=1}^{N}\left(d_k+d_{-k}\right)\cos(k\theta)-j\sum\limits_{k=1}^{N}\left(d_k-d_{-k}\right)\sin(k\theta)}$$

If we consider the special case where all poles of the equivalent transfer function $H_{NR}(z)$ are in quadrantal symmetry. This situation can be found, where poles of $H_1(z)$ are those of $H_2(z)$. So, the set of coefficients $\{d_k\}_{-N\le k\le N}$ can be considered as an autocorrelation function, where it will have the maximum at d_0 and the others coefficients are symmetric, such as: $d_k=d_{-k}$ for $k=1,\ldots,N$.

In this case, the denominator $\hat{D}(\theta)$ is a real function as follows:

(7) $$\hat{D}(\theta)=d_0+2\sum_{k=1}^{N} d_k\cos(k\theta)$$

Thus, $H_{NR}(e^{j\theta})$ can be expressed with real and imaginary parts as:

(8) $$H_{NR}\left(e^{j\theta}\right)=\left[\frac{c_0+\sum\limits_{k=1}^{N}\left(c_k+c_{-k}\right)\cos(k\theta)}{d_0+2\sum\limits_{k=1}^{N} d_k\cos(k\theta)}\right]-j\left[\frac{\sum\limits_{k=1}^{N}\left(c_k-c_{-k}\right)\sin(k\theta)}{d_0+2\sum\limits_{k=1}^{N} d_k\cos(k\theta)}\right]$$

$H_{NR}(e^{j\theta})$ can have zero phase shift if it is a real function, so its imaginary part equals to zero.

So, we have:

(9) $$\sum_{k=1}^{N}\left(c_k-c_{-k}\right)\sin(k\theta)=0$$

Finally, (9) can be expressed as a linear constraint over the set of the coefficients $\{c_k\}_{-N\le k\le N}$:

(10) $$\sum_{k=-N}^{N} c_k\sin(k\theta)=0$$

Formulation of the Chebyshev approximation problem

Let $F(\theta)$ be the desired frequency response of the NR digital filter, which is a specified function of frequency. $\delta(\theta)$ is the positive tolerance function of frequency such that:

(11) $$\delta(\theta)=\left|\frac{\hat{N}(\theta)}{\hat{D}(\theta)}-F(\theta)\right|$$

If we consider under the procedure of optimization that the maximum errors are respectively in the PB $\hat{\delta}=\max_{\theta\in PB}\left[\delta(\theta)\right]$, and in the SB $\delta=\max_{\theta\in SB}\left[\delta(\theta)\right]$.

Without lose of generality, the approximation problem consists of finding the coefficients c_k and d_k $(-N\le k\le N)$; such that we get the minimum of $\hat{\delta}$ / ($\theta\in PB$) and δ / ($\theta\in SB$) under the constraints:

(12) $$-\hat{\delta}\le\frac{\hat{N}(\theta)}{\hat{D}(\theta)}-F(\theta)\le 0 \text{ if } \theta\in PB$$

and

(13) $$0\le\frac{\hat{N}(\theta)}{\hat{D}(\theta)}-F(\theta)\le\delta \text{ if } \theta\in SB$$

In order to keep the same sign $\le$ of inequalities after multiplication by $\hat{D}(\theta)$, we suppose that $\hat{D}(\theta)>0$. In this situation, the constraints (12) and (13), which we call principle constraints, can be expressed as a set of linear inequalities in relation with the set of the coefficients $\{c_k\}$ and $\{d_k\}$ $(-N\le k\le N)$ by writing them in the form:

(14) $$\begin{cases}\hat{N}(\theta)-\hat{D}(\theta)F(\theta)\le 0\\ -\hat{N}(\theta)+\hat{D}(\theta)\left[F(\theta)-\hat{\delta}\right]\le 0 \text{ if } \theta\in PB\\ -\hat{D}(\theta)<0\end{cases}$$

and

(15) $$\begin{cases}\hat{N}(\theta)-\hat{D}(\theta)\left[F(\theta)+\delta\right]\le 0\\ -\hat{N}(\theta)+\hat{D}(\theta)F(\theta)\le 0 \quad \text{ if } \theta\in SB\\ -\hat{D}(\theta)<0\end{cases}$$

Additional inequalities, which we call supplemental constraints, can be used where we take in consideration the maximum of tolerances imposed by the specifications: δ_p and δ_s respectively in the PB and the SB, such that:

(16) $$-\hat{N}(\theta)+\hat{D}(\theta)\left[F(\theta)-\delta_p\right]\le 0 \text{ if } \theta\in PB$$

and

(17) $$\hat{N}(\theta)-\hat{D}(\theta)\left[F(\theta)+\delta_s\right]\le 0 \text{ if } \theta\in SB$$

Also, we take in consideration that $\hat{D}(\theta)>0$ and $\dfrac{\hat{N}(\theta)}{\hat{D}(\theta)}$ is a real function (in order to have zero phase shift), we impose also that: $\hat{N}(\theta)\ge 0$, in order to get positive magnitude frequency responses.

To simplify the problem of optimization, we consider that: $\dfrac{\hat{\delta}}{\delta}=K$. If we are interesting to optimize the maximum of error δ in the SB, we get the new formulation of the approximation problem as shown:

(18) $$\underset{over: c_k, d_k, \delta}{Minimize} \; (\delta)$$

under the constraints:

(19) $$\begin{cases} \hat{N}(\theta) - \hat{D}(\theta) F(\theta) \le 0 \\ -\hat{N}(\theta) + \hat{D}(\theta)\left[F(\theta) - K\delta\right] \le 0 \\ -\hat{D}(\theta) < 0 \\ -\hat{N}(\theta) \le 0 & \text{if } \theta \in PB \\ -\hat{N}(\theta) + \hat{D}(\theta)\left[F(\theta) - \delta_p\right] \le 0 \\ \sum\limits_{k=-N}^{N} c_k \sin(k\theta) = 0 \end{cases}$$

and

(20) $$\begin{cases} \hat{N}(\theta) - \hat{D}(\theta)\left[F(\theta) + \delta\right] \le 0 \\ -\hat{N}(\theta) + \hat{D}(\theta) F(\theta) \le 0 \\ -\hat{D}(\theta) < 0 \\ -\hat{N}(\theta) \le 0 & \text{if } \theta \in SB \\ \hat{N}(\theta) - \hat{D}(\theta)\left[F(\theta) + \delta_s\right] \le 0 \\ \sum\limits_{k=-N}^{N} c_k \sin(k\theta) = 0 \end{cases}$$

Linear programming formulation

In order to transform the above approximation problem (18), (19) and (20) into a linear programming, the maximum error *δ* in the SB is considered as a constant during the optimization procedure; also, an auxiliary variable λ is subtracted from the left side of each inequality constraint, forming the new linear programming formulation:

(21) The cost function to be minimized is: $y = \lambda$

under the constraints:

(22) $$\begin{cases} \hat{N}(\theta) - \hat{D}(\theta) F(\theta) - \lambda \le 0 \\ -\hat{N}(\theta) + \hat{D}(\theta)\left[F(\theta) - K\delta\right] - \lambda \le 0 \\ -\hat{D}(\theta) - \lambda < 0 \\ -\hat{N}(\theta) - \lambda \le 0 & \text{if } \theta \in PB \\ -\hat{N}(\theta) + \hat{D}(\theta)\left[F(\theta) - \delta_p\right] - \lambda \le 0 \\ \sum\limits_{k=-N}^{N} c_k \sin(k\theta) = 0 \end{cases}$$

and

(23) $$\begin{cases} \hat{N}(\theta) - \hat{D}(\theta)\left[F(\theta) + \delta\right] - \lambda \le 0 \\ -\hat{N}(\theta) + \hat{D}(\theta) F(\theta) - \lambda \le 0 \\ -\hat{D}(\theta) - \lambda < 0 \\ -\hat{N}(\theta) - \lambda \le 0 & \text{if } \theta \in SB \\ \hat{N}(\theta) - \hat{D}(\theta)\left[F(\theta) + \delta_s\right] - \lambda \le 0 \\ \sum\limits_{k=-N}^{N} c_k \sin(k\theta) = 0 \end{cases}$$

It is clearly that the approximation problem in (21), (22) and (23) can have a solution if and only if the minimum value of λ is 0; otherwise, no solution to the approximation problem is found, and either $F(\theta)$, or *δ* or both must be modified in order to obtain a solution.

Procedure of optimization

To determine the smallest value of *δ* such that the linear programming (21), (22) and (23) has a solution, an iterative procedure should be used since *δ* enters into the design constraints. Before, let assume that the sum of the maximum of the passband ripple $\hat{\delta} = K\delta$ and stopband ripple *δ* should be less than or equal to 1 [16]. If we let δ^* denote the minimum value of *δ* for which the approximation problem has a solution. We have:

(24) $$0 < \delta^* \le \frac{1}{K+1}$$

The initial lower $\delta^i_- = 0$ and upper $\delta^i_+ = 1/(K+1)$ bounds can be used to locate δ^* by a binary search procedure, when at each step *i*, *δ* is calculated by a geometric mean ($\delta = \sqrt{\delta^i_- \delta^i_+}$). Note that, the geometric mean can reduce dynamic between these two lower δ^i_- and upper δ^i_+ bounds in comparison with the arithmetic mean. So, we get a fast binary search [16]. The procedure of optimization can be described as follows:

Step 1: initialization
Let: $\delta^i_- = 10^{-15}$ (since $\delta^i_- < 10^{-15}$ is unrealistic), $\delta^i_+ = 1/(K+1)$ and $\delta = \sqrt{\delta^i_- \delta^i_+}$

Step 2: Solve the above linear programming (21), (22) and (23) with the value of *δ*. If the cost function $y=0$, a solution to the approximation problem exists and $\delta^* < \delta$. In this case, we can reduce the upper bound and we set $\delta^i_+ = \delta$. Otherwise, no solution to the approximation problem exist for this value of *δ* and $\delta^* > \delta$. In this case, the lower bound is increased and we set $\delta^i_- = \delta$.

Step 3: set $\delta = \sqrt{\delta^i_- \delta^i_+}$, and repeat step 2 until:

$\sigma = |\delta^i_+ - \delta^i_-|$ reaches a predetermined accuracy. Generally, we stop processing where σ reaches a spacing floating point number [16].

Finally, polynomials $\hat{N}(z)$ and $\hat{D}(z)$ in (3) can be factorized according to the final coefficients $\{c_k\}_{-N \le k \le N}$ and $\{d_k\}_{-N \le k \le N}$ that are associated with the optimal value δ^*.

Poles and zeros inside the unit circle are those of the causal minimum phase transfer function $H_1(z)$; where, whose outside the unit circle are assigned to the noncausal maximum phase time reversed transfer function $H_2(z^{-1})$. If zeros lie on the unit circle, the assignment to $H_1(z)$ or $H_2(z^{-1})$ can be done such that the pairs of zeros in complex conjugate lead as possible to flat magnitude characteristics for the separate causal transfer functions $H_1(z)$ and $H_2(z)$.

Note that the NR filter order (2N) can be fixed to a prescribed value, or initially estimated to its minimum value according to the specifications as in Kormylo and Jain design [2].

Additionally, if the desired performances are not satisfied because the NR filter order *2N* is under-estimated, then it can be increased in order to repeat again the described algorithm interactively until the required performances can be found.

Examples of design

In this section, two design examples are shown. The linear programming in (21), (22) and (23) can be solved by using the linprog() command from the optimization Toolbox of Matlab [17], with a double precision arithmetic on 64 bits machine. The commonly Simplex method is not suitable to solve a large scale problem as the filter design; so, we use this command with the option of interior point methods [17]. It seems more adequate in getting acceptable solution.

Results are compared with those of Kormylo and Jain's design [2]. The Willson and Orchard's design [11] is not considered here since the separate transfer functions $H_1(z)$ and $H_2(z)$ do not exhibit flat magnitude characteristics.

The desired frequency response $F(\theta)$ can be defined in both of the PB and the SB edges as follows:

(25) $$F(\theta)=\begin{cases}1 & if\ \theta\in PB\\ 0 & if\ \theta\in SB\end{cases}$$

The required design specifications for the NR filters of examples 1 and 2 are given respectively in Tables 1 and 3. The obtained results are summarized respectively in Tables 2 and 4. Note that the total filter order can be estimated depending on the specifications.

Table 1. Lowpass digital filter specifications for example 1

Parameters	Values
Total filter order is estimated for each design	NA
Maximum PB attenuation Ap in dB	0.5
Minimum SB attenuation As in dB	32
PB edge θ_p in normalised frequency	0.5π
SB edge θ_s in normalised frequency	0.6π

Table 2. Design results for example 1 (LowPass filter)

Method	Proposed	Design in [2]
Total filter order	08	08
Optimal Ap (dB)	0.47	0.5
Optimal As (dB)	32.6	32
Iterations num.	56	NA
Max phase error (Rad)	0.3×10^{-14}	1.8×10^{-14}

Table 3. Bandpass filter specifications for example 2

Parameters	Values
Total filter order is estimated for each design	NA
Maximum PB attenuation Ap in dB	0.5
Minimum SB attenuation As in dB	50
PB edge θp in normalised frequency	0.2π
PB edge θp in normalised frequency	0.3π
SB edge θs in normalised frequency	0.5π
SB edge θs in normalised frequency	0.7π

Table 4. Design results for example 2 (Bandpass filter)

Method	Proposed	Design in [2]
Total filter order	12	12
Optimal Ap (dB)	0.49	0.5
Min. As (dB) in lower SB	94.4	50
Min. As (dB) in higher SB	50	50
Iterations num.	53	NA
Max phase error (Rad)	0.228×10-11	0.121×10-11

Also, note that the proposed design imposes fixed maximum tolerances δ_p and δ_s respectively in the PB and the SB with a fixed ratio $k=\delta_p/\delta_s$ and fixed PB and SB edges.

In order to get equitable comparison with Kormylo and Jain's design, the elliptic filter devoted to implement the NR filter can be designed according to fixed maximum tolerances δ_p and δ_s with a fixed PB edge and a fixed ratio *k*, however, the SB edge can be a floating parameter. Designing an elliptic filter with a relaxed transition band by fixing the SB edge in the elliptic design is not suitable here, despite of reaching better attenuation in the SB. Because by the fixed k in the specifications, we get, a worse tolerance out of δ_p in the PB.

For example 1, the proposed design leads to identical transfer functions $H_1(z)$ and $H_2(z)$. We get an extra loss of 0.6 dB in the SB and better attenuation in the PB (Table 2) for the magnitude response of the tandem connection $H_1(z)H_2(z^{-1})$. The proposed design yields 6 times smaller maximum phase error. The overall magnitude response of the NR Lowpass filter, the pole-zero patterns and the phase response are illustrated respectively in Figs. 2, 3 and 4.

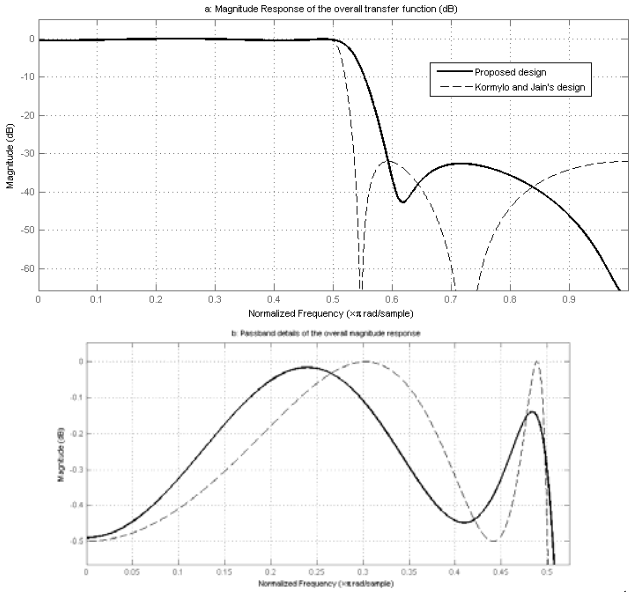

Fig.2. Magnitude responses of the tandem connection $H_1(z)H_2(z^{-1})$ yielded from both proposed design (thick solid line), and that of Kormylo and Jain (dashed line) for example 1. (a) Overall magnitude response, (b) Passband details

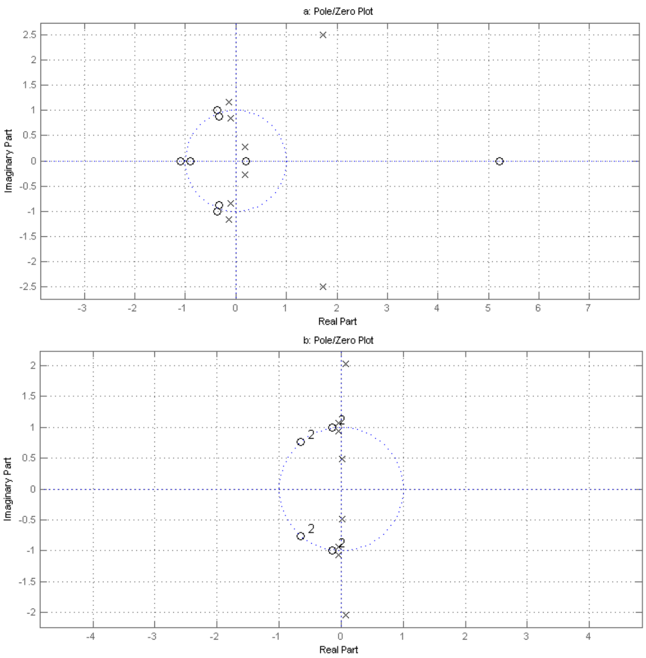

Fig. 3. Pole/zero patterns of the tandem connection $H_1(z)H_2(z^{-1})$ for example 1 (a) Proposed NR filter, (b) NR filter of Kormylo and Jain.

For example 2, the overall magnitude response of the NR Bandpass filter and the phase response are illustrated respectively in Figs. 5 and 6. The separate magnitude responses of $H_1(z)$ and $H_2(z)$ with flat characteristics in the PB are shown in Figs. 7 and 8. Here, we get different separate causal transfer functions $H_1(z)$ or $H_2(z)$.

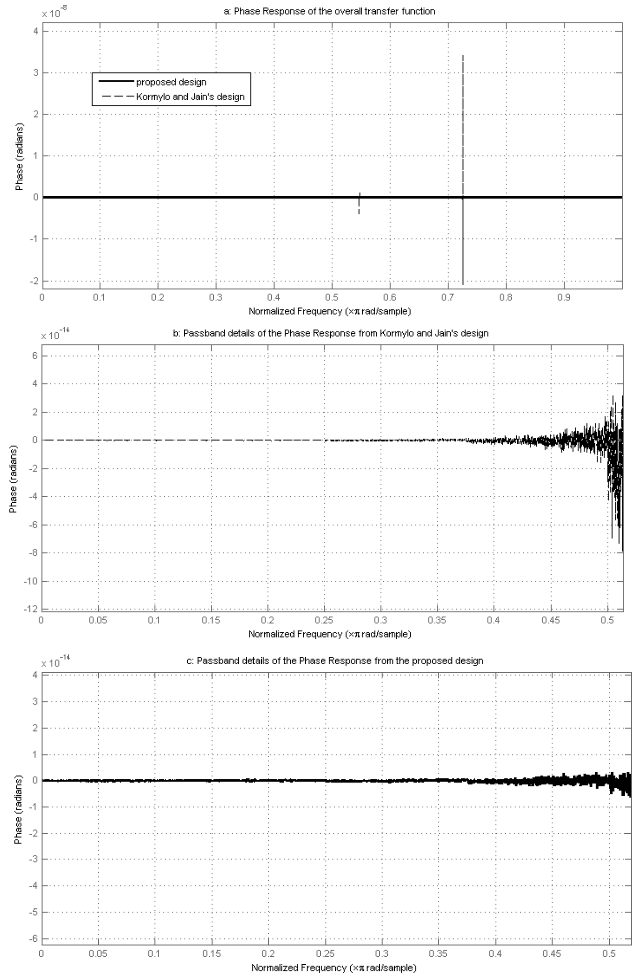

Fig. 4. Phase responses of the tandem connection $H_1(z)H_2(z^{-1})$ for example 1 (a) Overall phase response, (b) Passband details from Kormylo and Jain's design (dashed line) (c) Passband details from the proposed design (thick solid line)

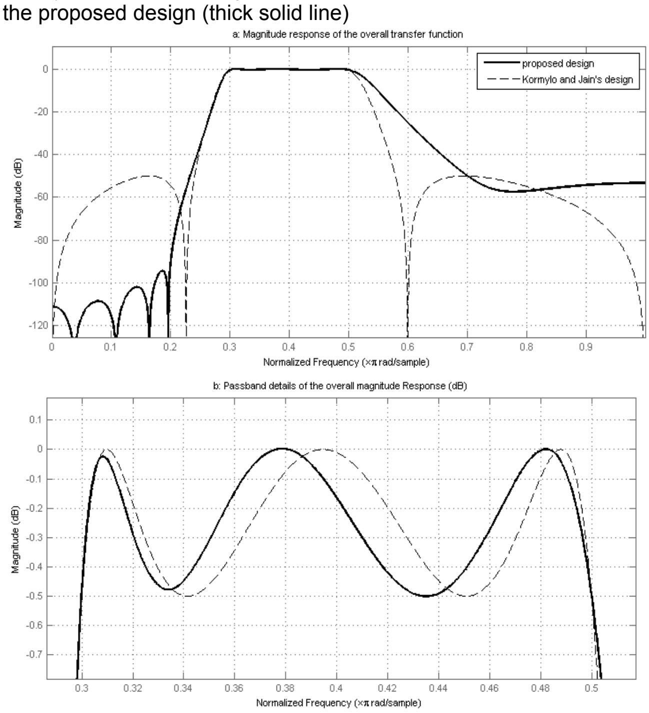

Fig.5. Magnitude responses of the tandem connection $H_1(z)H_2(z^{-1})$ yielded from the proposed design (thick solid line) and Kormylo and Jain design (dashed line) for example 2, (a) Overall magnitude response, (b) Passband details.

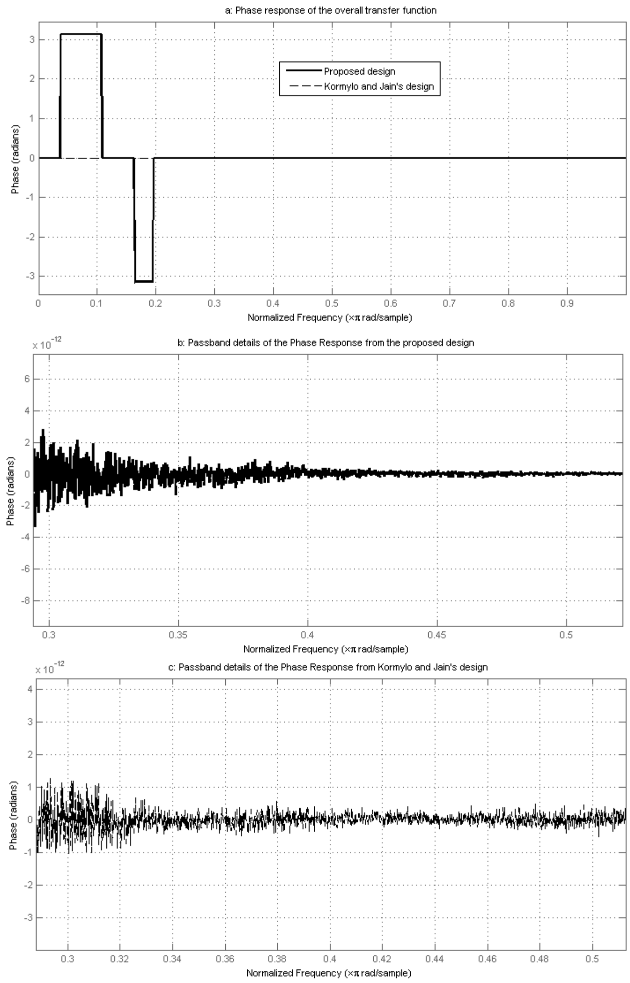

Fig.6. Phase responses of the tandem connection $H_1(z)H_2(z^{-1})$ yielded from the proposed design (thick solid line) and Kormylo and Jain design (dashed line) for example 2, (a) Overall phase response, (b) Passband details from the proposed design (c) Passband details from Kormylo and Jain's design.

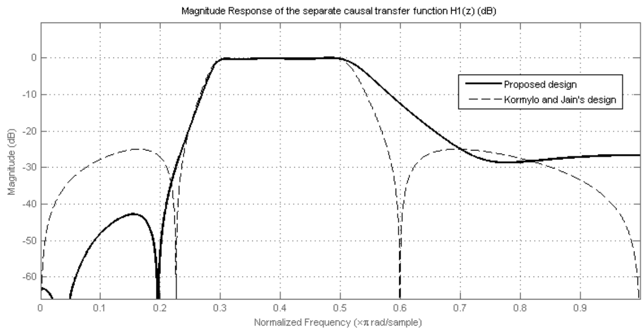

Fig.7. Magnitude responses of the separate causal transfer function $H_1(z)$ for example 2

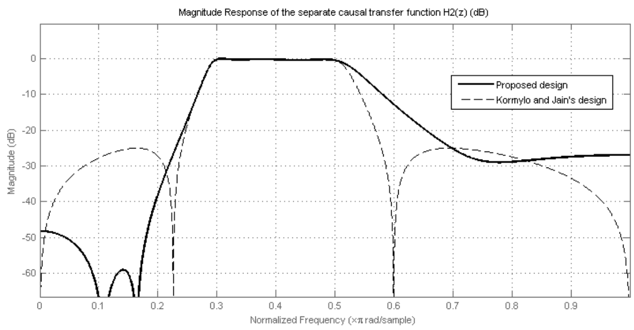

Fig.8. Magnitude responses of the separate causal transfer function $H_2(z)$ for example 2

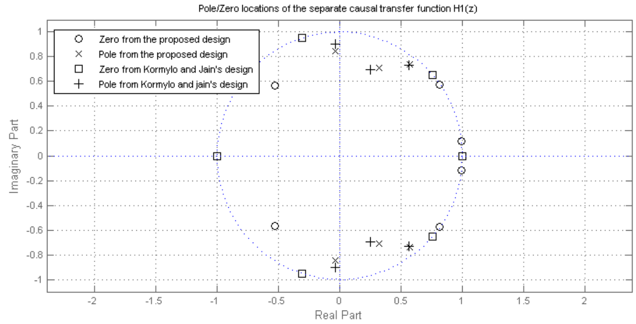

Fig.9. Pole/zero patterns of the separate causal transfer function $H_1(z)$ for example 2, including those of the proposed NR filter and Kormylo and Jain' design.

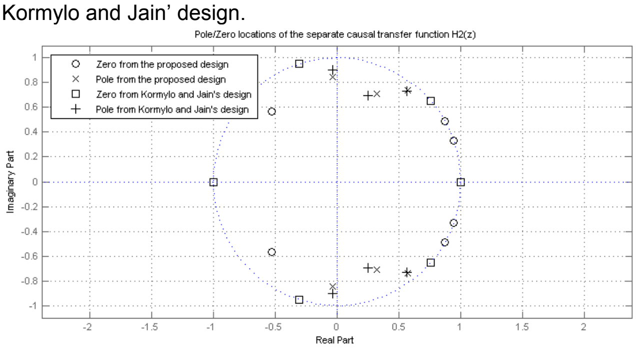

Fig.10. Pole/zero patterns of the separate causal transfer function $H_2(z)$ for example 2, including those of the proposed NR filter and Kormylo and Jain' design.

Their pole-zero patterns are presented respectively in Figs. 9 and 10. We get respectively an extra loss of 44.4 dB in the lower SB and 0.1 dB in the higher SB for the magnitude response of the tandem connection $H_1(z)H_2(z^{-1})$. The maximum phase response is practically zero.

Conclusions

An optimal technique for designing NR digital filters was presented. The design was based on the Chebyshev approximation problem. It was formulated as a linear programming under linear constraints of the zero phase shift. It was solved iteratively using a well-known binary search technique. Two examples of filter design were illustrated with identical and non-identical separate causal transfer functions. We demonstrate trough the proposed design that the optimal magnitude response of the NR filter can be different and better over previous classical Kormylo and Jain design. An additional loss in the Stop-Band was reached for the magnitude response of the tandem connection $H_1(z)H_2(z^{-1})$ with a better attenuation in the Pass-Band. Also, we get flat magnitude characteristics of the separate causal transfer functions $H_1(z)$ and $H_2(z)$. Results in Tables 2 and 4 demonstrate that the given NR filter had an accurate zero phase shift. The proposed optimal design can be used for large magnitude specifications using linprog() command with the option of interior point methods.

Acknowledgments

The author would like to thank the DGRSDT for funding and supporting this research.

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