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Multivariate Modelling of the LPTV Circuits in the MAOPCs Software Environment

***Abstract**. the results of applying the system UDF MAOPCs to solve optimization problems and statistical analysis of linear periodically time-varying (LPTV) circuits have been presented in the paper. LPTV circuits are linear electric circuits with time-varying parameters and the speed of parameter variation is commensurate with the speed of variation of the circuit signals. The mathematical basis of the system UDF MAOPCs is the frequency symbolic method, which provides the formation of the transfer functions of the circuit with the parameters specified by the symbols. This greatly simplifies the formation of their derivatives and sensitivity to these parameters. The results of optimization and statistical analysis of a single-circuit parametric amplifier have been presented.*

***Streszczenie.** W artykule przedstawiono wyniki zastosowania systemu UDF MAOPCs do rozwiązywania problemów optymalizacyjnych i analizy statystycznej liniowych układów okresowo zmiennych w czasie (LPTV). Obwody LPTV są liniowymi obwodami elektrycznymi o zmiennych w czasie parametrach, a szybkość zmian parametrów jest współmierna do szybkości zmian sygnałów obwodu. Matematyczną podstawę systemu UDF MAOPCs stanowi metoda symboliczna częstotliwości, która zapewnia tworzenie transmitancji obwodu o parametrach określonych przez symbole. To znacznie upraszcza tworzenie ich pochodnych i wrażliwość na te parametry. Przedstawiono wyniki optymalizacji i analizy statystycznej jednoobwodowego wzmacniacza parametrycznego. (**Modelowanie wielowymiarowe obwodów LPTV w środowisku oprogramowania MAOPC**)*

Keywords: circuit analysis computing, linear periodically time-varying circuits, frequency symbolic method, optimization.
Słowa kluczowe: LPTV – liniowe periodyczne obwody, .analiza pobwpodów elektryznych

Introduction

The design of modern electronic devices involves optimal procedures for suppression of "complex" interferences, formed by the combined action of Gaussian noise, intersymbol interferences, phase fluctuations and a number of other negative factors.

Design of signal receivers from distant objects is of high importance. That's why radio equipment developers pay close attention to low-noise receivers. Most often, these are devices that perform signal processing based on changes of the parameters of some of their elements over time. And this change occurs at a rate commensurable with the rate of change of the signals which are in the circuit. Circles that simulate such devices are called linear periodically time-varying (LPTV) circuits. LPTV circuits are linear electric circuits with time-varying parameters and the speed of parameter variation is commensurate with the speed of variation of the circuit signals.

The ability of LPTV circuits to convert the spectrum of input signals and perform parametric amplification of signals at low noise plays an important role in modern electronics [1,2,3,4,5,6,7]. Over the last decade, interest in the study of LPTV circuits has grown significantly due to the emergence of high-temperature superconductors and the possibility of their use in parametric amplifiers, high-quality resonators, filters, long lines, delay lines, etc. [1,2,3,4,5,6,7].

The need to design such LPTV circuits stimulated the development of appropriate software products. One such product is the system user-defined functions (UDF) MAOPCs [8,9], which is designed for computer multivariate analysis and modelling of LPTV circuits. The peculiarity of this system is that it allows the developer to optimize the tolerances of LPTV circuits and make statistical analysis. And this favourably distinguishes it from existing models for modelling devices with time-varying parameters [10].

The mathematical basis of the MAOPCs environment is the frequency symbolic method (FS method). In [10,11] it is shown that the FS-method is one of the most effective means of modelling and analysis of fixed LPTV circuits in the frequency domain. The method is based on solving the L. A. Zadeh equation of the circle by approximating the conjugate [10] transfer functions $W(x_1,...,x_n,s,t)$ of LPTV circuits, for example, by a trigonometric Fourier polynomial:

$$W(x_1,...,x_n,\Omega,t)= \tag{1}$$
$$=W_{\pm 0}(x_1,...,x_n,s)+\sum_{i=1}^{k}\begin{bmatrix} W_{-i}(x_1,...,x_n,\omega)\cdot\exp(-ji\Omega t)+ \\ +W_{+i}(x_1,...,x_n,\omega)\cdot\exp(+ji\Omega t)\end{bmatrix},$$

where $x_1,...,x_n$ are the symbolic parameters of the elements of the circle, $s=j\omega$ – is the complex variable, t- time, k- the number of harmonic components in the polynomial, $\Omega=2\pi/T$, $T-$ the period of change of the parameter of the parametric element of the circle under the action of the pump signal.

The MAOPCs environment is described in research papers [8, 9]. The system architecture is based on the principles of the MATLAB software environment [12].

Considering that the tasks of optimization and statistical analysis in the design of devices are central, their implementation in the MAOPCs environment has been considered.

Solving the problems of optimization and statistical analysis of LPTV circuits requires consideration of the following features:

a) the dependence of the transfer functions and characteristics of the circuit on time;

b) the need to control the stability of the circle when changing the parameters of its elements (changing the parameters can lead to a loss of stability of the circle).

The research paper deals with the specified features of LPTV circuits when solving optimization and statistical analysis problems in the MAOPCs environment.

Optimization peculiarities of LPTV circuits in the MAOPCs environment

The solution of the optimization problem are such finite values of the varied parameters $x_1,...,x_n$, that provide the minimum (maximum) value of the target function $F(x_1,...,x_n,s,t)$ under given limits [13,14].

In the work for optimization of LPTV circuits the general additive criterion of optimality is chosen [13,14], which is the sum of squares of deviations of initial characteristics from technical requirements. In our case, the objective function $F(x_1,...,x_n,s,t)$ for the selected values of the complex variable $s_i=j\omega_i$ (here $j=\sqrt{-1}$ of the frequency band under study and time values t_j (here $j=1,2,...$), selected on the period $(0-2\pi/\Omega)$ of the change of the transfer function, is [13,14]:

(2) $$F(x_1,...,x_n,\omega,t)=\sum_{i=1}^{p}\sum_{j=1}^{q}\left(M_F(x_1,...,x_n,\omega_i,t_j)-M_0(\omega_i,t_j)\right)^2,$$

where $M_0(\omega_i,t_j)$ and $M_F(x_1,...,x_n,\omega_i,t_j)$ is the goal function and the characteristic function, respectively [15]. The goal function $M_0(\omega_i,t_j)$ is set by the researcher as a set of values for the selected values of the variables ω_i and t_j. The characteristic function $M_F(x_1,...,x_n,\omega_i,t_j)$ is determined by the modulus of the parametric transfer function $W(x_1,...,x_n,\omega_i,t_j)$ of the parametric circuit, which according to the FS method [10,11] is formed in advance in the form of approximation (1) with symbolic parameters $x_1,...,x_n$ for current values ω_i, t_j.

As mentioned above, the peculiarity of the problems of parametric circuits optimization (mostly parametric amplifiers) is that they require stability control [16,17]. Therefore, the range of change of the varied parameters $x_1,...,x_n$ is chosen so that it is inside the range of stability of the steady-state mode of the circle in the range of the same variable parameters $x_1,...,x_n$.

The formation of the objective function and characteristic function in the MAOPCs environment is performed taking into account the considered peculiarities. Further optimization of LPTV circuits is performed using standard MATLAB software tools.

Conditional optimization in the MAOPCs environment

After reviewing and analysing the optimization functions of the MATLAB software environment, two functions have been considered: "patternsearch" [18] from "Global Optimization Toolbox" package and "fmincon" [19] from "Optimization Toolbox" package, because these functions allow perform conditional optimization of the function of many variables.

The task to investigate what results will be obtained when optimizing one of the most common parametric circuits - single-circuit parametric amplifier has been set. The conditional optimization functions "patternsearch" and "fmincon" have been used for different starting points.

Computer experiment. Optimization of parametric amplifier in MAOPCs environment

The optimization of a single-circuit parametric amplifier from Fig.1. is carried out in the computer experiment.

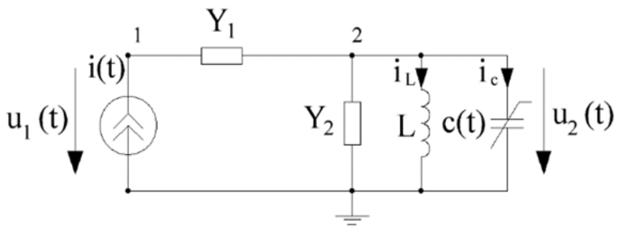

Fig.1. Single-circuit parametric amplifier

$i(t)=I_m\cdot\cos(\omega\cdot t+\varphi),\ c(t)=c_0\cdot(1+m\cdot\cos(\Omega\cdot t))$,

$c_0=10\cdot10^{-12}F,\quad m=0.05,\quad \Omega=4\cdot\pi\cdot10^8 rad/s,\quad Y_1=0.25S,$

$Y_2=0.0004S,\ \ L=0.2533\cdot10^{-6}H$, $s=j\omega, \omega=2\cdot\pi\cdot10^8 rad/s.$

Task of computer experiment. Carry out optimization and determine the values of c_0^* and m^* which provide a minimum of the objective function $F(c_0,m)=F_{\min}$ which is formed for each frequency point ω_i from a given range $1.7\cdot\pi\cdot10^8:0.02\cdot\pi\cdot10^8:2.5\cdot\pi\cdot10^8$ rad/s and for each time point t_j from a given range $0:0.05\cdot10^{-9}:5\cdot10^{-9}s$ for the parametric transfer function $Z_1(s,t)=U_1(s,t)/I(s)$ of a single-circuit parametric amplifier from Fig. 1 provided its asymptotic stability. Optimization should be conducted in the MATLAB software environment using the "fmincon" and "patternsearch" functions and the MAOPCs environment.

A computer experiment performing. According to the procedure optimization of parametric circuit under the condition of their stability control [16], the following steps are made:

Step 1. According to the *"Stability"* function [8,9] of the MAOPCs environment, the denominator $\Delta_G(s,m,c_0)$ of the normal parametric transfer function $G(s,\xi)$ [16,17] of the amplifier is formed at the symbolic values of the parameters m and c_0.

Step 2. According to the *"Stability"* function [8,9] of the MAOPCs environment and the denominator $\Delta_G(s,m,c_0)$ for each value c_0 from a number of values of a given range the limit value m, is determined at which the stability changes to instability, and the dependence $m_{\lim}=f(c_0)$. is formed. As a result for the amplifier the so-called "stability map" in coordinates of parameters m and c_0 is built.

Step 3. According to *FunctionOfZoneStability* function [8,9] of the MAOPCs environment, the dependence $m_{\lim}=f(c_0)$ is approximated by a polynomial of the fourth degree and an analytical expression is defined which confines the range of stability of the amplifier in the form:

(3) $$\begin{gathered}m<-0.1783e^{47}\cdot c_0^4+0.6767e^{36}\cdot c_0^3-\\-0.9458e^{25}\cdot c_0^2+0.5746e^{14}\cdot c_0-127\end{gathered}.$$

The limit values of the parameter m form the limit of the stability range (Fig. 2). The resulting inequality is used when applying the optimization functions "patternsearch" and "fmincon".

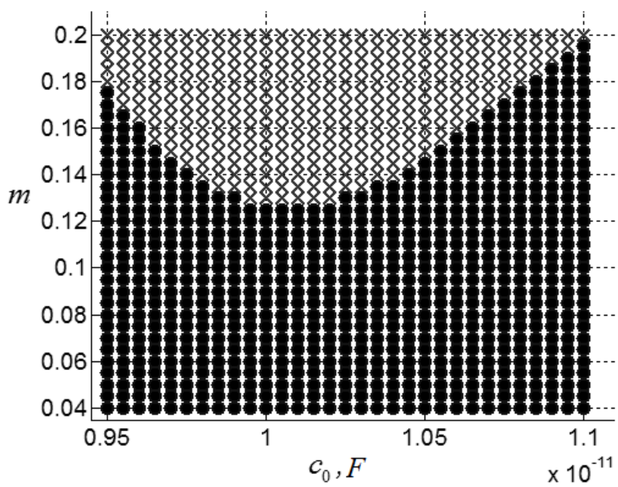

Fig.2. "Stability map" in the coordinates of the parameters c_0 and m. The black area defines the stability area.

Step 4. The *FSM* function [8,9] of the MAOPCs environment determines the parametric transfer function $Z_1(s,t)$ which is the basis for the optimization of the amplifier. The *FormOfFunOfGoal* function [8,9] of the

MAOPCs environment specifies the goal function $M_0(\omega_i,t_j)$. In this example, it is formed in such a way that the amplifier can implement it. Values c_0 and m are confined by the values which provide physical realization of a parametric element $c(t)$. These variables should be defined as a result of further optimization as c_0^* and m^*. The objective function $M_0(\omega_i,t_j)$ is defined as a modulus of the transfer function of the amplifier $\left|Z_1(\omega_i,t_j)\right|$.

Step 5. According to the *FormOfFunCharacteristic* function [8,9] of the MAOPCs environment, the characteristic function $M_{Z_1}(c_0,m,\omega_i,t_j)$ of the single-circuit parametric amplifier is formed in the form of a modulus of the transfer function $\left|Z_1(m,c_0,\omega_i,t_j)\right|$ where m and c_0 are given by symbols.

Step 6. According to the *FormOfObjectiveFun* function [8,9] of the MAOPCs environment, the objective function of the form (2) is formed at the given values ω_i and t_j: $1.7\cdot\pi\cdot10^8$ $-2.5\cdot\pi\cdot10^8$ rad/s with step $0.02\cdot\pi\cdot10^8$ rad/s and 0 $-5\cdot10^{-9}$ s with step $0.05\cdot10^{-9}$ s, respectively.

Step 7. Both optimization functions "patternsearch" and "fmincon" providing the stability of the amplifier (3) provide the optimization and the values of parameters c_0 and m are determined, which provide a minimum of the objective function $F(c_0,m)=F_{\min}$ which is formed for each frequency point in a given range $1.7\cdot\pi\cdot10^8:0.02\cdot\pi\cdot10^8:2.5\cdot\pi\cdot10^8$ rad/s and for each time point t_j in a given range $0:0.05\cdot10^{-9}:5\cdot10^{-9}s$.

Results of a computer experiment. According to the optimization function "fmincon" when the stability condition of the amplifier is observed (3) and the selected initial values (start point, Fig.3) $c_0=0.9\cdot10^{-11}F$, $m=0.01$ of the varied parameters for 15 iterations, the value of the objective function $F(c_0,m)=7.151\text{e}5$ provided $c_0^*=0.99\cdot10^{-11}$ and $m^*=0.01$ is determined. This value of the objective function is not the minimum value. Thus, the optimization function "fmincon" has incorrectly defined the minimum of the objective function (Fig.3, "FMINCON solution").

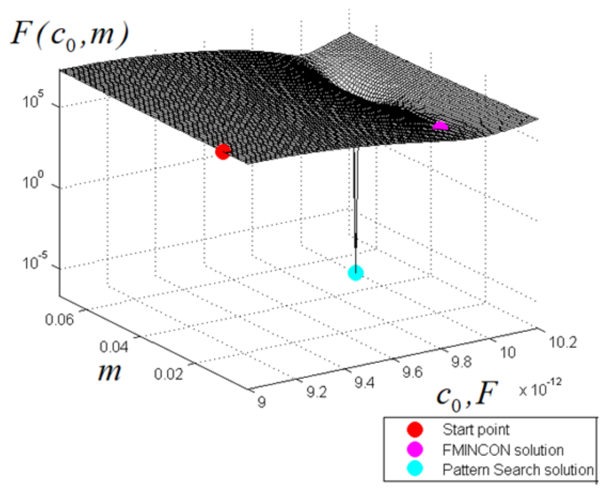

Fig.3 The objective function $F(c_0,m)$ in the coordinates c_0 and m.

According to the optimization function "patternsearch" when the same condition of the amplifier stability (3) and at the same initial values (start point Fig. 5) for 7 iterations, the minimum $F(c_0,m)=2.158\text{e-}7$ provided $c_0^*=1\cdot10^{-11}$ and $m^*=0.05$. So, the optimization function "patternsearch" correctly defined the minimum of the objective function (Fig. 3, "Pattern Search solution").

Table 1 shows the search results of the minimum by two methods for different starting points.

TABLE 1. the search results of the minimum by two methods for different starting points

Start point	Optimization results, «fmincon»	Optimization results, «patternsearch»
$c_0=0.9\cdot10^{-11}F$ $m=0.01$	$c_0=0.99\cdot10^{-11}F$ $m=0.01$ $F(c_0,m)=7.151\text{e}5$ Minimum is not defined	$c_0=1\cdot10^{-11}F$ $m=0.05$ $F(c_0,m)=2.158\text{e-}7$ Minimum defined
$c_0=0.95\cdot10^{-11}F$ $m=0.04$	$c_0=0.95\cdot10^{-11}F$ $m=0.03801$ $F(c_0,m)=6.4389\text{e}6$ Minimum is not defined	$c_0=1\cdot10^{-11}F$ $m=0.05$ $F(c_0,m)=2.158\text{e-}7$ Minimum defined
$c_0=0.95\cdot10^{-11}F$ $m=0.1$	0 $c_0=0.95\cdot10^{-11}F$ $m=0.03803$ $F(c_0,m)=6.4389\text{e}6$ Minimum is not defined	$c_0=1.003\cdot10^{-11}F$ $m=0.11842$ $F(c_0,m)=1.2213e6$ Minimum is not defined
$c_0=1.05\cdot10^{-11}F$ $m=0.01$	$c_0=1.05\cdot10^{-11}F$ $m=0.1378$ $F(c_0,m)=4.3735\text{e}6$ Minimum is not defined	$c_0=1\cdot10^{-11}F$ $m=0.05$ $F(c_0,m)=2.158\text{e-}7$ Minimum defined
$c_0=1.05\cdot10^{-11}F$ $m=0.04$	$c_0=1.05\cdot10^{-11}F$ $m=0.1375$ $F(c_0,m)=4.3733\text{e}6$ Minimum is not defined	$c_0=1\cdot10^{-11}F$ $m=0.05$ $F(c_0,m)=2.158\text{e-}7$ Minimum defined
$c_0=1.05\cdot10^{-11}F$ $m=0.1$	$c_0=1.05\cdot10^{-11}F$ $m=0.1374$ $F(c_0,m)=4.3722\text{e}6$ Minimum is not defined	$c_0=1.003\cdot10^{-11}F$ $m=0.13485$ $F(c_0,m)=1.7211e6$ Minimum is not defined
$c_0=1.1\cdot10^{-11}F$ $m=0.01$	$c_0=1.1\cdot10^{-11}F$ $m=0.1891$ $F(c_0,m)=5.2957\text{e}6$ Minimum is not defined	$c_0=1\cdot10^{-11}F$ $m=0.05$ $F(c_0,m)=2.158\text{e-}7$ Minimum defined
$c_0=1.1\cdot10^{-11}F$ $m=0.04$	$c_0=1.1\cdot10^{-11}F$ $m=0.1888$ $F(c_0,m)=5.3701\text{e}6$ Minimum is not defined	$c_0=1.006\cdot10^{-11}F$ $m=0.06538$ $F(c_0,m)=1.3062e5$ Minimum is not defined
$c_0=1.1\cdot10^{-11}F$ $m=0.1$	$c_0=1.09\cdot10^{-11}F$ $m=0.1895$ $F(c_0,m)=5.3384\text{e}6$ Minimum is not defined	$c_0=1.006\cdot10^{-11}F$ $m=0.06385$ $F(c_0,m)=1.1840e5$ Minimum is not defined

$c_0 = 1.01 \cdot 10^{-11} F$ $m = 0.06$	$c_0 = 1.01 \cdot 10^{-11} F$ $m = 0.0539$ $F(c_0, m) = 1.8944\text{e}6$ Minimum is not defined	$c_0 = 1 \cdot 10^{-11} F$ $m = 0.05$ $F(c_0, m) = 2.158\text{e-}7$ Minimum defined

From Table 1 it follows that none of the selected optimization methods guarantees the determination of the minimum of the objective function. The "patternsearch" function defined the required minimum in six cases, the "fmincon" function - never. It can be assumed that the "patternsearch" function is more reliable in such tasks than the "fmincon" function.

Features of the statistical analysis of LPTV circuits in the MAOPCs environment

Method of statistical analys. It is obvious that in practice the parameters of LPTV circuits are not equal to the nominal values. These parameters are random variables with defined statistical characteristics. The scatter of parameters leads to the scatter of characteristics of LPTV circuits. Therefore, these characteristics are also random variables, but with unknown statistical characteristics.

Tolerance analysis is a problem that is solved at the design stage of the device and lies in the determination of the deviation effect of the parameters of its elements from the nominal values on the deviation of the device characteristics [13,14,15]. This task specifies the structure of the device, the nominal values of its parameters and their tolerances. To solve the problem of analyzing the tolerances of parametric circles, the method of moments [13] is chosen, which is based on the representation of the relative deviation of the circle function in the form of a truncated Taylor series [14]:

$$\delta\left|W(x_1,...,x_n,s,t)\right| \cong \sum_{i=1}^{n} S_{x_i}^{|W(x_1,...,x_n,s,t)|} \cdot \delta x_i + \frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} S_{x_i,x_j}^{|W(x_1,...,x_n,s,t)|} \cdot \delta x_i \cdot \delta x_j\right), \tag{4}$$

where $\delta\left|W(x_1,...,x_n,s,t)\right|$ is the relative deviation of the modulus of the parametric transfer function determined by the frequency symbolic method; δx_i, δx_j are relative deviations of element parameters x_i and x_j from nominal values, respectively; n is a number of parameters of LPTV circuit elements; $S_{x_i}^{|W(x_1,...,x_n,s,t)|}$ and $S_{x_i,x_j}^{|W(x_1,...,x_n,s,t)|}$ are sensitivity functions which depend on the time of the first and second degrees [15], respectively. An important feature of this method of solving the problem of tolerance analysis is that it does not require statistical characteristics of the parameters of the elements, and uses only information about the tolerances of the parameters and sensitivity functions.

When solving the problem of tolerance analysis of LPTV circuits, it should be taken into account that all the parameters of the elements within the specified limits change simultaneously, and it is not known in advance whether the LPTV circuit is stable within these limits. Therefore, before analyzing the tolerances, it is necessary to assess the stability of the studied LPTV circuit within the specified limits of simultaneous change of all parameters of the elements and to draw a conclusion about the stability of the circuit within these limits. Stability assessment is carried out on the denominator $\Delta(s, x_1, ..., x_n)$ of the normal parametric transfer function which is determined by the frequency symbolic method [16,17].

The problem of analyzing the tolerances of LPTV circuits is solved in the environment of the system UDF MAOPCs and using standard MATLAB software tools.

Computer experiment of statistical research of parametric amplifier in the MAOPCs environment

Results of a Computer experiment tasks. To analyze the tolerances of a single-circuit parametric amplifier from Fig.3. To determine the relative changes in the parameters m, c_0, L, Y_1, Y_2 of the elements that provide a change in the relative deviation $\delta\left|Z_1(t)\right|$ of the modulus of the parametric transfer function $Z_1(s,t) = U_1(s,t)/I(s)$ of the circuit within the limits $\pm 1.5\%$ under the condition of its stability. The limits of change of each of the parameters should be selected as follows: $m = 0.04 : 0.06$, $Y_1 = 0.2375 : 0.2625\ S$, $c_0 = 9.5 \cdot 10^{-12} : 10.5 \cdot 10^{-12} F$, $L = 0.2433 \cdot 10^{-6} : 0.2633 \cdot 10^{-6} H$, $Y_2 = 0.00038 : 0.00042\ S$.

Performing of a computer experiment. According to the procedure of analysis of tolerances, the following steps are made [15]:

Step 1. According to the function *"Stability"* [8, 9] of the MAOPCs environment, the denominator $\Delta_g(m, c_0, Y_1, Y_2, L, s)$ of the normal parametric transfer function $G(s,\xi)$ of the current $I(s)$ in the voltage $U_2(s,t)$ for the amplifier is formed at the symbolic values of the parameters m, c_0, Y_1, Y_2, L, s of the elements and at $k = 1$:

$$\begin{aligned}
\Delta_g(m,c_0,Y_1,Y_2,L,s) &= (0.6e^{-25} \cdot L^3 \cdot {c_0}^3 \cdot m^2 - \\
&-0.12e^{-24} \cdot L^3 \cdot {c_0}^3) \cdot s^6 + (0.6e^{-25} \cdot L^3 \cdot {c_0}^2 \cdot m^2 \cdot Y_2 - \\
&-0.36e^{-24} \cdot L^3 \cdot {c_0}^2 \cdot Y_2) \cdot s^5 + (-0.36e^{-24} \cdot L^3 \cdot {Y_2}^2 \cdot c_0 + \\
&+0.60e^{-25} \cdot {c_0}^2 \cdot L^2 \cdot m^2 - 0.39e^{-6} \cdot L^3 \cdot {c_0}^3 - \\
&-0.36e^{-24} \cdot {c_0}^2 \cdot L^2 + 0.19e^{-6} \cdot L^3 \cdot {c_0}^3 \cdot m^2) \cdot s^4 + \\
&+(-0.12e^{-24} \cdot L^3 \cdot {Y_2}^3 - 0.75e^{-6} \cdot L^3 \cdot {c_0}^2 \cdot Y_2 + \\
&+0.96e^{-7} \cdot L^3 \cdot {c_0}^2 \cdot m^2 \cdot Y_2 - 0.72e^{-24} \cdot L^2 \cdot Y_2 \cdot c_0) \cdot s^3 + \\
&+(-0.36e^{-24} \cdot c_0 \cdot L - 0.3e^{12} \cdot L^3 \cdot {c_0}^3 - 0.57e^{-6} \cdot L^3 \cdot {Y_2}^2 \cdot c_0 - \\
&-0.96e^{-7} \cdot {c_0}^2 \cdot L^2 \cdot m^2 - 0.36e^{-24} \cdot L^2 \cdot {Y_2}^2 + \\
&+0.15e^{12} \cdot L^3 \cdot {c_0}^3 \cdot m^2) \cdot s^2 + (-0.30e^{12} L^3 \cdot {c_0}^2 \cdot Y_2 - \\
&-0.19e^{-6} \cdot L^3 \cdot {Y_2}^3 - 0.36e^{-24} \cdot L \cdot Y_2) \cdot s - \\
&-0.19e^{-6} \cdot L^2 \cdot {Y_2}^2 + 0.39e^{-6} \cdot c_0 \cdot L - \\
&-0.3e^{12} \cdot {c_0}^2 \cdot L^2 - 0.12e^{-24}.
\end{aligned} \tag{5}$$

In expression (5), the parameter Y_1 is absent. This means that it does not affect the stability of the circle.

Step 2. The denominator (5) is formed, but for $k = 3$, which provides satisfactory accuracy and the function *"Stability"* [8,9] of the MAOPCs environment at symbolic values of element parameters m, c_0, Y_2 L stability is estimated within the change of each of the parameters specified in the problem. The results of the stability assessment indicate the stability of the single-circuit parametric amplifier when the parameters m, c_0, L, Y_2 are changed within the specified limits and are given in [15]. Based on the obtained results of the stability assessment, all further studies are conducted within the values of the parameters m, c_0, L, Y_2, at which the amplifier in Fig.3 is stable. Therefore, the given limits of change of each of the parameters m, c_0, Y_2 L of the elements are turned into relative changes δm, δc_0, δY_2, of the parameters of the elements and are evaluated in per cents $\delta m = 6\%$, $\delta c_0 = 2\%$, $\delta L = 2\%$, $\delta Y_1 = 5\%$, $\delta Y_2 = 5\%$. These elements change values are the maximum allowable for this experiment.

Step 3. The *"TrFunc"* and *"FSM"* functions of the MAOPCs environment determine the parametric transfer function $Z_1(s,t)$ which is be the basis for the tolerances analysis of a single-circuit parametric amplifier.

Step 4. According to the symbolic expression $Z_1(s,t)$ which provides satisfactory accuracy if $k=3$ and the *"SensFO"* and *"SensSO"* functions of the MAOPCs environment the sensitivity functions $S_m^{|Z_1|}$, $S_{c_0}^{|Z_1|}$, $S_{Y_1}^{|Z_1|}$, $S_{Y_2}^{|Z_1|}$, $S_L^{|Z_1|}$ of the first degree and the sensitivity functions $S_{m,m}^{|Z_1|}$ $S_{m,c_0}^{|Z_1|}$, $S_{m,Y_1}^{|Z_1|}$, $S_{m,Y_2}^{|Z_1|}$, $S_{m,L}^{|Z_1|}$, $S_{c_0,Y_1}^{|Z_1|}$, $S_{c_0,Y_2}^{|Z_1|}$, $S_{c_0,L}^{|Z_1|}$, $S_{c_0,c_0}^{|Z_1|}$, $S_{L,L}^{|Z_1|}$, $S_{L,Y_1}^{|Z_1|}$, $S_{L,Y_2}^{|Z_1|}$, $S_{Y_1,Y_1}^{|Z_1|}$, $S_{Y_1,Y_2}^{|Z_1|}$, $S_{Y_2,Y_2}^{|Z_1|}$ of the second degree of the modulus of the parametric transfer function $Z_1(s,t)$ are determined.

Step 5. According to the *"RelativeDeviation_FirstOrder", "RelativeDeviation_SecondTerm"* and *"RelativeDeviationOfFunction"* functions of MAOPCs environment, and sensitivity functions, the relative deviation $\delta|Z_1(t)|$ of the modulus of parametric transfer function $Z_1(t)$ is calculated at relative changes $\delta m = 6\%$, $\delta c_0 = 2\%$, $\delta L = 2\%$, $\delta Y_1 = 5\%$, $\delta Y_2 = 5\%$ of the parameters m, c_0, L, Y_1, Y_2 of elements:

$$
\begin{aligned}
&\delta|Z_1(t)| = S_m^{|Z_1|}\cdot\delta m + S_{c_0}^{|Z_1|}\cdot\delta c_0 + S_{Y_1}^{|Z_1|}\cdot\delta Y_1 + \\
&S_{Y_2}^{|Z_1|}\cdot\delta Y_2 + S_L^{|Z_1|}\cdot\delta L + \frac{1}{2}\cdot\Big(S_{m,m}^{|Z_1|}\cdot\delta m\cdot\delta m + \\
&+S_{m,c_0}^{|Z_1|}\cdot\delta m\cdot\delta c_0 + S_{m,Y_1}^{|Z_1|}\cdot\delta m\cdot\delta Y_1 + S_{m,Y_2}^{|Z_1|}\cdot\delta m\cdot\delta Y_2 + \\
&+S_{m,L}^{|Z_1|}\cdot\delta m\cdot\delta L + S_{c_0,Y_1}^{|Z_1|}\cdot\delta c_0\cdot\delta Y_1 + S_{c_0,Y_2}^{|Z_1|}\cdot\delta c_0\cdot\delta Y_2 + \\
&+S_{c_0,L}^{|Z_1|}\cdot\delta c_0\cdot\delta L + S_{c_0,c_0}^{|Z_1|}\cdot\delta c_0\cdot\delta c_0 + S_{L,L}^{|Z_1|}\cdot\delta L\cdot\delta L + \\
&+S_{L,Y_1}^{|Z_1|}\cdot\delta L\cdot\delta Y_1 + S_{L,Y_2}^{|Z_1|}\cdot\delta L\cdot\delta Y_2 + \\
&+S_{Y_1,Y_1}^{|Z_1|}\cdot\delta Y_1\cdot\delta Y_1 + S_{Y_1,Y_2}^{|Z_1|}\cdot\delta Y_1\cdot\delta Y_2 + S_{Y_2,Y_2}^{|Z_1|}\cdot\delta Y_2\cdot\delta Y_2\Big).
\end{aligned}
\tag{6}
$$

Results of a computer experiment. Making calculations (6) for given relative changes in the parameters of the elements, it has been noticed that the relative deviation of the modulus of the transfer function is significantly greater than ±1.5%. In fact, solving the problem of synthesis of tolerances, the following relative changes $\delta m = 2.5\%$, $\delta c_0 = 0.01\%$, $\delta L = 0.01\%$, $\delta Y_1 = 1\%$, $\delta Y_2 = 1\%$ of the parameters m, c_0, L, Y_1, Y_2 have been determined that provide a given change in relative deviation $\delta|Z_1(t)|$. This follows from Fig. 6, which shows the relative deviation $\delta|Z_1(t)|$ in time formed by expression (6) by the function *Graph_3D* for certain relative changes in the parameters of the elements.

Computer experiment completed. As a result, the relative deviation $\delta|Z_1(t)|$ of the modulus of the parametric transfer function $Z_1(t)$ of a single-circuit parametric amplifier is a function that depends on time. And this is because the transfer function $Z_1(t)$ also depends on time.

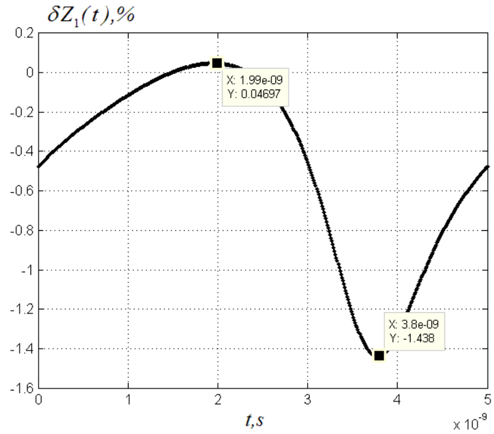

Fig.6. Time dependence of the relative deviation $\delta|Z_1(t)|$ at a defined relative change in the parameters of the elements

Conclusion

1. Symbolic transfer functions of LPTV circuits formed by the system UDF MAOPCs are a good basis for optimization and statistical analysis of these circuits in the MATLAB software environment.
2. The presence of parametric elements in a circle under certain conditions can lead to a loss of its stability. Therefore, the peculiarity of solving optimization problems and statistical analysis of parametric circuits is that their stability should be controlled during the solution.
3. According to the formed transfer functions by means of MATLAB it is convenient to form their derivatives on the parameters set by symbols. Therefore, in the optimization or statistical analysis of parametric circles, methods that require calculations of the gradients of the objective functions or the sensitivity of the circles can be used.
4. The frequency symbolic method does not limit the number of circuit parameters specified by the symbols. Therefore, optimization functions of the MATLAB software environment of increased complexity can be chosen for optimization.
5. Procedures of optimization and statistical analysis of LPTV circuits do not limit the number of varied parameters (parameters specified by symbols), as this is not limited by the frequency symbolic method.
6. As shown by computer experiments, the optimization functions of the MATLAB environment do not always find the optimal values of the objective function. Therefore, the resulting solutions should be checked for other start points or by other methods.

From the above conclusions it follows that the system UDF MAOPCs, which is designed for simulation LPTV circuits, has good prospects for use in the multivariate simulation of electronic devices of wide profile, represented by models with parametric elements.

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