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Bi-objective robust design optimization for LED lens design

***Abstract**. The primary objective of this paper is to propose bi-objective robust design optimization models for LED lens design. The viewing angle and the luminance uniformity are used as the two optical quality objectives. Based on the experimental designs, the response surface methodology is applied to identify the functional relationships between input factors (i.e., design parameters) and their two output responses. Then, the dual response model is applied to determine the optimal solutions. In addition, the ε-constrained method is developed to identify the optimal design parameter settings. Moreover, the weighted sum model based on the quality loss function is proposed to consider the trade-off between two conflicting objectives. The final results show that the proposed models are far more effective than the existing method in LED lens design.*

***Streszczenie.** Głównym celem tego artykułu jest zaproponowanie dwóch dwuobiektywnych, solidnych modeli optymalizacji konstrukcji dla konstrukcji soczewek LED. Kąt widzenia i jednorodność luminancji są używane jako dwa optyczne cele jakości. W oparciu o projekty eksperymentalne zastosowano metodologię powierzchni odpowiedzi w celu zidentyfikowania funkcjonalnych zależności między czynnikami wejściowymi (tj. parametrami projektowymi) a ich dwiema odpowiedziami wyjściowymi. Następnie opracowywany jest model podwójnej odpowiedzi w celu określenia optymalnych rozwiązań. Ponadto zaproponowano metodę z ograniczeniami ε w celu identyfikacji optymalnych ustawień parametrów projektowych. Kompromis między dwoma sprzecznymi celami może być zagrożony przy użyciu proponowanych modeli. Ostateczne wyniki pokazują, że proponowane modele są znacznie bardziej efektywne niż dotychczas stosowane metody projektowania soczewek LED. (**Dwuobiektywowa, solidna optymalizacja konstrukcji soczewki LED**)*

Keywords: LED lens design, robust design, bi-objective optimization.
Słowa kluczowe: Konstrukcja soczewki LED, solidna konstrukcja, optymalizacja dwuobiektowa.

Introduction

The light emitting diode (LED) has been widely used in lighting systems due to its advantages of long lifetime, low energy consumption, and being environmentally friendly. Therefore, LEDs are now becoming alternative generation light sources. However, the light distribution of LED chips, which is considered Lambertian with the radiance property, is quite different from traditional light sources, such as incandescent, halogen, and fluorescent lamps [1]. LEDs release energy in the form of light based on the combination of electrons and holes. The distribution of light on the target surface may be discontinuous in practical applications. In addition, several contours of light may arise on the target surface. The angular extent of light emitted by an unmodified LED is near $\pm$ 60^0. The highest luminous intensity of a LED source is concentrated at 90^0 to the epitaxial axis [2]. For different practical applications of LEDs in lighting, the two characteristics of unmodified LEDS consisting of light distribution and efficiency may not guarantee the specific requirements. To solve the problems, the secondary lenses of LEDs need to be designed to change achieve the proper light distribution with higher efficiency.

In the literature, several works have proposed several methods to identify the optical design process of LED lenses. A freeform LED lens design for the illumination of uniformity near 90% using numerical computations is proposed [3]. Both numerical simulations based on the Monte Carlo ray tracing method and experiments are used to design a compact freeform lens for application-specific light-emitting diode packaging [4]. For LED street lighting, a new energy mapping method is provided for the optimal design of a freeform lens to generate uniform illumination of over 95% on the target plane [5]. An efficient and practical design method for a LED-based reflector-array lighting module for specific illuminance distribution is proposed [6]. Authors in [7] develop the simultaneous multiple surface design method of diffractive optical surfaces besides refractive and reflective ones. A facile automotive headlamp optical configuration is proposed to solve the problem of high cost and low optical efficiency of current LED-based headlamps [8]. A uniform illumination design by configuration of LEDs and optimization of LED lens is presented for large-scale color-mixing applications [9]. Another research direction is to use statistical and optimization methods to design optical lenses for time-saving in the manufacturing process. The functional-link-based neuro-fuzzy network with immune particle swarm optimization is used to compensate for the backlight images [10]. A search method using a neural network algorithm and computer simulation is proposed to improve the surface profile of injection molding optic lens [11]. A hybrid dynamic pre-emptive and competitive neural-network approach is proposed to solve the multi-objective dispatching problem for TFT-LCD manufacturing [12]. Spectral power distribution is estimated using neural network models [13]. The lens array for LED backlight in the LCD imaging engine of the helmet-mounted display is designed and optimized [14]. The simulated annealing algorithm is used to design an LED array for achieving a good uniform illumination distribution on the target plane [15]. An irradiance array scheme is proposed to achieve the optimization design of an irradiance array for LED uniform rectangular illumination [16]. LED color is predicted using a boosting neural network model for a visual-MIMO system [17]. The neural network is developed to estimate the performance of an optical system based on the errors of its individual components [18].

In recent years, the optical design process using optical analysis software packages can save time, costs, and manpower. Based on these software packages, several design strategies can be implemented to obtain the desired parameters of LED lens shapes. In the early stage, the trial-and-error method is usually used to build the leans dimensions and parameters. However, this method may be time-consuming to identify the optimal lens. To solve this optimization issue of LED lens design, the Taguchi method is used in several works. The Taguchi orthogonal array, signal-to-noise (SN) ratio, and principal component analysis are used to optimize the multiple quality characteristics of an LED pocket-sized projection display [19]. A combination of Taguchi methods, principal component analysis (PCA), and fuzzy theory is integrated to solve the multiple quality characteristics in optimization experiments of a miniature optical engine with high light efficiency and uniformity [20]. The Taguchi L_{25} orthogonal array, back-propagation neural network, and genetic algorithm are used to determine the optimum design for LED lens design with 135^0 viewing angle and 93.35% uniformity [21]. The Taguchi L_{25} orthogonal array, back-propagation neural network, genetic algorithm, and particle swarm are combined to provide a procedure for optimization of optical design for developing an LED lens module [22]. The Taguchi method and PCA are combined in an optical lens design for LED lighting with over 92% light efficiency and an improvement in uniformity [2]. The Taguchi method determines the optimal values of optical uniformity factors of the backlight module [23]. The Taguchi philosophy consists of two basic steps: the experimental designs (or, the design of experiments) and the SN ratio. Although this philosophy is widely recognized in quality engineering, the Taguchi two-step method has caused several controversy and debate [24]–[27]. Therefore, the Taguchi and response surface philosophies are combined to establish the dual response approach where the process mean and variance are separately estimated as functions of control factors [28]. Based on that milestone, the robust design (RD) methodology is generated with three sequential steps, i.e., the experimental design or design of experiments (DoE), estimation, and optimization. The primary goal of the DoE step is to exploit the information about the relationship between the input factors and output responses. The main purpose of the estimation step is to estimate the functional relationship between these variables. Finally, the optimal settings of the input factors are determined in the last one.

In the LED lens design optimization issue, the viewing angle and the luminance uniformity are considered two quality characteristics. Each quality characteristic is examined as a single response approach in the RD viewpoint. The optimal values of six factors including lens bottom width, lens height, diameter of diffusion mechanism, radius of lens negative camber, radius of lens positive camber, and radius of curved surface of diffusion mechanism are identified to satisfy the optimization targets: (1) maximize the viewing angle and (2) maximize luminance uniformity. Therefore, the primary purpose of this paper is to solve the bi-objective robust design optimization models for LED lens design. The response surface methodology (RSM) is used to estimate the functional relationships between the viewing angle and the luminance uniformity and their six input factors. The simultaneous maximization of the viewing angle and luminance uniformity may cause conflict. Therefore, the dual response model is applied to solve this issue. The ε- constrained method is developed to identify the LED lens design. Furthermore, the weighted sum model based on the quality loss function is established to consider the trade-off between the viewing angle and the luminance uniformity.

The remainder of this paper is organized as follows. In Section 2, the proposed RD optimization models are presented. In Section 3, the case study of LED lens design in lighting is conducted. The conclusions are given in Section 4.

Bi-objective robust design optimization models

Response surface methodology

In statistics, RSM is usually utilized to analyze and estimate the functional relationship between input factors and their corresponding output responses. The coefficients in the input-output functional relationship are usually estimated using the conventional least squares method, as introduced by Box and Wilson [29]. The various development status and future directions of RSM are discussed in [30]. In the matrix form, the output response **y** can be established as a function of input factors **x** as:

(1) $$\mathbf{y} = \mathbf{x}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where β is a column vector of the coefficients and vector ε is the random error. The estimated function model is represented as follows:

(2) $$\hat{\mathbf{y}} = \hat{\boldsymbol{\beta}}\mathbf{x}$$

where the coefficients are estimated using the least squares method as

(3) $$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y} .$$

Dual response model

The dual response model proposed by Vining and Myers is used to consider the trade-off between process mean and variance. In this paper, the viewing angle and the luminance uniformity are two objectives of optimization. The primary goal of LED lens design is to maximize the viewing angle and the luminance uniformity simultaneously. Therefore, the dual response optimization model in this paper can be modified and represented as follows:

(4) $$\begin{aligned} &\max \quad \hat{y}_1(\mathbf{x}) \\ &\text{subject to } \hat{y}_2(\mathbf{x}) = \tau_2 \end{aligned}$$

where τ_2 is the target value of the estimated function of the luminance uniformity (i.e., $\hat{y}_2(\mathbf{x})$).

ε-constrained method

For multi-objective optimization issues, the main purpose of this optimization model is to transform one of the objectives into a constraint. The absolute deviation of one objective and its corresponding target must be less than a small predefined bias to relax the solutions. In this paper, the bi-objective optimization model based on the ε-constrained method for two objectives can be represented as follows:

(5) $$\begin{aligned} &\max \quad \hat{y}_2(\mathbf{x}) \\ &\text{subject to } \left|\hat{y}_1(\mathbf{x}) - \tau_1\right| \le \varepsilon_1 \end{aligned}$$

(6) $$\begin{aligned} &\max \quad \hat{y}_1(\mathbf{x}) \\ &\text{subject to } \left|\hat{y}_2(\mathbf{x}) - \tau_2\right| \le \varepsilon_2 \end{aligned}$$

where τ_1, ε_1, and ε_2 are the target and upper bounds on the absolute deviation of the viewing angle and the luminance uniformity from their desirable target values, respectively.

Weighted sum model

The weighted sum model is probably the most popular method used to solve bi-objective optimization. By assigning different values of weights of two objectives, a better compromise between the viewing angle and the luminance uniformity can be obtained. The weighted sum model using the quality loss function in this case study can be shown as:

(7) $$\begin{aligned} &\max \quad \omega\left[\hat{y}_1(\mathbf{x}) - \tau_1\right]^2 + (1-\omega)\left[\hat{y}_2(\mathbf{x}) - \tau_2\right]^2 \\ &\text{subject to} \quad 0 \le \omega \le 1 \end{aligned}$$

where ω is the weight.

Case study

A case study given by Chen *et al.* [21] is utilized to demonstrate the applicability of the above RD optimization models in identifying the optimum of LED lens design. The six design parameters are lens bottom width (A), lens height (B), diameter of diffusion mechanism (C), radius of lens negative camber (D), radius of lens positive camber (E), and radius of curved surface of diffusion mechanism (F). In the first stage, the setting range values of these six factors are divided into five levels. An L_{25} (5^6) orthogonal array is conducted where ”5” denotes the levels of each factor and ”6” denotes the number of factors. According to the results of the first stage, the radius of lens positive camber (E) is determined to be fixed at 3.0 mm for a wide viewing angle design. The setting range values of the five factors are adjusted to achieve better parameters. Therefore, the second L_{25} (5^5) orthogonal array for five control factors with five levels is conducted. No. 26 to No. 30 are the combination for tests derived from random numbers within Taguchi experimental parameters. Consequently, an L_{30} orthogonal array is executed and represented in Table 1. In their work, the genetic algorithm (GA) is used to solve the optimization issues. Then, the optimal solutions are compared to the Taguchi two-step method.

Table 1. Second experimental data table for case study

Input factors [mm]					Output responses	
A (x_1)	B (x_2)	C (x_3)	D (x_4)	F (x_5)	Viewing angle [^{0}] (y_1)	Luminance uniformity [%] (y_2)
0.650	4.900	3.800	2.500	2.350	129.00	88.95
0.650	4.950	3.850	2.525	2.375	131.00	89.62
0.650	5.000	3.900	2.550	2.400	130.00	90.15
0.650	5.050	3.950	2.575	2.425	130.00	90.62
0.650	5.100	4.000	2.600	2.450	132.00	90.29
0.675	4.900	3.850	2.550	2.450	131.00	87.97
0.675	4.950	3.900	2.575	2.350	129.00	91.63
0.675	5.000	3.950	2.600	2.375	129.00	92.32
0.675	5.050	4.000	2.500	2.400	130.00	91.71
0.675	5.100	3.800	2.525	2.425	128.00	88.54
0.700	4.900	3.900	2.600	2.425	134.00	88.92
0.700	4.950	3.950	2.500	2.450	132.00	88.89
0.700	5.000	4.000	2.525	2.350	133.00	88.87
0.700	5.050	3.800	2.550	2.375	130.00	90.67
0.700	5.100	3.850	2.575	2.400	130.00	91.00
0.725	4.900	3.950	2.525	2.400	129.00	89.79
0.725	4.950	4.000	2.550	2.425	131.00	89.76
0.725	5.000	3.800	2.575	2.450	134.00	87.76
0.725	5.050	3.850	2.600	2.350	128.00	91.94
0.725	5.100	3.900	2.500	2.375	130.00	92.03
0.750	4.900	4.000	2.575	2.375	131.00	89.85
0.750	4.950	3.800	2.600	2.400	130.00	90.09
0.750	5.000	3.850	2.500	2.425	130.00	89.43
0.750	5.050	3.900	2.525	2.450	133.00	89.46
0.750	5.100	3.950	2.550	2.350	134.00	88.08
0.710	5.070	3.830	2.535	2.360	129.00	90.76
0.750	5.010	3.860	2.540	2.370	128.00	89.88
0.690	4.990	3.890	2.545	2.380	129.00	89.89
0.680	4.970	3.920	2.560	2.390	130.00	89.06
0.740	4.930	3.940	2.570	2.410	129.00	88.67

By using RSM, the estimated functions of viewing angle and luminance uniformity are represented as follows:

$$\begin{aligned}\hat{y}_1(\mathbf{x}) &= 3540 - 2773x_1 + 1110x_2 + 983x_3 \\ &-1393x_4 - 3984x_5 + 557x_1^2 - 65x_2^2 \\ &+96x_3^2 + 428x_4^2 + 515x_5^2 + 402x_1x_2 \\ &+341x_1x_3 - 572x_1x_4 + 47x_1x_5 - 118x_2x_3 \\ &-46x_2x_4 - 70x_2x_5 - 436x_3x_4 - 108x_3x_5 + 879x_4x_5\end{aligned} \tag{8}$$

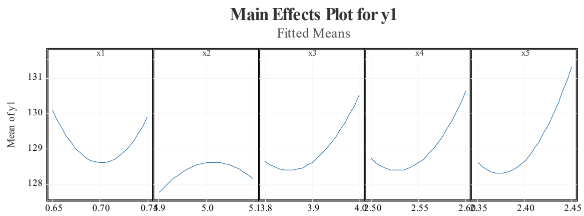
$$\begin{aligned}\hat{y}_2(\mathbf{x}) &= -2084 + 835x_1 - 108x_2 - 47x_3 \\ &+1175x_4 + 629x_5 + 201x_1^2 + 56x_2^2 + 46.3x_3^2 \\ &+271x_4^2 - 195x_5^2 - 47x_1x_2 - 15x_1x_3 - 601x_1x_4 \\ &+292x_1x_5 - 35x_2x_3 - 102x_2x_4 - 9x_2x_5 \\ &-247x_3x_4 + 209x_3x_5 - 270x_4x_5\end{aligned} \tag{9}$$

The corresponding analysis of variance (ANOVA) tables of the two estimated functions are shown in Tables 2 and 3, respectively. The main and interaction effects plots of five input factors on the two output responses are shown in Figures 1 and 2, respectively.

Table 2. ANOVA table for the estimated function of the viewing angle

Source	DF	Adj SS	Adj MS	F-value	P-value
Regression	20	70.5569	3.52784	1.53	0.262
x_1	1	3.5291	3.52914	1.53	0.248
x_2	1	0.9731	0.97310	0.42	0.533
x_3	1	0.9285	0.92851	0.40	0.542
x_4	1	0.8759	0.87588	0.38	0.553
x_5	1	4.8719	4.87192	2.11	0.181
x_1*x_1	1	2.4546	2.45457	1.06	0.330
x_2*x_2	1	0.2780	0.27802	0.12	0.737
x_3*x_3	1	1.2931	1.29308	0.56	0.474
x_4*x_4	1	1.5801	1.58010	0.68	0.430
x_5*x_5	1	2.9003	2.90034	1.25	0.292
x_1*x_2	1	1.0783	1.07829	0.47	0.512
x_1*x_3	1	0.4098	0.40980	0.18	0.684
x_1*x_4	1	0.8503	0.85034	0.37	0.559
x_1*x_5	1	0.0125	0.01247	0.01	0.943
x_2*x_3	1	0.5023	0.50226	0.22	0.652
x_2*x_4	1	0.1668	0.16679	0.07	0.794
x_2*x_5	1	0.1120	0.11203	0.05	0.831
x_3*x_4	1	2.1690	2.16902	0.94	0.358
x_3*x_5	1	0.9600	0.96000	0.42	0.535
x_4*x_5	1	9.2538	9.25381	4.00	0.076
Error	9	20.8098	2.31220	R-sq	
Total	29	91.3667		77.22%	

a)



b)

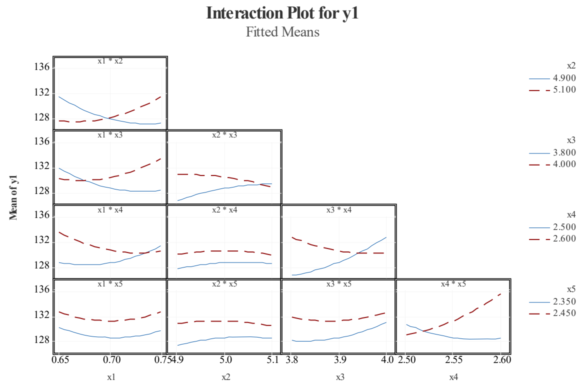


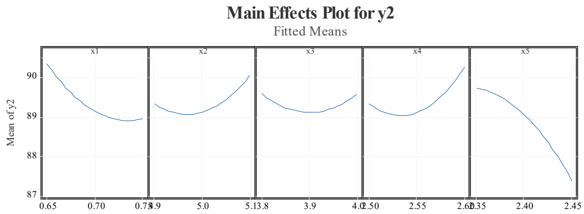
Fig.1. Main (a) and interaction (b) effects plot for viewing angle

Table 3. ANOVA table for the estimated function of the luminance uniformity

Source	DF	Adj SS	Adj MS	F-value	P-value
Regression	20	70.5569	3.52784	1.53	0.262
x_1	1	3.5291	3.52914	1.53	0.248
x_2	1	0.9731	0.97310	0.42	0.533
x_3	1	0.9285	0.92851	0.40	0.542
x_4	1	0.8759	0.87588	0.38	0.553
x_5	1	4.8719	4.87192	2.11	0.181
x_1*x_1	1	2.4546	2.45457	1.06	0.330
x_2*x_2	1	0.2780	0.27802	0.12	0.737
x_3*x_3	1	1.2931	1.29308	0.56	0.474
x_4*x_4	1	1.5801	1.58010	0.68	0.430
x_5*x_5	1	2.9003	2.90034	1.25	0.292
x_1*x_2	1	1.0783	1.07829	0.47	0.512
x_1*x_3	1	0.4098	0.40980	0.18	0.684

x_1*x_4	1	0.8503	0.85034	0.37	0.559
x_1*x_5	1	0.0125	0.01247	0.01	0.943
x_2*x_3	1	0.5023	0.50226	0.22	0.652
x_2*x_4	1	0.1668	0.16679	0.07	0.794
x_2*x_5	1	0.1120	0.11203	0.05	0.831
x_3*x_4	1	2.1690	2.16902	0.94	0.358
x_3*x_5	1	0.9600	0.96000	0.42	0.535
x_4*x_5	1	9.2538	9.25381	4.00	0.076
Error	9	20.8098	2.31220	R-sq	
Total	29	91.3667		77.22%	

a)



b)

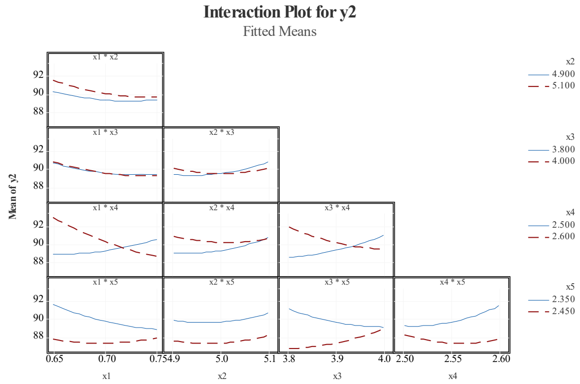


Fig.2. Main (a) and interaction (b) effects plot for luminance uniformity

Table 4. Comparisons of the optimal parameter combinations for the optical quality

Method	Input factors [mm]				
	A	B	C	D	F
Taguchi method	0.700	4.900	3.900	2.600	2.425
Taguchi method	0.675	5.000	3.950	2.600	2.375
GA by Chen *et al.*	0.688	5.024	3.924	2.588	2.386
Dual response model	0.650	4.900	3.800	2.600	2.4432
ε-constrained method (Equation 5)	0.650	4.900	3.800	2.600	2.4495
ε-constrained method (Equation 6)	0.650	4.900	3.800	2.600	2.4444
Weighted sum model	0.650	4.900	3.800	2.600	2.4500

To consider the trade-off between two output responses, the estimated functions of viewing angle and luminance uniformity in Equations (8) and (9) are combined into the different RD optimization models. The goal of the optimization issue is modified by using a predefined target for each objective function. The target values of the viewing angle and luminance uniformity are specified 138 (^{0}) and 96 (%), respectively, for all RD optimization models. The upper bounds of ε_1 and ε_2 in the ε-constrained method are set at 1.3 and 0.1, respectively. For the weighted sum model, a better compromise between the viewing angle and luminance uniformity can be obtained with the weight ω = 0.4. The comparisons of the optimal parameter combinations for the optical quality and the optimal values of the viewing angle and luminance uniformity in this paper are shown in Tables 4 and 5, respectively.

Table 5. Comparisons of the optimal values of the viewing angle and luminance uniformity

Method	Output responses	
	Viewing angle [0]	Luminance uniformity [%]
Taguchi method	134	88.92
Taguchi method	129	92.32
GA by Chen *et al.*	134.67	92.95
Dual response model	135.853279	95.99999
ε-constrained method (Equation 5)	136.700162	95.45025
ε-constrained method (Equation 6)	136.00605	95.89999
Weighted sum model	136.76750	95.40700

As shown in Table 5, the Taguchi method can provide relatively high viewing angle and luminance uniformity of LED lens. The genetic algorithm (GA) proposed by Chen *et al.* can provide a better compromise between viewing angle (134.67^0) and luminance uniformity (92.95%) compared to the Taguchi method. Furthermore, the proposed RD optimization models including the dual response model, the two ε-constrained methods, and the weighted sum model can provide better optimal solutions compared to the GA method proposed by Chen *et al.* In comparison, the proposed dual response model, the ε-constrained method (Equation 5), the ε-constrained method (Equation 6), and weighted sum model can improve the viewing angle by 0.8787%, 1.5075%, 0.9921%, and 1.5575%, respectively. Similarly, the proposed dual response model, the ε-constrained method (Equation 5), the ε-constrained method (Equation 6), and the weighted sum model can improve the luminance uniformity by 3.2813%, 2.6899%, 3.1737%, and 2.6434%, respectively.

Conclusions

In this paper, the RD methodology consisting of three sequential steps, i.e., experimental design, estimation, and optimization, is proposed to determine the optimal values of lens size parameters in LED lens design. Based on the experimental design data, RSM is used to estimate the functional relationship between input factors and their corresponding output responses (i.e., the viewing angle and luminance uniformity). Furthermore, several bi-objective RD optimization models are proposed to consider the trade-off between the viewing angle and luminance uniformity of LED lens. The final results show that the proposed RD optimization models can provide better optimal solutions than the existing methods. Therefore, the efficiency of the proposed optimization models can be verified.
For further study, the multi-objective RD optimization models based on the goal programming methods will be investigated to solve the multiple quality characteristics issue.

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