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Uncertainty budget by evaluation of the coil factor in accordance with the standard EN 61000-4-8

Abstract. *Procedure for establishing the uncertainty budget by evaluation of the coil factor in accordance with the standard EN 61000-4-8 is presented. In addition to errors of the measurement instruments i.e.: field probe and ampere meter, contributions of uncertainty of probe positioning in the centre of the coil and error due to truncation of the magnetic permeability by recalculation of the magnetic induction to the field strength is presented. It is performed with analytical formulas. Dominant component of the budget is uncertainy of the measurement of magnetic induction. It confirms correctness of the budget.*

Streszczenie. *W artykule jest przedstawiona procedura wyznaczania bilansu niepewności współczynnika cewki zdefiniowanego w normie EN 61000-4-8. Oprócz błędów przyrządów pomiarowych tzn.: sondy pola magnetycznego i amperomierza, uwzględniono niepewność ustawienia sondy w centrum cewki oraz błąd obcięcia wartości przenikalności magnetycznej przyjęty w przeliczeniu indukci magnetycznej na natężenie pola magnetycznego. Współczynniki wrażliwości wynikające z tego obliczono wyrażono wzorami analitycznymi. Dominującym składnikiem bilansu jest błąd pomiaru indukcji magnetycznej, co potwierdza prawidłowość budowy budżetu niepewności.* ***(Budżet niepewności wyznaczania współczynnika cewki według normy EN 61000-4-8)***

Keywords: coil factor, measurement intrumentation uncertainy MIU, sensitivity coeffcient, type A and B uncertainty, rule of root of sums of squares RSS
Słowa kluczowe: współczynnik cewki, niepewność pomiarowa przyrządów, współczynnik wrażliwości, niepewność typu A i B, zasada pierwiastka z sumy kwadratów

Introduction

Immunity against power frequency magnetic field is within the scope of EMC testing. The test set-up, according to [1] is shown in Fig 1. Magnetic field is generated with the square coil. The equipment under test (EUT) must be immersed centrally in the coil and oriented with three orthogonal positions ξ, η, ζ tied to the EUT.

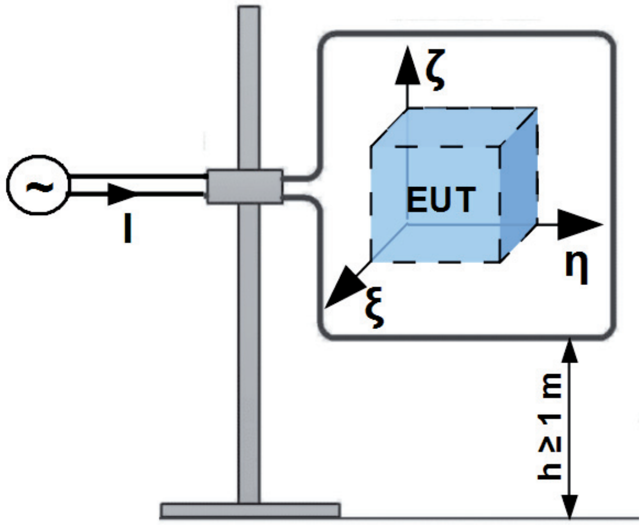

Fig. 1. Set-up for immunity test against power frequency magnetic field.

Severity level by testing is defined with the strength of magnetic field in the centre of the coil plane, by absence of the EUT.

It is much easier to control current driven in the coil instead of strength of accompanied magnetic field. Coil factor F_c defined in the standart [1] as the ratio of the magnetic field strength in the centre of the plane $H(0,0,0)$, as shown in Fig. 2 to the current driven in the coil I

$$F_c = \frac{H(0,0,0)}{I}, \tag{1}$$

facilitates it.

Analytical formula for distribution of magnetic field strength along symmetry axis of the coil is as follows

$$H_z(0,0,z) = \frac{4Ia^2}{\pi(4z^2+a^2)\sqrt{4z^2+2a^2}}, \tag{2}$$

Therefore

$$F_c = \frac{2\sqrt{2}}{\pi a} \tag{3}$$

For the coil with the side length $a = 1\ m$ theoretical coil factor is $F_c = 0.900\ [\,1/m\,]$

Motivation for elaboration of the uncertainty budget of the coil factor is absence of this task in standard [1].

Set-up for evaluation of the coil factor

Set-up for evaluation of the coil factor is shown in Fig. 2.

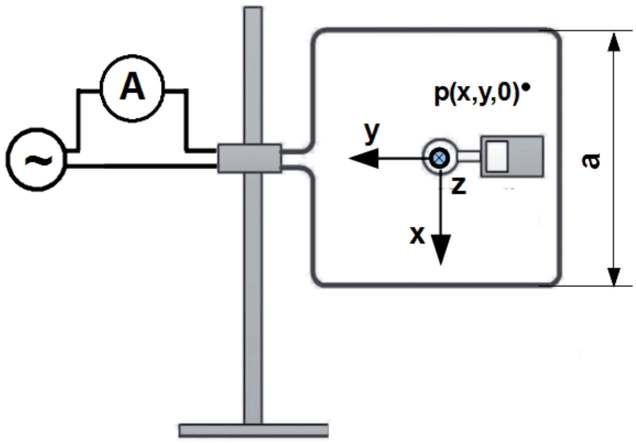

Fig. 2. Set-up for establishing the coil factor.

Implementation of the set-up is illustrated in Fig. 3. It consists of:

- programable AC source Model 61501 from Chroma-Source, Inc.,
- isotropic field probe ESM-100 from Maschek Elektronik,
- ampere meter DMM 4020 from Tektronix,
- magnetic field coil INA 702 from TESEQ AG.

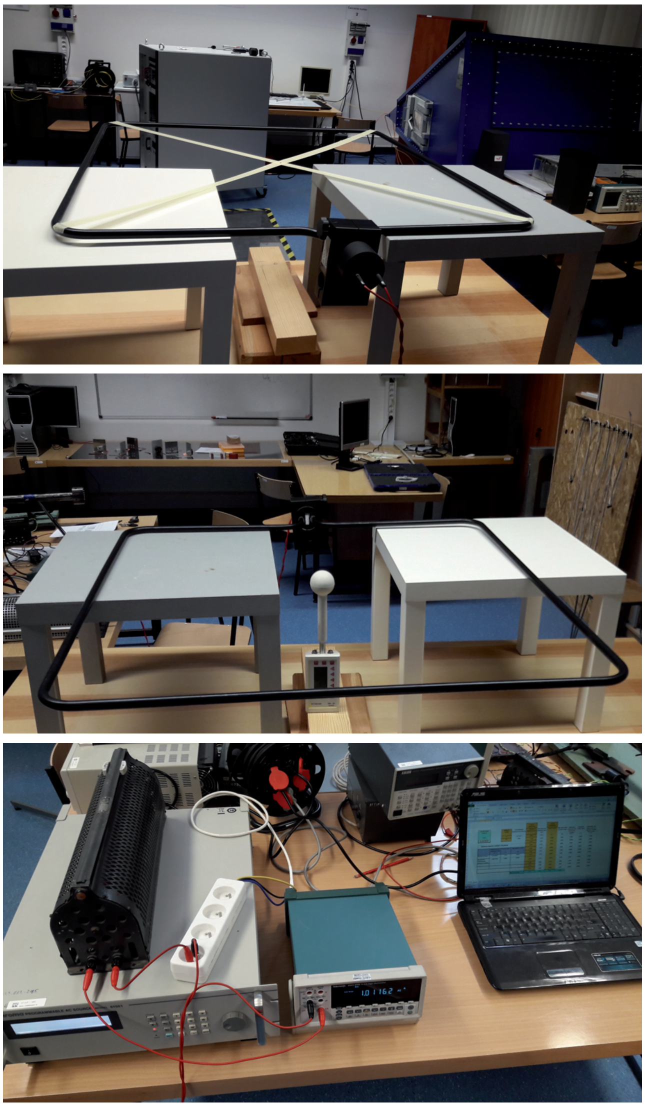

Fig. 3. Set-up for evaluation of the coil factor from top to bottom: marking the coil centre, positioning of the probe, power source and the ampermeter

Contributors of the uncertainty budget

- Measurement of the magnetic field strength

Isotropic field probe ESM-100 from Maschek Elektronik measures magnetic induction B.

The first uncertainty contribution stems from randomness of the measurement sample consised of N measurements. The measurement of the sample of currents as well magnetic inductance is equal to $N = 10$. It is called type A uncertainty. Measured is only one component of the field, alinged with symmetry axis z. Notation B without any additional indication is used for it. The estimate of the mean value yields

(4) $$\bar{B} = \frac{1}{N}\sum_{i=1}^{N} B_i$$

and the estimate of the standard uncertainty of the mean value yields

(5) $$s(\bar{B}) \approx \sqrt{\frac{1}{N(N-1)}\sum_{i=1}^{N}(B_i - \bar{B})^2}$$

Remaining contributors, type B uncertainties, are estimates of systematic errors.

According to the calibration report expanded uncertainty of the used probe is $U_{B_{cal}} = 8\ \%$.

Measurement of magnetic induction in the centre of the coil depends also on accuracy of positioning of the field probe. It is split here into thre components: positioning along the symmetry axis z and positioning in the centre of the coil plane in respect to x and y axis.

For calculation of derivative of magnetic induction B with respect to position along the symmetry axis z, product of field strength dependent on position at the axis $H_z(0, 0, z)$ and magnetic permeability which is burden with uncertainty due to truncation can be used

(6) $$B(\mu_0, z) = \mu_0 H_z(0, 0, z)$$

yielding

(7) $$dB = H_z d\mu_0 + \mu_0 \frac{\partial H_z}{\partial z} dz = H_z d\mu_0 + \mu_0 dH_z$$

where derivative of magnetic field strength dH_z can be calculated with Eq. (2)

(8) $$\frac{\partial H_z}{\partial z} dz = -8zH_z \left(\frac{1}{4z^2 + a^2} + \frac{1}{4z^2 + 2a^2} \right) dz$$

Finally

(9) $$dB = H_z d\mu_0 - 8zB \left(\frac{1}{4z^2 + a^2} + \frac{1}{4z^2 + 2a^2} \right) dz$$

With division by B, relative derivatie yields

(10) $$\frac{dB}{B} = \frac{d\mu_0}{\mu_0} - 8z \left(\frac{1}{4z^2 + a^2} + \frac{1}{4z^2 + 2a^2} \right) dz$$

From the point of view of statistics, Eq. (10) is sum of two independent, uncorelated random phenomena. The first is truncation of value of magnetic permeability taken by uncertainty estimation. Its maximal error is denoted with $U_{\mu_0} = max(d\mu_0/\mu_0)$ and uniform probability distribution can be assigned to it.

The second is positioning of the probe along symmetry z-axis. There is constant probability that the probe is placed in arbitrary point of the segment $[-U_z,\ U_z]$, where U_z is accuracy ability of the positioning arrangement. It means that uniform probability is associated with this random event. The maximal error, i.e. the worst case is when it happens to situate the probe at the edge of the segment, by $z = -U_z$ or $z = U_z$.

Propagation of the maximal, absolute error of the probe positioning U_z to the maximal, relative error of induction $U_B(U_z)$ is the second summand in Eq. (10) by setting $dz = U_z$

(11) $$U_B(U_z) = -8U_z^2 \left(\frac{1}{4U_z^2 + a^2} + \frac{1}{4U_z^2 + 2a^2} \right)$$

Relation formulated with Eq. (11) is illustrated in Fig. 4.

The derivative of magnetic induction B with respect to position x, y at the coil plane can be derived from following equation

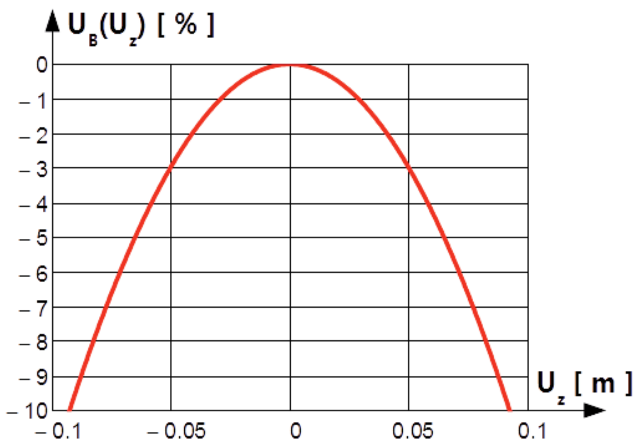

Fig. 4. Maximal error by positioning along symmetry z-axis.

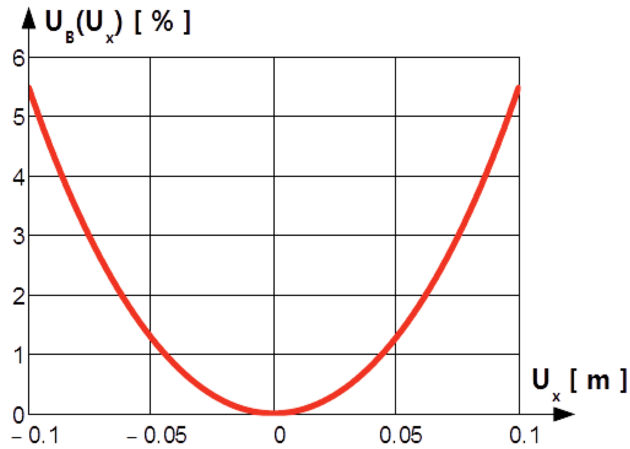

Fig. 5. Maximal error by positioning along x-axis.

(12) $$H_z(x,y,0) = \frac{I}{2\pi}\left[\frac{\sqrt{(a+2x)^2+(a+2y)^2}}{(a+2x)(a+2y)} + \frac{\sqrt{(a+2x)^2+(a-2y)^2}}{(a+2x)(a-2y)} + \frac{\sqrt{(a-2x)^2+(a+2y)^2}}{(a-2x)(a+2y)} + \frac{\sqrt{(a-2x)^2+(a-2y)^2}}{(a-2x)(a-2y)}\right]$$

with introducing relation Eq. (6).

Alike as in case of z-variation, there is constant probability of random event consisted in positioning the probe in arbitrary point of the square with sides $[-U_x,\ U_x]$ and $[-U_y,\ U_y]$ around the centre, where $U_x = U_y$ is accuracy ability of the positioning arrangement. The maximal error, i.e. the worst case is when it happens to situate the probe at the edge of the segment, by $x = -U_x$ or $x = U_x$ and $y = -U_y$ or $y = U_y$. Such situation is sum of two independent random events with uniform distributions. Therefore they are considered separately and combined as the root of sum of squares RSS.

Propagation of the maximal, absolute error of the probe positioning U_x to the maximal, relative error of measured induction $U_B(U_x)$ leads to too complex formula to be presented here. It is calculated with Mathcad15. Maximal error by positioning along x-axis $U_B(U_x)$ is shown in Fig. 5. The same relation is by dependence in respect to y-axis.

As in case of z-variation $U_B(U_x)$ and $U_B(U_y)$ must be combined with uncertainty U_{μ_0} originating from truncation.

Finally, combined standard uncertainty by measurement of magnetic induction is as follows

(13) $$u(\bar{B}) = \sqrt{s^2(\bar{B}) + \frac{U^2_{B_{cal}}}{4} + 3\frac{U^2_{\mu_0}}{3} + \frac{U^2_B(U_x)}{3} + \frac{U^2_B(U_y)}{3} + \frac{U^2_B(U_z)}{3}}$$

Divisor equal to 2 by expanded uncertainty $U_{B_{cal}}$ originates from the accredited calibration protocol of the field probe. Divisors equal to $\sqrt{3}$ by remaining systematic contributors are due to assumption of uniform probability distribution.

The uncertainty budget of magnetic induction performed in this paper is gathered in Table 1.

For U_{μ_0} truncation of sixth significant digit was used. Note, that in Eq. (13) contribution of truncation it taken three times into account. It is accomplished with sensitivity coeffcient equal to 3 in Table 1.

For positioning uncertainty, accuracy of positioning arrangement equal to $U_x = U_y = U_z = 5\ cm$ was taken.

Standard combined uncertainty of magnetic field strength can be derived from the $H(B)$ relation

(14) $$H = \frac{B}{\mu_0}$$

Derivative of magnetic field strength yields

(15) $$dH = -\frac{B}{\mu_0^2}d\mu_0 + \frac{1}{\mu_0}dB$$

By division with H, relative derivatie yields

(16) $$\frac{dH}{H} = -d\mu_0 + dB$$

As the sum of two uncorelated random events, standard combined uncertainty of magnetic field strength $u(\bar{H})$ must be calculated as the root of sum of squares RSS

(17) $$u(\bar{H}) = \sqrt{\frac{U^2_{\mu_0}}{3} + u^2(\bar{B})}$$

In Eq. (17) uncertainty due to truncation is taken into account once more. It results with slight change of combined standard uncertainty of magnetic field strength versus combined standard uncertainty of magnetic induction

(18) $$u(\bar{H}) = 4.479\ \%$$

- Current measurement

The first contribution stems from randomness of the measurement sample consised of N measurements of current. As noted earlier, the measurement of the sample of currents and magnetic inductance is equal to $N = 10$. The estimate of the mean value yields

Table 1. Combined standard uncertainty by measurement of magnetic induction B.

No.	Measurand	Origin	Expanded uncertainty $[\%]$	Probability distribution	Divisor	Sensitivity coefficient $[-]$	Standard uncertainty $[\%]$
1	Type A	Measurement sample	0.083	Normal	1	1	0.083
2	EMS-100 Maschek	Calibration protocol	8.000	Normal	2	1	4.000
3	Magnetic permeability	Truncation of fifth significant digit	0.048	Rectangular	$\sqrt{3}$	3	0.083
4	Probe positioning along x-axis	Analytical formula	1.279	Rectangular	$\sqrt{3}$	1	0.738
5	Probe positioning along y-axis	Analytical formula	1.279	Rectangular	$\sqrt{3}$	1	0.738
6	Probe positioning along z-axis	Analytical formula	2.975	Rectangular	$\sqrt{3}$	1	1.718
	combained standard uncertainty						4.478

(19)
$$\bar{I}= \frac{1}{N}\sum_{i=1}^{N} I_i$$

and the estimate of the standard uncertainty of the mean value yields

(20)
$$s(\bar{I}) \approx \sqrt{\frac{1}{N(N-1)}\sum_{i=1}^{N}(I_i- \bar{I})^2}$$

Current is measured with Digital Multimeter Tektronix DMM 4020. According to the data sheet its measurement uncertainty is $0.3\ \%$ of reading and $0.06\ \%$ of range, by the current range $2\ A$ and frequency range $45\ Hz\ -\ 2\ kHz$. The first component depends on individual element i of the measurement sample. In other words, it is type A uncertainty. The second is systematic error of the multimeter, i.e. type B uncertainty. There is no procedure in [3] for handling the error which is combination of type A and B.

Coil factor presented here was established by the current level close to $2\ A$, i.e. close to the chosen measurement range. Therefore systematic error by current measurement equal to $U_{I_{dat}} = 0.36\ \%$ is taken. It is the worst case and it is not big violation of rules.

Combined standard uncertainty by measurement of current is as follows

(21)
$$u(\bar{I}) = \sqrt{s^2(\bar{I}) + \frac{U^2_{I_{dat}}}{3}}$$

Divisor equal to $\sqrt{3}$ is due to assumption of uniform probability distribution which is recommended in [3] by uncertainty declared in the data sheets of the measurement equipments.

Combined uncertainty of the coil factor F_c

The coil factor calculated as quotient of Eq. (4) and Eq. (19) from the experimental data yields $F_c = 0.891\ [\,1/m\,]$. It is very close to theoretical value $F_c = 0.900\,[\,1/m\,]$.

The coil factor depends on measurement uncertainty of magnetic field strenght and current $F_c[u(\bar{H}), u(\bar{I})]$. Both input quantities are burdened with uncertainties.

(22)
$$F_c[u(\bar{H}), u(\bar{I})] = \frac{H[u(\bar{H})]}{I[u(\bar{I})]},$$

Total derivative of the coil factor is as follows

(23)
$$dF_c = \frac{\partial F_c}{\partial H}dH + \frac{\partial F_c}{\partial I}dI = \frac{1}{I}dH - \frac{H}{I^2}dI$$

By refering relation represented with Eq. (23) to the measured coil factor F_c and application of the rule of root of sum of squares RSS yields

(24)
$$u(F_c) = \sqrt{u^2(\bar{I}) + u^2(\bar{H})}$$

The total uncertainty budget of the coil factor is illustrated with the fishbone diagram in Fig. 6.

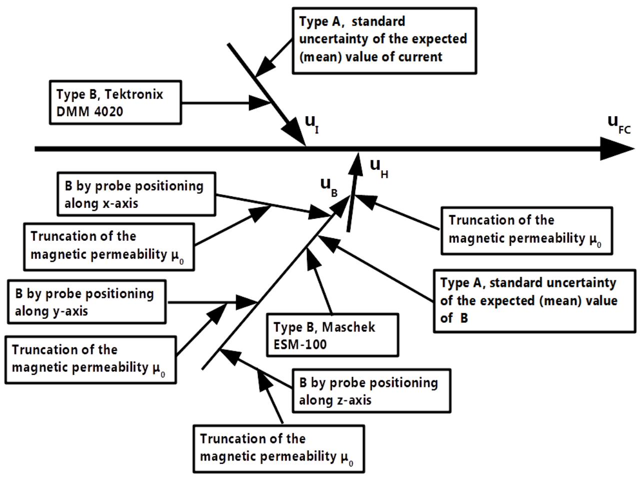

Fig. 6. Fishbone diagram of the uncertainty budget.

For the combined standard uncertainies of magnetic field strength, presented in Eq. (18) and for combined standard uncertainty of current presented in Table 2, combined standard uncertainty of the coil factor is equal to $u(F_c) = 4.484\%$. Expanded uncertainty of the coil factor is equal to

Table 2. Combined standard uncertainty by current measurement.

No.	Measurand	Origin	Expanded uncertainty $[\,\%\,]$	Probability distribution	Divisor	Sensitivity coefficient $[\,-\,]$	Standard uncertainty $[\,\%\,]$
1	Type A	Measurement sample	0.008	Normal	1	1	0.008
2	DMM 4020 Tektronix	Data sheet	0.360	Rectangular	$\sqrt{3}$	1	0.208
	combained standard uncertainty						0.208

$U(F_c) = 8.967\%$. It means that there is 95% level of confidence that the measurement result $F_c = 0.891\ [\,1/m\,]$ is within the range smaller than $U(F_c) = \pm 8.967\%$ to the true coil factor.

Conclusions

Procedure for establishing the uncertainty budget by evaluation of the coil factor in accordance with the standard [1] is presented. In addition to errors of the measurement instruments i.e.: field probe and ampere meter, contributions of uncertainty of probe positioning in the centre of the coil is presented. It is performed with analytical formulas for field distribution around the coil. This approach differes form [4] where the sensitivity coefficients were established with changing the probe position about the finite distance.

Sensitivity coefficients of relative quanities are always equal to 1.

Approach to the uncertainy of the current measurement presented here is not far away from the rules presented in [3]. More sophisticated approach requires advanced statistical tools.

Dominant component in the uncertainty budget is uncertainy of the measurement of magnetic induction. It confirms correctness of the budget.

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