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Performance Improvement of Aircraft pitch angle using the Fractional Order Adaptive PID Controller

Abstract. *Fractional calculus has been rediscovered by scientists and engineers in the last two decades, and applied in an increasing number of fields, namely control theory. The current research work presents the use of the fractional adaptive PID controller approach optimized by a genetic algorithm to improve the performances (rise time, setting time, overshoot, and mean absolute error) for aircraft by introducing a fractional order integrator and differentiator in the classical feedback adaptive PID controller. To validate the arguments, the effectiveness and performance analysis of the proposed fractional order adaptive PID controller optimized by a genetic algorithm have been studied in comparison to the classical adaptive PID controller. Numerical simulation and analysis are presented to verify the best controller. The fractional order adaptive PID gives the best results in terms of settling time, rise time, overshoot, and mean absolute error. This approach can also be generalized to other fractional and integer systems in order to improve their performances and noise rejection.*

Streszczenie. *Rachunek ułamkowy został na nowo odkryty przez naukowców i inżynierów w ciągu ostatnich dwóch dekad i stosowany w coraz większej liczbie dziedzin, a mianowicie w teorii sterowania. Obecna praca badawcza przedstawia zastosowanie podejścia adaptacyjnego regulatora PID ułamkowego zoptymalizowanego przez algorytm genetyczny w celu poprawy wydajności (czas narastania, czas ustawiania, przeregulowanie i średni błąd bezwzględny) dla samolotów poprzez wprowadzenie integratora i różniczkowania ułamkowego rzędu w klasycznym adaptacyjnym regulatorze PID ze sprzężeniem zwrotnym . Aby potwierdzić te argumenty, przeprowadzono analizę skuteczności i wydajności proponowanego adaptacyjnego regulatora PID ułamkowego rzędu zoptymalizowanego algorytmem genetycznym w porównaniu z klasycznym adaptacyjnym regulatorem PID. Przedstawiono symulację i analizę numeryczną w celu weryfikacji najlepszego sterownika. Adaptacyjny PID ułamkowego rzędu daje najlepsze wyniki pod względem czasu ustalania, czasu narastania, przeregulowania i średniego błędu bezwzględnego. To podejście można również uogólnić na inne systemy ułamkowe i całkowite w celu poprawy ich wydajności i tłumienia szumów.* (***Poprawa wydajności kąta nachylenia samolotu za pomocą adaptacyjnego kontrolera PID rzędu ułamkowego***)

Keywords: Fractional systems, Aircraft System, Fractional Adaptive PID controllers, Optimization.
Słowa kluczowe: Systemy ułamkowe, system lotniczy, kontrolery frakcyjno-adaptacyjne PID, optymalizacja.

Introduction

Fractional calculus was introduced as a branch of mathematics after the first communication was made on a fractional-order (FO) derivative with speculation between L'Hospital and Leibniz [1,2]. The applications of fractional order differentiation have attracted the attention of researchers from a wide variety of science disciplines, especially from the fields of applied sciences [3-5]. In 1997, Podlubny put forward the idea of FOPID controllers. In comparison to the conventional PID controller, Podlubny and others have also shown how this sort of controller responds more quickly when used for the control of fractional order systems [6-8].

It is well known that to address the parametric uncertainty in both linear and non-linear systems, the adaptive control method is one of the best control techniques being used so far. However, the focus was on the use of integer order systems to implement the adaptive control method [9, 10]. Monje and *al* [3] have reported the use of fractional calculus in conventional systems and control. The adaptive function projective and feedback control schemes have also been reported regarding synchronization of chaotic systems with fractional order in addition to the development of adaptive sliding-mode controller for such systems [11]. The effect of uncertain fractional order in chaotic systems can be controlled by adopting various practical methods such as an adaptive fractional-order switching-type control method, an adaptive fuzzy sliding-mode control method, or a synchronization control method [12].

A genetic algorithm is one of the very important techniques used in computing to find true or approximate solutions to optimization and search problems. The genetic algorithm's fitness function design is crucial since it has a significant impact on the output that is intended. Each individual fitness value is calculated by applying the fitness function to it [13-15].

The main contribution of this work is the use fractional adaptive PID controller approach optimized by genetic algorithm to improve the performances (rise time, setting time and overshoot) for Aircraft system by introducing fractional order integrator and differentiator in the classical feedback adaptive PID controller. The optimizing parameters are obtained by using the fitness function.

The manuscript is organized as follows: first, we have discussed the fundamentals of a fractional order system, followed by the study of algorithms for an integer and fractional adaptive PID controller. Afterwards, the results obtained with performance analysis from the simulation applied to the aircraft system using the integer and fractional adaptive PID controllers are presented. Lastly, the conclusion and future perspectives of the study are given.

Fractional Order Systems

It is well-known that calculus is used to generalize the derivation or integration of various functions. However, a subfield of calculus is called fractional calculus, which normally uses non-integer order for the generalization of derivatives or integrals of a function [16-19]. fractional approximation techniques are authorized to employ easily fractional order systems in many different application areas such as control theory [20,21], renewable energy [22] , economical systems [23]... etc.

There are several characteristics that would decide the relative merits of any approximation, such as differentiation order, frequency behavior, time responses, etc. Several approximations are discussed here with respect to their comparative analysis with others [24,25]. The available approximations belong to two different domains (frequency and time domains), which are specified as s-domain and z-domain, respectively. The approximations in frequency and time domains are also termed continuous and discrete approximations [26, 27].

The Oustaloup method is based on the function approximation from as;

(1) $$G_f(s) = S^{\alpha} \qquad , \alpha \in R^{+}$$

By taking into account the rational function:

(2) $$G_f(s) = K\prod_{k=1}^{N}\frac{s+w_k^{'}}{s+w_k}$$

However, the poles, zeros, and gain can beevaluated as;

$w_k^{'} = w_b . w_u^{(2k\text{-}1\text{-}\gamma)/N}$, $w_k = w_b . w_u^{(2k\text{-}1+\gamma)/N}$, $K = w_h^{\gamma}$

Where w_u represents the unity gain in frequency and the central frequency in a geometrically distributed frequency band. Let $w_u = \sqrt{w_h w_b}$, where, w_h and w_b represent the upper and lower frequencies, respectively. γ and N are the orders of derivative and filter, respectively.

Optimization through genetic algorithms

A genetic algorithm is a method for locating precise or approximate answers to optimization problems [28]. The essential steps in the genetic algorithm process are encoding, evaluation, cross-over, mutation, and decoding. The initial population is chosen at random, and each person's fitness is then calculated. The design of fitness function is particularly important in genetic algorithm Because the desired output significantly depends on the design of the fitness function. The steps of the algorithm of GA given in [28] are:

1. Choose the initial population
2. Each individual in the population should have their fitness evaluated.
3. Repeat
    3.1. Choose the most qualified people to reproduce
    3.2. Create a new generation through hybridization and mutation, then produce children.
    3.3. evaluation of the fitness of the progeny.
    3.4. Children should be used to replace the least desirable population.
4. Until termination

Aircraft mathematical model

During flight, the aircraft can be manipulated into three rotational axes. Rotation along the vertical axis is called yaw, along the longitudinal axis is called roll and along the lateral axis is called pitch. The axis meets in the gravity center of the aircraft.

The pitch angle mathematical model is shown in Figure 1 [28].

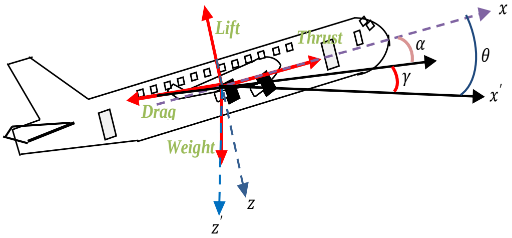

Fig.1. Pitch control description

The aircraft dynamic behavior is given by the following set of relations [28]:

(3) $$\frac{d\alpha}{dt} = \mu\Omega\sigma\left[-(C_L + C_D)\alpha + \frac{1}{\mu - C_L}q - (C_M sin\gamma)\theta + C_L\right]$$

(4) $$\frac{dq}{dt} = \frac{\mu\Omega}{2i_{yy}}[(C_M - \eta(C_L + C_D))\alpha + (C_m + \sigma(1 - \mu C_L))q + (\eta C_W sin\gamma)\delta]$$

(5) $$\frac{d\theta}{dt} = \Omega q$$

where: $\alpha, \theta, \gamma, \delta$: Attack, pitch, flight path and elevator deflection angles respectively; C_T, C_D, C_L, C_W, C_M Thrust, drag, lift, weight and pitch moment coefficients respectively, σ, η: Constants ; q: Pitch rate ;ρ: Air density; S: Wing platform area; m: Mass of the aircraft. i_{yy}: Inertia normalized moment

The aircraft pitch angle transfer function is given in Equation (6).

$$G_P(s) = \frac{\theta(s)}{\Delta(s)}$$

(6) $$G_P(s) = \frac{1.151\,s + 0.1774}{s^3 + 0.739\,s^2 + 0.921s}$$

Where: $\Delta(s)$: is the elevator deflection; $\theta(s)$: is the pitch angle

Integer Adaptive PID Controller

The integer adaptive feedback control law is given by the equation 7, [13]:

(7) $$u(t) = -k_c[k_1(t)e(t) + I\{k_2(t)e(t)\} + D(k_3(t)e(t))]$$

With:

$k_1(t) = k_p(t) + \alpha_1 k_i(t) + \alpha_3 k_d(t)$
$k_2(t) = \alpha_2 k_i(t)$,$k_3(t) = \alpha_4 k_i(t)$, $k_p(t) = e^2(t)$
$k_i(t) = I\{e^2(t)\}$ and $k_d(t) = D\{e^2(t)\}$
$e(t) = y(t) - r(t)$
Where k_c, α_1 and α_2 are positive constants.

The ordinary schematic representation of the overall system is shown in figure 2.

We can see the extreme simplicity of the control system.

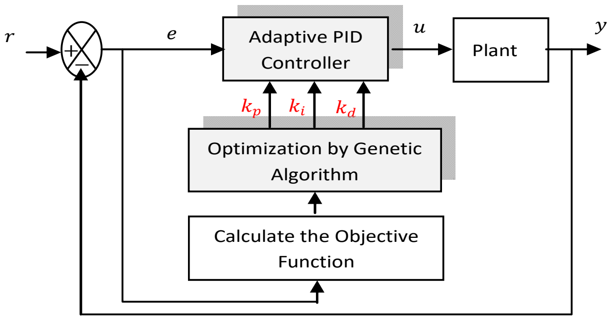

Fig. 2. Classical Adaptive PID Control System.

Fractional adaptive PI$^{\lambda}$D$^{\mu}$ Controller

The Fractional adaptive feedback control law is given by the equation (8):

(8) $$u(t) = -k_c\left[k_1(t)e(t) + I^{\lambda}\{k_2(t)e(t)\} + D^{\mu}(k_3(t)e(t))\right]$$

With:

$k_1(t) = k_p(t) + \alpha_1 k_i(t) + \alpha_3 k_d(t)$,$k_2(t) = \alpha_2 k_i(t)$,
$k_3(t) = \alpha_4 k_i(t)$
and $k_p(t) = e^2(t)$, $k_i(t) = I^{\lambda}\{e^2(t)\}$,$k_d(t) = D^{\mu}\{e^2(t)\}$
$e(t) = y(t) - r(t)$

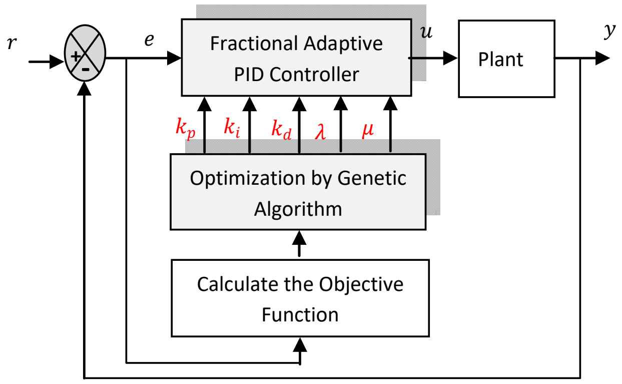

Fig. 3. Fractional Adaptive PID Control System

Results and Discussion

The error between the computed and measured output is used to determine the fitness function F that will be used for minimization. It is given by the Mean Absolute Error (MAE) as follows:

$$MAE = \frac{\sum_{i=1}^{N} y(i)-r(i)}{N} \tag{9}$$

Where y is the measured output system and r is the desired output.

Figure 4 shows the Pitch angle of the Aircraft using the Integer adaptive PID Controller with the following optimized parameters values: $k_p = 114.8231, k_i = 4.4575, k_d = 49.0933$:

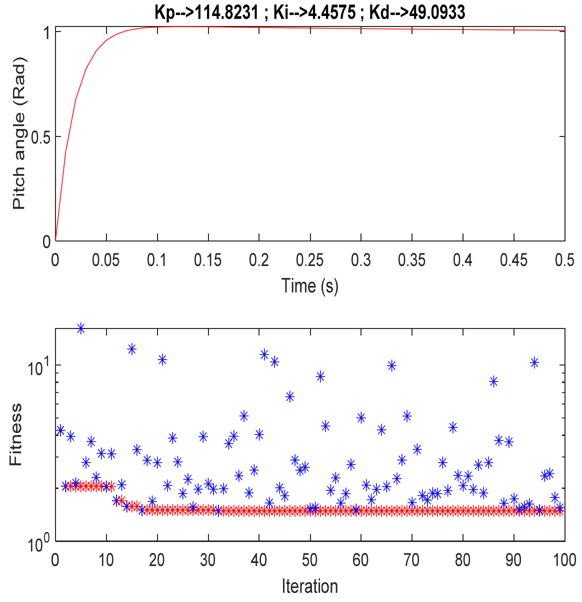

Fig. 4. Pitch angle of the Aircraft using the Integer adaptive PID Controller

Figure 5 shows the Pitch angle of the Aircraft using the Fractional adaptive PID Controller with the following optimized parameters values:

$k_p = 6.5582, k_i = 0.15435,\ k_d = 673.5805,\ \lambda = 0.20115,$
$\mu = 0.99731$

The performance analysis of the proposed fractional order adaptive PID controller and the classical adaptive PID controller is given by the following table:

Table 1. Transient Response Stability Parameters of Aircraft System

Controllers	Overshoot [%]	Setting time [s]	Rise time [s]	Mean Absolute Error (Rad)
APID	1.7242	0.0595	0.0373	0.0012
FAPID	0.0533	0.0193	0.0106	0.0005

We remark that the fractional adaptive PID Controller for Aircraft system give the good improvement of overshoot, setting time , rise time and mean absolute error comparatively to the Integer adaptive PID Controller results.

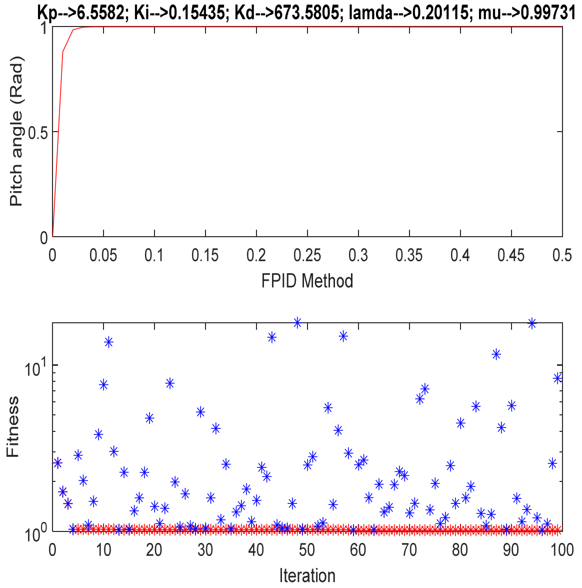

Fig. 5. Pitch angle of the Aircraft using the Fractional adaptive PID Controller.

Conclusion

In this paper, we study the performance analysis using the integer adaptive PID controller and the fractional adaptive PID controller optimized by a genetic algorithm applied to an aircraft system. The Fractional approach allows to improve the good performances of overshoot, setting time, rise time and mean absolute error comparatively to the Integer adaptive PID Controller results.

The simulation studies show good performance of the proposed approach and confirm its superiority over an integer adaptive PID controller.

In future work, we will investigate the generalization of the fractional adaptive control approach to other fractional systems in order to improve their robustness and noise rejection.

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