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Modelling the effect of excitation frequency on the shape of hysteresis loop in permalloy

***Abstract**. In the paper the effect of excitation frequency on the shape of hysteresis loops in permalloy was investigated and modelled. Two models were considered – one based on effective field with term depending on magnetization derivative, and the other based on viscosity-related equation superimposed over a static hysteresis model. In the first approach only a qualitative agreement with measurements was obtained, whereas the second one offers a better agreement, although it exhibits specific anvil-like distortion in the second and the fourth quadrant of the H-B coordinate system. The "anvils" are a direct consequence of singularity in the viscosity equation when saturation is approached.*

***Streszczenie.** W artykule zbadano i zamodelowano wpływ częstotliwości wymuszenia na kształt pętli histerezy permaloju. Rozważano dwa modele – jeden oparty na polu efektywnym ze składnikiem zależnym od pochodnej namagnesowania, drugi oparty na równaniu lepkościowym nałożonym na model histerezy statycznej. W pierwszym podejściu uzyskano jedynie jakościową zgodność z pomiarami, natomiast drugie oferuje lepszą zgodność, choć z pojawiają się w nim pętle z „kowadłami" w drugiej i czwartej ćwiartce układu współrzędnych H-B. „Kowadła" te są bezpośrednią konsekwencją osobliwości w równaniu lepkościowym w stanie nasycenia. (**Modelowanie wpływu częstotliwości wzbudzenia na kształt pętli histerezy w permaloju**)*

Keywords: magnetic hysteresis, effective field, viscosity-like equation, *T*(*x*) model, permalloy.
Słowa kluczowe: histereza magnetyczna, pole efektywne, równanie lepkościowe, model *T*(*x*), permaloj.

Introduction

The shape of hysteresis loop may be affected by a number of phenomena, to mention e.g. temperature, mechanical stress or demagnetization fields [1]. The present paper focuses on the effect of excitation frequency which impacts eddy currents generated in conductive soft magnetic materials (SMMs) [1,2]. It is a follow-up and an extension of concepts presented in previous publications [3-7].

In the paper we consider two possible approaches to take into account rate dependence of hysteresis, which is an effect noticeable at increased excitation frequency. The first approach is based on the introduction of an appropriate extension to the "effective field", the second one is based on the viscosity-related relationship considered in a number of papers by Zirka *et al.* [8-10]. For description of hysteresis loop branches we use the phenomenological *T*(*x*) model advanced by Takács [11].

The concept of "effective field" plays a fundamental role in contemporary solid state physics. It denotes an additional field of any origin, introduced into a theoretical model in order to capture in an approximate way the effect of some complex phenomenon (e.g. mutual interactions between magnetic moments) or a factor, which is otherwise difficult to be handled analytically. An example of such interpretation of the "effective field" is the century-old paper by Weiss [12], who analysed interactions between magnetic moments in ferromagnets. The introduction of effective field concept allowed him to explain the phenomenon of spontaneous magnetization. A direct consequence of Weiss' theory was the postulate that within ferromagnetic sample there exist randomly directed regions of uniform magnetization (magnetic domains). In spite of its highly simplified character, the Weiss' theory allowed one to consider qualitatively the most important physical properties of ferromagnets, e.g. spontaneous magnetization, the existence of Curie point. This approach made it possible to explain the dependence of magnetization on temperature. It was the first mathematical description of phase transitions in the material. On the basis of Weiss' theory Heisenberg developed a quantum-based description of exchange interactions between electrons on partially filled atomic layers, which nowadays remains one of the cornerstones of contemporary ferromagnetism theories. The Weiss' theory is being analysed by researchers still today, its particular role is especially noticeable as far as the studies of phase transitions are concerned [13].

Briefly speaking, the "effective field" should be understood as a framework to describe a cooperative action of numerous elementary contributions. In the language of control science the "effective field" implies a positive feedback in the system, what in ferromagnetism may be written as a self-consistent relationship

$$(1)\qquad M(t) = \Gamma\left(H_{\text{eff}}(t)\right) = \Gamma\left(H(t), M(t)\right),$$

where Γ denotes the hysteresis operator and the „effective field", $H_{\text{eff}}(t)$ is a function both of the applied field $H(t)$ and magnetization $M(t)$ itself [14].

The phenomenological *T*(*x*) model uses a combination of hyperbolic tangent and linear functions to describe magnetization curves. The model is able to describe both closed hysteresis loops as well as more complicated magnetization curves e.g. biased loops or reversal curves of arbitrary order [15]. The linear function is used to describe the reversible magnetization process, this term is usually skipped in the considerations for simplicity. Takács uses in his book [11] an abstract notation in dimensionless units, however it is straightforward to introduce an interpretation of the quantities x and y as reduced magnetization and reduced "effective field", respectively [16].

The "effective field" accounting for rate dependence of magnetization curves may be written in a general form using physical units as

$$(2)\qquad H_{\text{eff}}(t) = H(t) + \alpha M(t) - \operatorname{sign}\left(\mathrm{d}M/\mathrm{d}t\right)\beta\left|\frac{\mathrm{d}M}{\mathrm{d}t}\right|^{\nu},$$

where α, β and ν are constants to be determined from measurement. The value of fractional exponent ν may be recovered from the experimentally determined coercive field strength H_c as a function of frequency f.

As pointed out previously, yet another approach to consider rate dependent effects in hysteresis modelling is to avail of a viscosity-related equation, which is superimposed over a quasi-static hysteresis model. The relationship may be written as

$$H(t) = H_{stat}(M) + \left[\frac{1}{r(M)}\frac{dM}{dt}\right]^{1/\nu}, \tag{3}$$

where $r(M) = K[1 - (M/M_s)^2]$, K is a parameter, M_s is saturation magnetization and the (possibly fractional) exponent $1/\nu$ accounts for model dynamics. Again, it can be noticed that this relationship is related to experimentally determined $H_c = H_c(f)$ dependence if one puts $M = 0$. Let us remark that in fact the afore-described approach assumes that the effects from hysteresis and eddy currents may be separated.

A comparison of both approaches combined with the *T*(*x*) model was carried for non-standard excitation conditions (triangular flux density waveform) in [3]. The material under study was grain-oriented electrical steel used as core material for a resistance spot welding transformer. It was found that the second approach allowed one to describe magnetization curves over a wider frequency range than the first one. The aim of the present paper is to verify this supposition for other soft magnetic materials examined under standard excitation conditions.

Ref. [4] proved that both approaches belonged to the class of Chua-type dynamic hysteresis models. A good performance of the hybrid Takács-Zirka description was achieved for hysteresis loops of non-oriented electrical steels. Ref. [5] provided additional verification of the Zirka's approach for hysteresis loops of a Metglas core. It was found that the fractional value of the exponent could be assumed constant in a wide range of flux densities.

The paper [6] put the concept of anhysteretic curve in the *T*(*x*) model in the spotlight. It was found that the equation of inverse anhysteretic curve i.e. for the relationship $H_{an} = H_{an}(M)$ may be given in a closed algebraic form within the *T*(*x*) framework, provided the linear term accounting for reversibility effects is skipped.

Neglection of linear term in the analysis does not imply that the modelled hysteresis loops are devoid of some reversibility "fingerprint". In fact, for any closed loop, either symmetric or biased one, one can determine a corresponding local anhysteretic curve as the middle curve between its branches [18]. This statement is in the spirit of Far-Eastern philosophy (the *yin* and *yang* concept) and it explains the concept of anhysteretic surfaces, envisaged by Sablik and Langman [19]. Takács' assumption that hysteresis loop tips are the loci of anhysteretic curves [20] offers a unified framework for analysis of magnetization curves. It should be remarked that in the widely used Jiles-Atherton formalism [21] there exists a severe bug due to misplaced positions of the loop tip and the corresponding point on the anhysteretic curve; this model deficiency has to be patched *post-hoc* by the introduction of artificially introduced pseudo-parameter δ_M, which suppresses the irreversible magnetization component after field reversals [22,23].

Ref. [7] was devoted to description of quasi-static hysteresis loops of non-oriented electrical steel sheets for two principal directions using the *T*(*x*) model. The computational flowchart applied previously in [6] for grain-oriented electrical steel was followed for this important SMM. The morphology of non-oriented and grain-oriented steels is completely different, the latter material features much larger grains [24], what in turn results in better magnetic properties (e.g. lower coercivity and losses). It was found that the *T*(*x*) model might be applicable also to non-oriented steels, however it generally underestimated losses and the modelled minor hysteresis loops were narrower than their measured counterparts.

Material and methods

In the present paper we examine the performance of both approaches derived from *T*(*x*) framework for description of hysteresis loops of a permalloy core.

Nickel-iron alloys, commonly referred to as permalloys, are relatively old group of soft magnetic materials [25-27], however the information on modelling of their properties is rather scarce. In dependence on processing route and nickel content these materials exhibit unique magnetic properties:

- alloys containing 35-40% Ni feature high values of resistivity, around 7.5 × 10^{-5} Ωm, small initial relative permeability μ_i = 2000 − 3000, maximum permeability μ_{max} = 8000 − 20000 and saturation polarization around 1.3 T;
- alloys containing around 50% Ni exhibit highest values of saturation polarization, average values of permeability and lowest resistivity within this group of soft magnetic materials;
- alloys containing 79-80% Ni exhibit highest values of permeability, lowest coercivity values; however their saturation polarization is also low (around 0.7 T) [28].

The interest of engineering community in the last group of permalloys is still observable today. In these materials it is possible to control precisely their electromagnetic properties; moreover, they can be easily processed mechanically, what is an advantage when parts of magnetic circuits with complex shapes are required [29].

79-80% NiFe alloys exhibit practically rectangular hysteresis loops in quasi-static magnetization conditions and at mains frequency. For higher frequencies the shape of hysteresis loop is greatly affected by eddy currents generated in the conductive material, cf. Fig. 1. As evident from Figure 2 the measured $H_c = H_c(f)$ dependencies for this material may be described with straight lines. The slopes of the trend lines depend on B_m; thus in the subsequent analysis we assume that ν = 1.

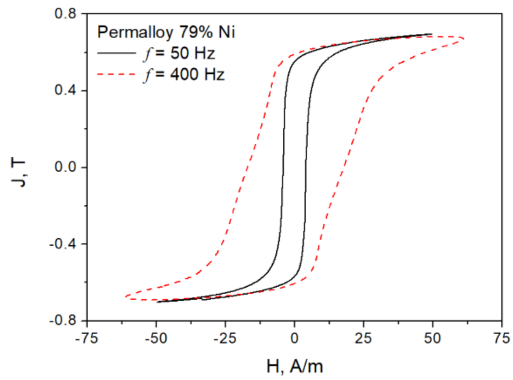

Fig.1. Experimental hysteresis loops for two frequencies

The $H_c = H_c(f)$ relationship is indeed related to rate dependence, what can be proven using a simple reasoning for the major hysteresis loop, described with the *T(x)* model, in which the effective field contains an additional term, as in expression (1).

Let us recall that the ascending branch of the major loop is given as

$$M = M_s \tanh\frac{H_{eff} - H_{c0}}{A}, \tag{4}$$

and magnetization M is then swept from $-M_s$ to M_s in accordance with $M = -M_s\cos 2\pi ft$. The coercivity point is reached for $t = T/4$, magnetization is then zero. Simple transformation of relationships (1) and (4) allows one to write $H_c(f) = H_{c0} + \beta^* f$, where $\beta^* = 2\pi M_s\beta$.

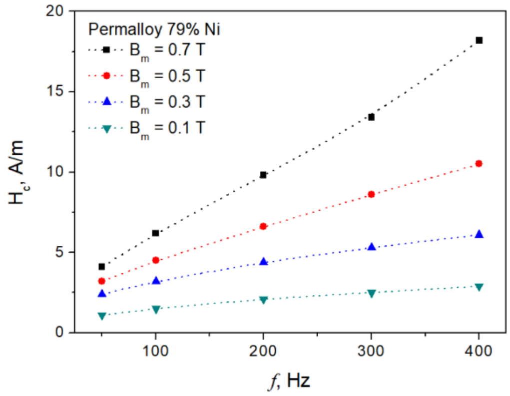

Fig.2. Experimental dependencies of coercive field strength vs. frequency for different amplitudes of magnetic flux density

The mains frequency was the lowest one considered during modelling, thus the experimental hysteresis loop at this frequency (shown in black in Fig. 1) was used as the starting point in the analysis and treated as an approximation of quasi-static excitation conditions. The parameter of quasi-static $T(x)$ model were determined from the condition that the modelled hysteresis loop follows the measured one (a nonlinear regression problem, the sum of squared deviations in a number of reference points was the optimization criterion). The measurements were carried out on a cylinder-shaped sample in accordance with the IEC 60404-6 standard.

Modelling results are depicted in Figures 3 ($T(x)$ model with rate-dependent term in the effective field) and 4 (hybrid Takács-Zirka approach) for excitation frequency equal to f = 400 Hz.

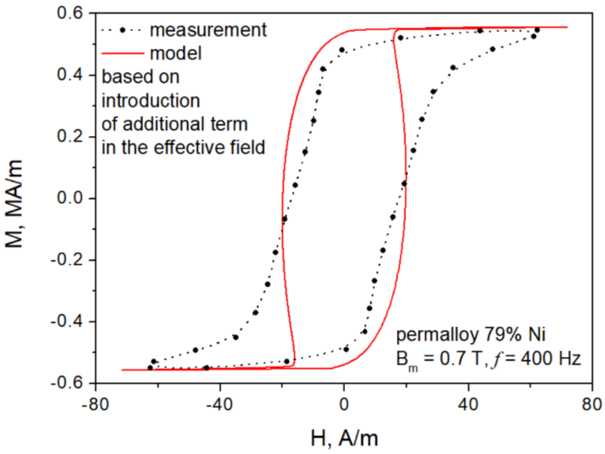

Fig.3. A comparison of measured and modelled major hysteresis loop ($T(x)$ model with effective field containing the rate-dependent term)

It can be stated that the second approach outperforms the first one. The criterion for assessment of model accuracy was not only the possibility to recover the experimental $H_c = H_c(f)$ dependence, but also an overall qualitative agreement between the shapes of measured and modelled hysteresis loops.

The first model offered only a qualitative agreement with experiment. For the second model we observed anvil-like „horns" in the second and the fourth quadrants of the coordinate system, which are the direct consequence of singularity of function $r(M)$ when saturation is approached. We suspect this effect could be suppressed by adding a small residual constant in the denominator of the relationship (3), however we did not want to introduce additional model parameters or "patches".

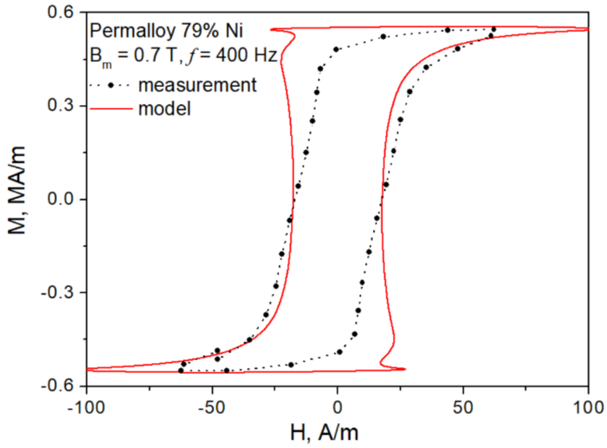

Fig.4. A comparison of measured and modelled major hysteresis loop ($T(x)$ model with effective field without the rate-dependent term + Zirka's relationship to account rate-dependence)

Another interesting observation which could be made from the analysis of results is that the measured slopes of hysteresis loops are smaller than their modelled counterparts. In order to find the reason for this unusual model behaviour we have determined the anhysteretic curves corresponding to experimental loops depicted in Fig. 1 as their middle curves [17]. We have found that in fact there exists some dependence of the slope of reconstructed anhysteretic curves on frequency, cf. Fig. 5, however the reason for this phenomenon is unknown. Neither of the analysed models accounted for this effect, thus in Figs. 3 and 4 there are noticeable discrepancies between measurement data and modelled curves. We believe that a potential explanation for this effect is that in fact there exist anhysteretic hyperplanes and not just surfaces, as envisaged by Sablik and Langman [18]. The external stimuli (e.g. applied stress, temperature) and excitation parameters (not just the amplitude, but also frequency) determine a unique projection of the hyperplane onto a single curve. This interpretation shall be the subject of subsequent studies.

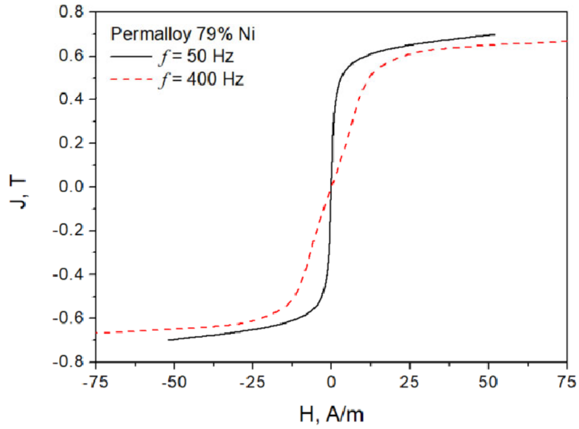

Fig.5. Reconstructed anhysteretic curves corresponding to hysteresis loops from Fig. 1

The general conclusion which can be drawn from the analysis is that the hybrid Takács-Zirka approach yields a better modelling accuracy than the $T(x)$ model with rate-dependent term in the effective field. This remark is consistent with conclusions from previous study [4], which however was devoted to another SMM and used non-standard excitation system.

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