

Radial Basis Function Networks Based Synergetic Terminal Controller For Piezoelectric Actuator With Colman-Hodgdon Hysteresis Model

Abstract. This article proposes an intelligent nonlinear control approach based on the concept of combining neural networks and the methodology of synergetic terminal control. We have introduced the notion of convergence in finite time through the improvement induced on the macrovariable. This approach is associated with the approximation criteria of unknown nonlinearities by a radial basis function neural network to a class of PEA piezoelectric actuator positioning mechanisms. RBF weights are adjusted using the terminal attractor concept in the PEA finite time control process. The stability of a closed-loop system is ensured by the Lyapunov method. The simulation results have demonstrated the robustness of the proposed approach and provide good results in terms of tracking the trajectory and provides a better overall performance than that of classical synergetic control.

Streszczenie. W artykule zaproponowano inteligentne podejście do sterowania nieliniowego, oparte na koncepcji łączenia sieci neuronowych i metodologii synergicznego sterowania terminalami. Wprowadziliśmy pojęcie zbieżności w skończonym czasie poprzez poprawę wywołaną makrozmienną. Podejście to wiąże się z aproksymacją kryteriów nieznanych nieliniowości za pomocą sieci neuronowej o promieniowej funkcji bazowej do klasy mechanizmów pozycjonowania siłowników piezoelektrycznych PEA. Wagi RBF są regulowane za pomocą koncepcji atraktora końcowego w procesie kontroli czasu skończonego PEA. Stabilność zamkniętej pętli zapewnia metoda Lapunowa. Wyniki symulacji wykazały solidność proponowanego podejścia i zapewniają dobre wyniki w zakresie śledzenia trajektorii oraz zapewniają lepszą ogólną wydajność niż w przypadku klasycznego sterowania synergicznego. (Funkcja podstawy promieniowej Synergiczny kontroler terminala oparty na sieci dla siłownika piezoelektrycznego z modelem histerezy Colmana-Hodgdona)

Keywords: Coleman-Hodgdon model, Identification with particle swarm optimization approach, Radial Basis Function Networks.
Słowa kluczowe: Model Colemana-Hodgdona, Identyfikacja metodą optymalizacji roju cząstek, Radialne sieci funkcyjne

Introduction

Scientific research and industry frequently call for precision positioning mechanisms with large millimeters order stroke and Micro/Nanoscale positioning resolution. Uncertainties such non-linearities, hysteresis phenomena, and drift that reduce positioning and/or tracking accuracy have spurred research in positioning systems control theory [1]. Furthermore, resonance vibrations can result in vibrational reactions that upset a system [2]. A system is typically controlled via its modeling, or by figuring out the relationships between its variables [3]. A mathematical model that is based on an experimental analysis, a theoretical analysis, or even a theoretical-experimental analysis can describe a system [4].

To model the system, one must first select the model's structural elements that best explain and forecast the system's behavior and are compatible with the synthesis of the control. Identification, or figuring out the parameters of the selected model, comes next in the second step [5]. Due to several considerations, such as measurement noise, the positioning systems in use today have unmodeled dynamics [6]. The motion detector essentially consists of a displacement sensor and hardware circuit [7]. For closed loop control, it may measure the output displacement and relay it back to the control system [8]. It is also a crucial component because the detector's performance directly affects the system's accuracy [9]. By mathematical modeling and control strategies, remarkable attempts have been made to account for the non-linearities of the piezoelectric actuator [10]. Consequently, it would seem that both theoretical and practical parts of nonlinear system control strategies are inadequate [11].

The structure of the internal dynamics of nonlinear systems is revealed by recent approaches to nonlinear systems, such as differential geometry theory [12]. Nonlinear inversion dynamics [13], backstepping [14]-[15], and

feedback linearization [16] are three nonlinear control design techniques that have been created and successfully used to solve control issues. Advanced nonlinear control design techniques typically rely on an analytical model of the dynamic system, which is either unreliable or unavailable in many control applications [17].

While the adaptive component automatically corrects for the modeling error created by the simplification, nonlinear adaptive control approaches also permit the use of a simplified dynamic model in the design of the controller [18]. The assumption that the unknown dynamics have a defined structure and that unknown parameters enter the dynamics linearly is a key flaw in these nonlinear adaptive control systems [19]. The employment of adaptive control methods in real applications is severely hampered by the linear parameterization of the unknown dynamics because it is challenging or impossible to fix the structure of the unknown non-linearities [20].

Therefore, smart control attracts great attention. It offers a fresh concept for resolving challenging system control issues. A standard and workable structure for the representation of nonlinear dynamical systems is provided by neural networks, which can approximate a broad class of nonlinear functions [21]. In order to improve applications when a mathematical model of a nonlinear system is available but imprecise or to create a model of the system from experimental data when a theoretical and fundamental model is impractical, neural networks are a strong tool [22]. While in adaptive control, the change of the neural network parameters is driven by a tracking error, which by definition contains no information about the entry, the weights of the neural network are updated according to input-output information in offline applications [23].

loop are bounded [24]. Approximation of nonlinear systems using function basis radial networks has some interesting advantages [25]. The general RBF network approximation capabilities provide the theoretical basis for

complex process representation. Also, the RBF network response is linear with respect to the network connection weights [26]. Provided that the other parameters, the centers of RBF, can be chosen appropriately. One relies mainly on the use of RBF network, in order to approximate the unknown non-linearities of an uncertain model [27]-[28]. More precisely, we will be interested in the terminal synergetic control associated with the approximation criteria of the non-linearities unknown by RBF to a class of piezo actuator PEA positioning mechanism.

1. Experimental investigation

1.1 Modeling and Identification of C-H model

The Coleman-Hodgdon model of Piezoelectric Actuator is described by the following equations :

$$(1) \quad x(U) = \begin{cases} (a_1 - a_2 e^{-a_3 V_f})U - a_4 \left(1 - \frac{2e^{-a_5 U}}{e^{-a_5 V_m} + e^{-a_5 V_M}} \right) & \text{Si } \dot{U} \geq 0 \\ (a_1 - a_2 e^{-a_3 V_f})U + a_4 \left(1 - \frac{2e^{a_5 U}}{e^{a_5 V_m} + e^{a_5 V_M}} \right) & \text{Si } \dot{U} < 0 \end{cases}$$

where x is the mechanism's output displacement, U is the control input and its derivative for the PEA, $V_f = V_M - V_m$ is the voltage range, and V_M, V_m are the maximum and minimum input voltages, and $a_1 - a_5$ are constant parameters. The application of the PSO algorithm in the determination of the optimal position has been described and validated by the experimental configuration used for the modeling and the identification of the C-H PEA model [29]. Figure 1 compares the C-H model's tracking performance to that of experiments when the piezoelectric positioning stage is driven by a sinusoidal input. The experimental results match the C-H modeling results very well.

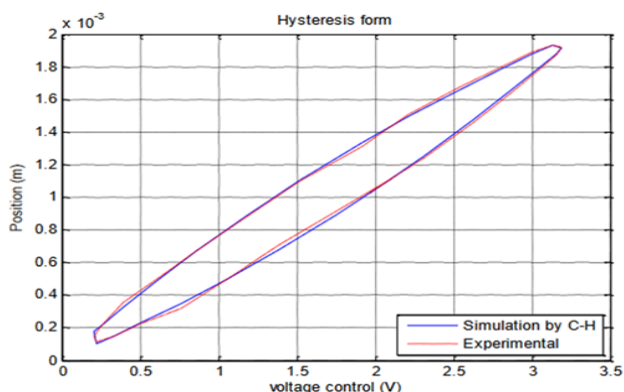


Fig.1. shows the experimental and PSO-based C-H model hysteresis with a 1 Hz input voltage

2. Description the control system

The dynamic of the nonlinear system can be expressed in the following state space model:

$$(2) \quad \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2, t) + g(x_1, x_2, t)U + d(t) \end{cases}$$

Where $x = [x_1, x_2]^T$ the state space vector, the vector of outputs is given by $y = [x_1, x_2]^T$, U , and $d(t)$. The control input and disturbances. The dynamic behavior of our system is described by a nonlinear C-H model. This model requires robust control and must be insensitive and stable to the variation of the parameters and disturbances. These expected characteristics of the control can be realized by a synthesis based on the technique of synergetic control [30].

The synergetic control theory is one of the new promoter options in modern and emerging control theory. The purpose of the control is to regulate the tracking of the trajectory of the position of the PEA [31].

We define the error

$$(3) \quad e = x_{\text{ref}} - x$$

Where x_{ref} denotes the desired trajectory, the choice of a terminal macro variable is defined by:

$$(4) \quad \psi = \dot{e} + \beta e^p$$

Where $\beta > 0$ is constant, and p and q are odd positive constants, such that $p > q$. To create a control law, RBF NN uses the macro variable's variable as input. A hidden layer, an output layer, and an input layer make up the Radial Basis Function Network. For each neuron in the hidden layer, the activation function is selected as the Gaussian function. These Gaussian functions' excitation values are chosen as the distances between the input value and the function's center point.

$$(5) \quad \varphi_i(\psi) = \exp\left(-\frac{|\psi - c_i|^2}{2b_i^2}\right)$$

Where c_i the central position of the i^{eme} neuron et b_i is the spreading factor of the corresponding Gaussian function.

The weight of the parameters w_i between the neurons of the hidden layer and the output layer are adjusted according to an adaptation law \dot{w}_i . The control law is:

$$(6) \quad U_{\text{RBF}} = \sum_{i=1}^n w_i \varphi_i(\psi) = \sum_{i=1}^n w_i \exp\left(-\frac{|\psi - c_i|^2}{2b_i^2}\right)$$

Where n is the number of neurons in the hidden layer.

Weight adjustment is established by gradient descent, the weight adaptation law is:

$$(7) \quad \dot{w}_i = -\rho \frac{\partial E}{\partial w_i} = -\rho \frac{\partial \psi \partial \dot{\psi}}{\partial w_i} = -\rho \frac{\partial \psi \partial \dot{\psi}}{\partial U} \frac{\partial U}{\partial w_i}$$

Then

$$(8) \quad \frac{\partial \psi \partial \dot{\psi}}{\partial U} = -\rho g \psi$$

$$(9) \quad \frac{\partial U}{\partial w_i} = \exp\left(-\frac{|\psi - c_i|^2}{2b_i^2}\right)$$

From where

$$(10) \quad \dot{w}_i = -\gamma \psi \exp\left(-\frac{|\psi - c_i|^2}{2b_i^2}\right)$$

Where $\gamma = \rho g$, the learning rate parameter

We derive the equation (4), and we use the system equation (2), we will have:

$$(11) \quad \begin{aligned} \dot{\psi} &= \ddot{e} + \beta \frac{q}{p} \|e\|_p^{q-1} \\ &= \dot{x}_2 - \dot{x}_{\text{ref}} + \beta \frac{q}{p} \|e\|_p^{q-1} \\ &= f(x_1, x_2, t) + g(x_1, x_2, t)U + d(t) - \dot{x}_{\text{ref}} + \beta \frac{q}{p} \|e\|_p^{q-1} \end{aligned}$$

Then, the control law is:

$$(12) \quad \bar{U} = \frac{1}{g(x)} \left[\ddot{x}_{\text{ref}} - f(x) - d(t) - \beta \frac{q}{p} \|e\|_p^{q-1} + \gamma \psi \right]$$

$\gamma > 0$

The control law may have some difference from the \bar{U} control law. From (2) and (8) we get:

$$(13) \quad \dot{\psi} = -T\psi + g(\tilde{w}^T \varphi + \epsilon)$$

2.1 Theorem.

$x_{ref} = 500$ For the Piezoelectric Actuator system (1), and if the control signal is designed as

$$(14) \quad U = U_{RFB} + \bar{U}$$

And the update law for weight vector is chosen by:

$$(15) \quad \dot{\hat{w}} = -\gamma \psi^T \beta \frac{q}{p} \|e\|^{q-p-1}$$

Where $\gamma > 0$, is the learning rate

The system errors and the error rate asymptotically converge to zero in finite time, the rate of convergence depending on the parameters γ , p and q .

$\tilde{w} = w^* - \hat{w}$ **2.2 Proof.**

Let \tilde{w} be the estimation error of the weight vector Consider the Lyapunov candidate function as:

$$(16) \quad V = \frac{1}{2} \psi^T \psi + \frac{1}{2\rho} \tilde{w}^T \tilde{w}$$

We derive (16), we get

$$(17) \quad \dot{V} = \psi^T \dot{\psi} + \frac{1}{\rho} \tilde{w}^T \dot{\tilde{w}}$$

$$\dot{V} = \psi^T \left[-T\psi + g(w^T \varphi + \epsilon) \right] + \frac{1}{\rho} \tilde{w}^T \dot{\tilde{w}}$$

We replace (15) in (17), we get:

$$(18) \quad \dot{V} = -T\psi^T \psi + g\psi^T \epsilon \leq \|\psi\|(-T\|\psi\| + g\epsilon)$$

$$(19) \quad \dot{V} \leq -T\|\psi\|^2 + \sigma\|\psi\|$$

$\sigma = g\epsilon$

Where

The influence of the $\alpha\|\Psi\|$ term is minimal, of the order of the minimum approximation error which is by design very small. As a result, all signals in the system to be controlled are bounded. Obviously, $e(0)$ is bounded, then $e(t)$ is also bounded. The reference signal x_{ref} is bounded, then the state of the system $x(t)$ is also bounded. To complete the proof and establish the asymptotic convergence of the tracking error. We need to prove that $\Psi \rightarrow 0$ when $t \rightarrow \infty$, assume $\|\Psi\| \leq \alpha$, then equation (18) can be rewritten as:

$$(20) \quad \dot{V} \leq -T\alpha\|\psi\| + \alpha|\sigma|$$

Integrating both sides of (20) leads to:

$$(21) \quad \int_0^t \|\psi\| d\tau \leq \frac{1}{T\alpha} \left[V(0) + |r(t)| \right] + T \int_0^t |\sigma| d\tau$$

Since $\Psi \in L_1$, from (19), the macro-variable is bounded and that $\Psi, \dot{\Psi} \in L_\infty$ consequently by the use of Barbalat (t)'s lemma[31], we also have $\Psi \rightarrow 0$ when $t \rightarrow \infty$, the system is stable and the error converges asymptotically to zero.

3. Results and discussion

To show the performance of the proposed terminal synergistic radial neural base function control. this part,

presents a numerical simulation carried out on the dynamic model of hysteresis C-H. The purpose of the control is to force the system output to follow a desired set point

micron meter. Figure 2.a, show rapid convergence of system states to their reference. From figure 2.b, shows that the tracking error is about 5.797×10^{-5} m. Figure 3.a shows the control signal which is bounded. As well as the system is stable that presented by the figure 3.b. To demonstrate the superior qualities of the suggested controller compared to the synergistic terminal controller. We have seen in figure 4, the results of the responses of the system in the cases a) Trajectory tracking response of position b) Control voltage. c) Tracking error d) the phase plane, by an application of a disturbance of a load of 10 N. Figure 5 compares the error position tracking inaccuracy between the synergistic terminal controller and the suggested control approach, the latter being inferior to the synergistic terminal controller.

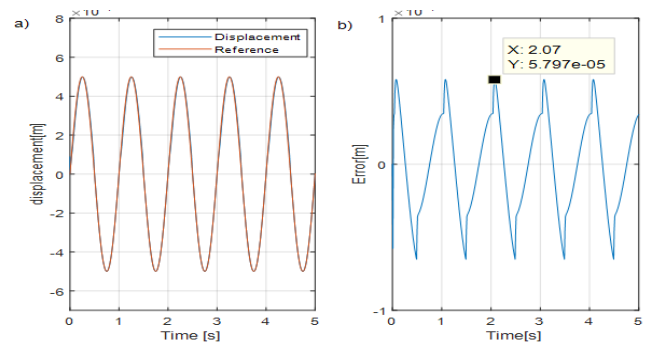


Fig.2. Results Simulation of the RBF network Synergetic Terminal Controller: a) Trajectory tracking response of the position b) Tracking error.

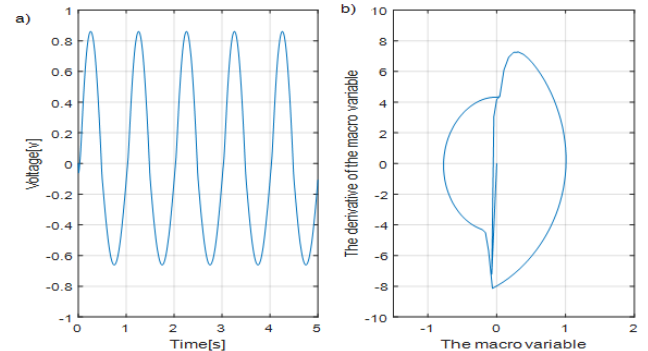


Fig.3. Results Simulation of the RBF network Synergetic Terminal Controller

r: a) Control voltage b) phase diagram

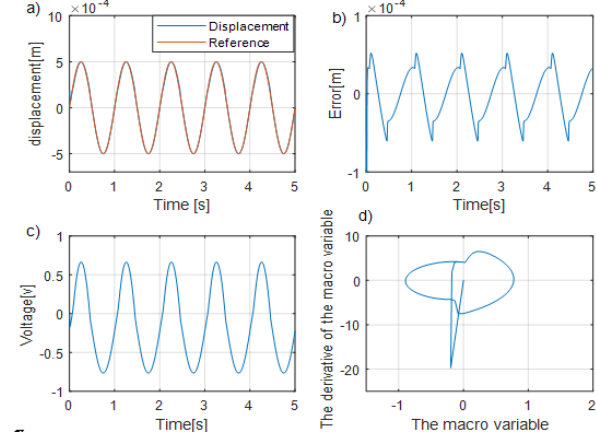


Fig.4. Simulation of the RBF network Synergetic Terminal Controller with a load of 10 N: a) Trajectory tracking response of the position b) Tracking error c) Control voltage d) phase diagram

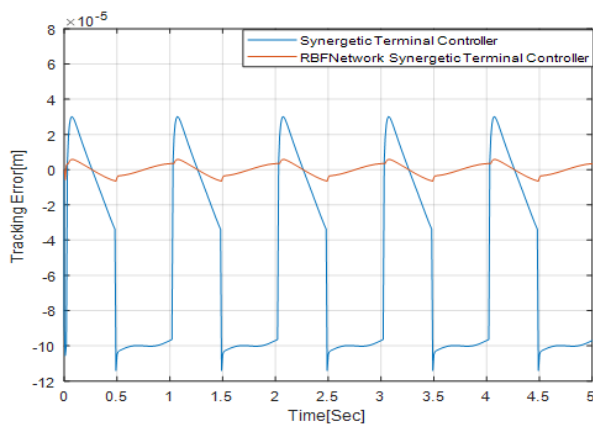


Fig.5. Evolution of the tracking error for the RBF network Synergetic Terminal Controller and the Synergetic Terminal Controller

4. Conclusion

The work considered in this article is a neural network of RBF type combined with the methodology of synergetic terminal control. The developed algorithm has been applied to the PEA. This interest is justified by the fact that piezoelectric actuator positioning systems are, for the most part, non-linear systems subject to modeling and parametric uncertainties, as well as to external disturbances. The algorithm is based on the RBF type neural network, with a hidden layer, to achieve the necessary approximations. The adjustment laws used made it possible to ensure the boundedness of the adjustable parameters. Lyapunov's analysis has guaranteed the stability of the control loop, under certain assumptions, even in the presence of uncertainties and external disturbances. Based on simulation results and performance comparison with traditional synergetic control. Our control strategy offers the best performance in terms of positioning tracking accuracy, small tracking errors, fast convergence and chattering rejection

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