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The mathematical modelling of transient processes in a three phase electric power system for a single phase short-circuit at the end of a long power supply line

Abstract. *A mathematical model of a three-phase power line loaded with a resistance and inductive receiver is developed in the paper. The power line is considered as a system with distributed electrical parameters with a calculated output voltage. This approach makes it possible to use the three-phase line model as a stand-alone, universal subsystem of the integrated electric power system. The results are presented of a computer simulation of transient processes as the line is switched on in the single-phase short-circuit mode at the end of the line.*

Streszczenie. *W pracy opracowano model matematyczny trójfazowej elektroenergetycznej linii zasilania obciążonej odbiornikiem o charakterze rezystancyjno-indukcyjnym. Linia elektroenergetyczna jest traktowana, jako układ o elektrycznych parametrach rozłożonych, w którym obliczono napięcie wyjściowe. Takie podejście umożliwiło opracowanie modelu linii trójfazowego, jako autonomicznego, uniwersalnego podsystemu scalonego układu analizowanej sieci elektrycznej. Przedstawiono wyniki symulacji komputerowej procesów nieustalonych w sieci elektrycznej w czasie włączania linii w tryb zwarcia jednofazowego na końcu linii.* (***Modelowanie matematyczne procesów nieustalonych w trójfazowym układzie elektroenergetycznym w stanie jednofazowego zwarcia na końcu długiej linii zasilania***).

Keywords: mathematical modelling, transient electromagnetic processes, long line equation, distributed parameters, three-phase power grid.
Słowa kluczowe: modelowanie matematyczne, elektromagnetyczne procesy nieustalone, równanie linii zasilania, parametry rozłożone, trójfazowa sieć elektryczna.

Introduction

The analysis of emergency modes in electric power systems is important when designing power supply systems. These modes are caused by the disruptions of power supply to consumers, the failures of relay protection and automation systems, a decreasing quality of electrical energy near consumers in post-fault modes, etc. Calculating these emergency modes at the stage of designing electric power supply systems helps to predict the possibility of such consequences for the adaptation of power supply systems. There are many causes of emergency modes, which are accompanied by complex transient processes. In fact, this paper analyses the transient electromagnetic processes in the elements of electrical networks during single-phase short circuits, because they cause most damage, nearly 85%, to power lines.

A mathematical modelling apparatus that does not require expensive live experiments is a possible method of analysing transient electromagnetic processes. Ultra-high voltage power lines are made at the lengths of hundreds of kilometers. Unlike in lower voltage lines, wave processes in such lines are more pronounced and exert a significant influence on the overall physics in the electric power systems [1-3]. Transient processes in a power line are normally analysed in two ways: using circular or field approaches. It is obvious that the latter method is more valuable, because it allows to reproduce transient processes on the basis of the fundamental laws of electrodynamics [4]. In terms of practical implementation, field models are much more complex because they are described not only by ordinary but also by partial differential equations, which require boundary conditions for solving. In the present paper, we consider the method of finding these conditions for the differential long transmission line equation. On the one hand, this approach makes it possible to calculate transients with a high degree of adequacy, on the other hand, to use the three-phase power line model autonomously when modelling electric power systems that include many other elements.

A review of recent research

There are a large number of scientific publications that deal with the analysis of transient processes in long power transmission lines. We will take a closer look at some that are relevant to the present paper.

In [5] presents a mathematical model of a power line based on the methods of "lost elements". In this case, the line model also considers frequency-dependent parameters, taking into account impact of the soil. In addition, the procedure of analytical integration of equations of electromagnetic state is suggested, which makes it possible to study the transient modes of the line operation.

In the article [6] proposes a mathematical model of an electric line whose key element is a long power line. The long line equation is employed to model transient processes across the line, solved by means of the first-type boundary conditions.

In the article [7] introduces a power line model that includes cascades of equivalent line circuits in series. The electromagnetic state of each cascade is described with ordinary differential equations.

A mathematical model of a three-phase power supply line is developed in [8]. The model enables calculation of phase currents and voltages along the line as a function of time on the basis of the state equations of the line equivalent circuit addressing phase and interphase parameters.

The testing of transient states in power lines using MatLab/Simulink software package deserves attention. In [9] develops a simulation model of a part of power line whose key element is a long power line of distributed parameters. A model is then built to test transient states in the line.

This review demonstrates a range of authors use the field approach to model the transients. This is subject to analytical integration [5], requires well-defined boundary conditions for the long transmission line equation [6] or the use of equivalent circuits [7, 8]. The line model of distributed parameters, available as part of MatLab/Simulink package, fails to address all the parameters of a distributed parameter line in order to simplify the calculations using D'Alembert's method [9], which may impair the adequacy of results.

The purpose of this paper is to test transient electromagnetic processes in long three-phase power lines in single-phase short circuits based on the field approaches to the modelling of the long power line.

The layout of main data

The equivalent circuit of a part of the power line is shown in Fig. 1. This part consists of a long three-phase power line loaded with a resistance and inductive receiver. The following are marked in Fig. 1: $i_1^{(k)}$, $i_N^{(k)}$, $i_H^{(k)}$ – phase currents at the start, end, and load, respectively; $u_1^{(k)}$, $u_2^{(k)}$ – phase voltages at the start and end of the line; $R_H^{(k)}$, $L_H^{(k)}$ – the resistance and inductance of the corresponding phase of the equivalent load; $k = A, B, C$ – the phase name.

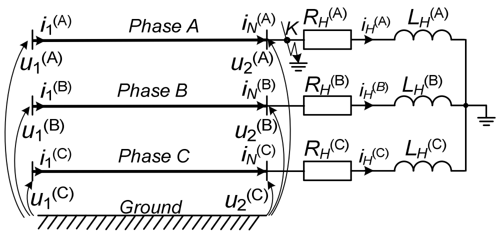

Fig.1. Computing equivalent circuit of the tested part of the power grid

In order to reduce the volume of the material, we will not discuss how we arrive at the equations of the electromagnetic state of the studied object. You can get an understanding of the method of obtaining similar equations in [10, 11]. Let us present the final equations of the electromagnetic state in a matrix-vector form:

$$(1)\quad \frac{\partial^2 \mathbf{u}}{\partial t^2} = \left(\mathbf{L}_0\mathbf{C}_0\right)^{-1}\left(\frac{\partial^2 \mathbf{u}}{\partial x^2} - \left(\mathbf{L}_0\mathbf{G}_0 + \mathbf{R}_0\mathbf{C}_0\right)\frac{\partial \mathbf{u}}{\partial t} - \mathbf{R}_0\mathbf{G}_0\mathbf{u}\right);$$

$$(2)\quad \mathbf{L}_H \frac{d}{dt}\mathbf{i}_H + \mathbf{R}_H\mathbf{i}_H - \mathbf{u}_2 = 0,$$

where [10]:

$$(3)\quad \mathbf{L}_0 = \begin{vmatrix} L_0 & M & M \\ M & L_0 & M \\ M & M & L_0 \end{vmatrix},\quad \mathbf{R}_0 = \begin{vmatrix} R_0+R_Z & R_Z & R_Z \\ R_Z & R_0+R_Z & R_Z \\ R_Z & R_Z & R_0+R_Z \end{vmatrix};$$

$$(4)\quad \mathbf{C}_0 = \begin{vmatrix} C_0+2C & -C & -C \\ -C & C_0+2C & -C \\ -C & -C & C_0+2C \end{vmatrix},\quad \mathbf{G}_0 = \begin{vmatrix} g_0+2g & -g & -g \\ -g & g_0+2g & -g \\ -g & -g & g_0+2g \end{vmatrix};$$

$$(5)\quad \mathbf{L}_H = \mathrm{diag}\left(L_H^{(A)}, L_H^{(B)}, L_H^{(C)}\right),\ \mathbf{R}_H = \mathrm{diag}\left(R_H^{(A)}, R_H^{(B)}, R_H^{(C)}\right);$$

The terminology in (1) – (5): $\mathbf{u}$ – the columnar vectors of the phase voltages of the respective elements, $\mathbf{i}$ – the columnar vectors of the phase currents of the respective elements; $\mathbf{u}_2$ – the columnar vectors of the phase voltages at the line's end; $\mathbf{i}_H$ – the columnar vector of the phase load current; R_0, g_0, C_0, L_0 – resistance, conductivity, capacitance, and inductance per unit length of the line, respectively; g, C – phase-to-phase conductivity and capacitance per unit length, respectively; M – mutual inductance per unit length; R_Z – earth resistance per unit length.

The determination of boundary conditions is the most important problem for solving (1). In this case, the voltage at the beginning of the line is known. However, it is unknown at the end of the line. Therefore, the problem is to find the boundary condition at the end of the line.

In [11-13], we suggest using the boundary conditions of the second and third types (Neumann and Poincaré boundary conditions) to solve the long transmission line equation, in particular, the equation that can be obtained from the second Kirchhoff law for electric circuits with distributed parameters:

$$(6)\quad -\frac{\partial \mathbf{u}}{\partial x} = \mathbf{R}_0\mathbf{i} + \mathbf{L}_0\frac{\partial \mathbf{i}}{\partial t}.$$

By discretising (1) and (6) with the straight-line method, using the notion of the central derivative, we obtain:

$$(7)\quad \frac{d\mathbf{v}_j}{dt} = \left(\mathbf{L}_0\mathbf{C}_0\right)^{-1}\left(\frac{1}{(\Delta x)^2}\left(\mathbf{u}_{j-1} - 2\mathbf{u}_j + \mathbf{u}_{j+1}\right) - \left(\mathbf{L}_0\mathbf{G}_0 + \mathbf{R}_0\mathbf{C}_0\right)\mathbf{v}_j - \mathbf{R}_0\mathbf{G}_0\mathbf{u}_j\right),\qquad \frac{d\mathbf{u}_j}{dt} = \mathbf{v}_j;$$

$$(8)\quad \frac{d\mathbf{i}_j}{dt} = \mathbf{L}_0^{-1}\left(\frac{1}{2\Delta x}\left(\mathbf{u}_{j-1} - \mathbf{u}_{j+1}\right) - \mathbf{R}_0\mathbf{i}_j\right),\qquad j = 2,\dots,N.$$

If (7) and (8) are written for the last discretisation node ($j = N$), we will see that in these equations there will be an unknown voltage in the fictitious node $\mathbf{u}_{N+1}$.

The method of finding this voltage is described and tested, but imperfect, because to formulate mathematical models given the different configurations of line connections with other elements of electric power systems, the fictitious voltage $\mathbf{u}_{N+1}$ will constantly change. This in turn creates certain inconveniences in the further organisation of the model in the form of a program code for the analysis of transient processes. Therefore, we suggest introducing $\mathbf{u}_2$ voltage to universalise and autonomise the mathematical model of the line (see Fig. 2).

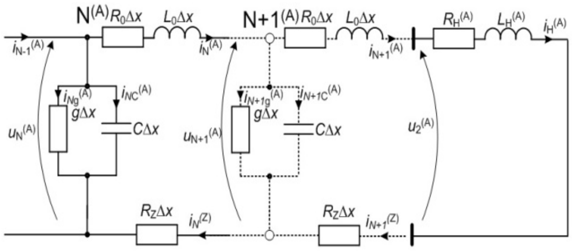

Fig.2. The equivalent circuit of the last discrete line section

Fig. 2 presents an equivalent circuit of the final discrete line section of the electric power transmission system analysed, where only the last discrete phase A node of the supply line, $N^{(A)}$, and fictitious node $N+1^{(A)}$ are marked.

The following equation will be written according to the second Kirchhoff law for the circuit in Fig. 2:

$$(9)\quad \frac{d\mathbf{i}_N}{dt} = \left[\Delta x\mathbf{L}_0\right]^{-1}\left(\mathbf{u}_N - \Delta x\mathbf{R}_0\mathbf{i}_N - \mathbf{u}_2\right).$$

Let us equate (8) written for the last discretisation node ($j = N$) and (9).

$$(10)\quad \mathbf{L}_0^{-1}\left(\frac{1}{2\Delta x}\left(\mathbf{u}_{N-1} - \mathbf{u}_{N+1}\right) - \mathbf{R}_0\mathbf{i}_N\right) = \left[\Delta x\mathbf{L}_0\right]^{-1}\left(\mathbf{u}_N - \Delta x\mathbf{R}_0\mathbf{i}_N - \mathbf{u}_2\right).$$

By extracting from (10) the voltage function of the fictitious node $\mathbf{u}_{N+1}$, we get:

$$(11)\quad \mathbf{u}_{N+1} = \mathbf{u}_{N-1} - 2\left(\mathbf{u}_N - \mathbf{u}_2\right).$$

Thus, using (11) as a function of the fictitious voltage $\mathbf{u}_{N+1}$, it is possible to avoid its replacement when changing the configuration of the power line connection circuit with other elements of the electric power system, which makes the mathematical model of the line autonomous and universal. However, now there is a need to find $\mathbf{u}_2$.

The current $\mathbf{i}_N$ is identical to the current $\mathbf{i}_H$ (see Fig. 1). Therefore, considering the initial conditions, we can write (equating the derivatives of currents (2) and (9)) as follows:

$$\text{(12)} \qquad \frac{d}{dt}\mathbf{i}_H = \frac{d\mathbf{i}_N}{dt} \Rightarrow \mathbf{L}_H^{-1}\left(\mathbf{u}_2 - \mathbf{r}_H\mathbf{i}_H\right) = \left[\Delta x\mathbf{L}_0\right]^{-1}\left(\mathbf{u}_N - \Delta x\mathbf{r}_0\mathbf{i}_N - \mathbf{u}_2\right),$$

wherefrom we get

$$\text{(13)} \quad \mathbf{u}_2 = \left[\mathbf{L}_H^{-1} + [\Delta x\mathbf{L}_0]^{-1}\right]^{-1}\left[[\Delta x\mathbf{L}_0]^{-1}\left(\mathbf{u}_N - \Delta x\mathbf{r}_0\mathbf{i}_N\right) + \mathbf{L}_H^{-1}\mathbf{r}_H\mathbf{i}_H\right].$$

The value of the current in the elements of the line can be calculated by discretising (6) with the straight-line method, but now applying the notion of the right derivative [10]:

$$\text{(14)} \qquad \frac{d\mathbf{i}_j}{dt} = \mathbf{L}_0^{-1}\left(\frac{1}{\Delta x}\left(\mathbf{u}_j - \mathbf{u}_{j+1}\right) - \mathbf{R}_0\mathbf{i}_j\right), \quad j = 1,\dots,N.$$

The following system of differential equations is subject to joint integration: (7), (14) including (3) – (5), (11), (13).

The results of computer simulation

An algorithm of the model realisation is constructed on the basis of the mathematical model for the purposes of computer simulation of transient electromagnetic processes in the tested part of the power grid (Fig. 1). The transients are studied when the supply line is switched on for asymmetric loading and further operation in the mode of single-phase short circuit at the end of the line. The simulation proceeds as follows: at the time t = 0 s, a phase turn-on of the line is simulated as the instantaneous voltage is zero. Once the system switches to its steady state at t = 0.13 s at the end of the power line (*K* in Fig. 1), a single-phase short circuit to ground is simulated in phase *A*.

The simulation is conducted for the parameters of the circuit in Fig. 1, which corresponds to a real 360.5 km long transmission line 750 kV. These are the parameters of the long line: R_0 = 1.9·10-5 Ω/m, L_0 = 1.665·10-6 H/m, C_0 = 1.0131·10-11 F/m, g_0 = 3.25·10 -11 Sm/m, C = 1.0122·10^{-12} F/m, g = 3.25·10^{-13} Sm/m, R_Z = 5·10^{-5} Ω/m, M = 7.41·10^{-7} H/m. The parameters of the equivalent asymmetric load are as follows: $R_H^{(A)}$ = 400 Ω, $R_H^{(B)}$ = 350 Ω, $R_H^{(C)}$ = 300 Ω, $L_H^{(A)}$ = 0.8 H, $L_H^{(B)}$ = 0.71 H, $L_H^{(C)}$ = 0.61 H. The line is supplied with the following voltages: $u_1^{(A)}$ = 643 sin(ωt) kV, $u_1^{(B)}$ = 643 sin(ωt – 120°) kV, $u_1^{(C)}$ = 643 sin(ωt + 120°) kV. The step of the spatial discretisation of partial differential equations using the straight-line method is equal to: Δx = = l/20 = 360.5/20 ≈ 18 km. Conventional differential equations were integrated using the second-order Giro method with a step of Δt = 27 μs.

The waveforms of phase currents and voltages at t = 0.005 s are illustrated in Fig. 3 and 4.

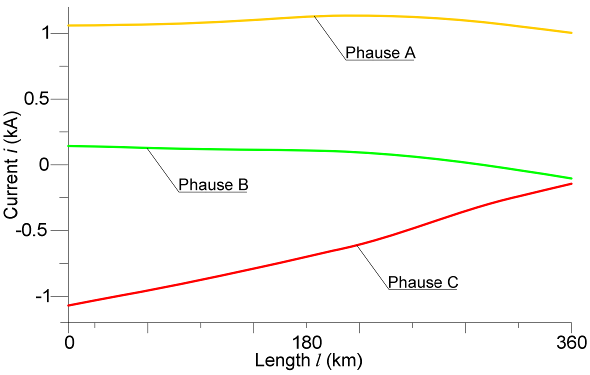

Fig. 3. The phase current waveforms as a function of the line's length

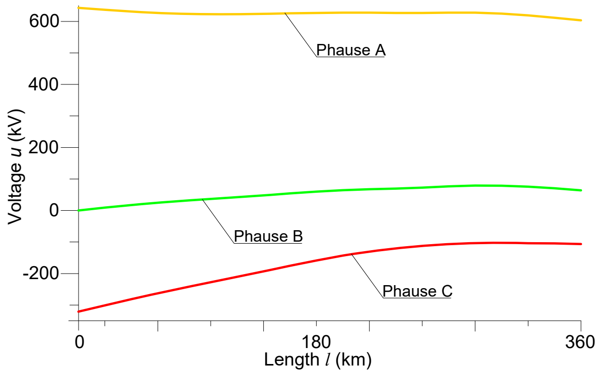

Fig. 4. The phase voltage waveforms as a function of the line's length

The waveforms of currents (Fig. 3) and voltages (Fig. 4) in the long power supply line are close to the linear function. Phase *A* current is ca. 1 kA virtually all along the line. Phase *B* current, on the other hand, is 180 A at the line's start, to gradually reduce to zero along the line. Phase *C* current is approximately – 1 kA at the line's start, to gradually reduce to zero along the line.

Phase *A* voltage is nearly identical along the long supply line, that is, around 615 kV. Phase *B* voltage is zero at the line's start and rises along the line to reach 50 kV. On the other hand, phase *A* voltage is -310 kV at the line's start and gradually falls to –100 kV at the line's end.

At 0.005 s, phase *B* of the line is not on yet, since its value is other than zero. The presence of phase *B* currents and voltages along the line is caused by electromagnetic induction.

Fig. 5 depicts transient waveforms of short-circuit phase currents at the line's end (short-circuiting currents) and of voltages in the middle of the line.

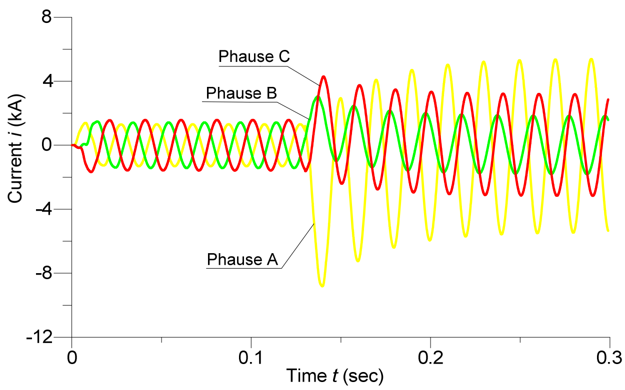

Fig.5. Transient states of short-circuit phase currents at the line's end

The current waveforms as a function of time show their amplitudes are approximately 1.4 kA as the system enters its steady state. The amplitudes of the currents are not identical because the line loading is asymmetrical. Following a single-phase short-circuiting in phase *A* at the end of the line at t = 0.13 s, the maximum current in phase *A* is 9 kA, in phase *B* – 3 kA, and in phase *C* – 4.2 kA. Steady short-circuiting takes place after about 0.13 s, with the current amplitudes in phase *A* 5.2 kA, phase *B* 1.8 kA, and phase *C* 3.6 kA.

Fig. 6 shows the transient waveforms of voltages mid-line. In the steady state, the amplitudes of phase voltages are around 620 kV. Following a short-circuit at the end of the line, the voltage amplitude in the steady state of phase *B* reduces to 400 kV, and of phases *A* and *C*, to around 600 kV.

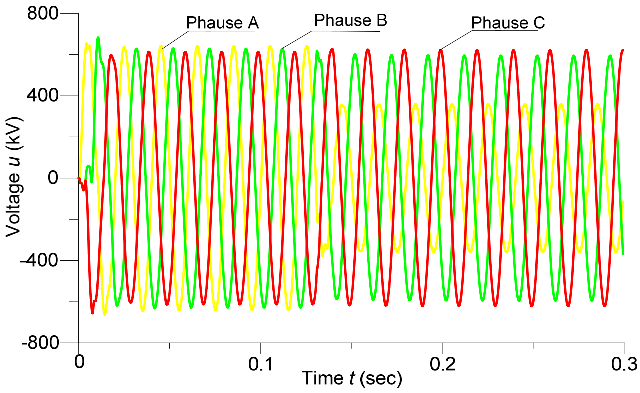

Fig.6. The transient waveforms of voltages mid-line

Fig. 7 and 8 present the waveforms of phase *A* current and phase *C* voltage, respectively. These 3D graphs illustrate both the temporal and spatial waveforms of the current and voltage across the line.

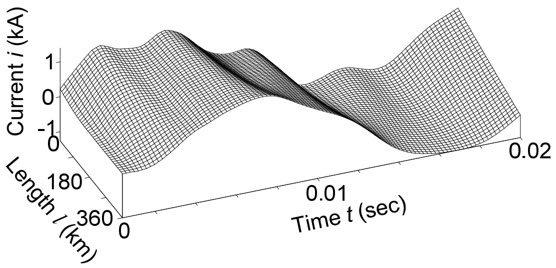

Fig.7. The space-temporal waveform of phase *A* current at the time $t \in (0; 0.02)$ s

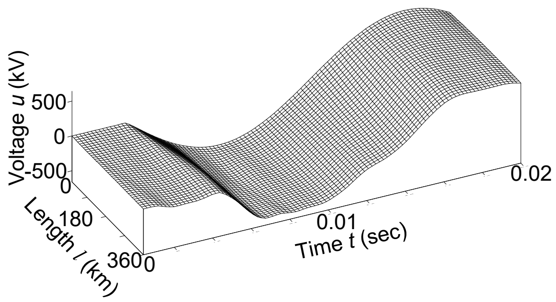

Fig. 8. The space-temporal waveform of phase *C* voltage at the time $t \in (0; 0.02)$ s

The waveforms in Fig. 7 and 8 vary along the power line as a function of time once the line is switched on. They should be analysed in comparison to Fig. 3 – 6.

Conclusion

The application of the second and third-type Neumann and Poincare boundary conditions to the solution of the equation of a three-phase power supply line allows for the testing of transient electromagnetic processes in long power lines based on the field approaches to modelling.

The introduction of the output voltage function to the mathematical model of a three-phase power supply line helps to analyse the object as an autonomous subsystem when modelling electric grids. In effect, the line model can be used in any power grid configuration, which simplifies the analysis of transient states in tested electric networks.

The mathematical model of a three-phase power supply line based on the field approaches can serve to analyse transients of temporal-spatial waveforms of currents and voltages at any point of a line. Such an approach allows for the analysis of wave processes across a line.

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