1. Madi Said¹, 2. Kherief Nacereddine Mohamed², 3. Mohand Said LARABI³,

Mechanical and Materials Engineering Laboratory (LGMM), Skikda 21000 Algeria (1) Normal High School of Technology Education ENSET azzaba Skikda 21000 Algeria (2) Automatic laboratory of University August 20, 1955 Skikda 21000 Algeria (2)

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Robust linear LQG control for a multi-variable wind turbine system subject to parametric perturbation

Abstract. This article presents a design approach to develop an optimal LQG/LTR controller for a multivariate system subject to parametric perturbations, specifically a wind turbine. The primary objective of a control system is to regulate the behavior of the output variables by manipulating the inputs of the system. To achieve better performance in dynamic operating conditions, the proposed control system should provide more accuracy and better overall performance. Based on the MIMO control theory, a linear-quadratic (LQR) controller is designed to optimize performance, followed by the design of a second-order Gaussian (LQG) controller using a Kalman filter to improve stability. The LQG controller is tested with different weighting matrices to meet the required performance and stability criteria. Typically, the parameters of the weighting matrices are manually modified through trial and error method. The Loop Transfer Recovery (LTR) method is utilized to enhance the performance of the stadium controller by restoring the dynamic characteristics of a closed loop. A simulation study of the wind turbine behavior was performed using MATLAB, and the obtained results were utilized to analyze and install the control system using MATLAB Simulink. The feasibility of the LTR method is demonstrated through comprehensive theoretical analysis. The proposed LTR controller is included to stabilizing output power in various wind conditions, proving its effectiveness over conventional controllers.

Abstrakcyjny. W artykule przedstawiono podejście projektowe mające na celu opracowanie optymalnego sterownika LQG/LTR dla systemu wielowymiarowego podlegającego zakłóceniom parametrycznym, w szczególności turbiny wiatrowej. Podstawowym celem systemu sterowania jest regulacja zachowania zmiennych wyjściowych poprzez manipulowanie wejściami systemu. Aby osiągnąć lepszą wydajność w dynamicznych warunkach pracy, proponowany system sterowania powinien zapewniać większą dokładność i lepszą ogólną wydajność. W oparciu o teorię sterowania MIMO zaprojektowano kontroler liniowo-kwadratowy (LQR) w celu optymalizacji wydajności, a następnie zaprojektowano kontroler Gaussa (LQG) drugiego rzędu wykorzystujący filtr Kalmana w celu poprawy stabilności. Kontroler LQG jest testowany z różnymi matrycami wagowymi, aby spełnić wymagane kryteria wydajności i stabilności. Zazwyczaj parametry macierzy wag modyfikowane są ręcznie metodą prób i błędów. Metoda Loop Transfer Recovery (LTR) służy do zwiększenia wydajności kontrolera stadionu poprzez przywrócenie charakterystyki dynamicznej zamkniętej pętli. Przeprowadzono badania symulacyjne zachowania turbiny wiatrowej w programie MATLAB, a uzyskane wyniki wykorzystano do analizy i montażu układu sterowania w programie MATLAB Simulink. Wykonalność metody LTR została wykazana poprzez wszechstronną analizę teoretyczną. Proponowany sterownik LTR ma za zadanie stabilizować moc wyjściową w różnych warunkach wietrznych, udowadniając swoją skuteczność w porównaniu z konwencjonalnymi sterownikami. (Solidne liniowe sterowanie LQG dla wielowymiarowego systemu turbin wiatrowych podlegającego zakłóceniom parametrycznym)

Keywords: Wind turbine; Quadratic linear regulator (LQR); Linear Quadratic Gaussian Synthesis (LQG); loop Transfer recovery (LTR); **Słowa kluczowe**: Turbina wiatrowa; Kwadratowy regulator liniowy (LQR); Liniowa kwadratowa synteza Gaussa (LQG); pętla Odzyskiwanie:

Introduction

Energy usage often reflects the level of development in a country. However, the current reliance on fossil fuels and the escalating energy consumption patterns have made it imperative to transition to sustainable and renewable sources of energy. The primary challenge encountered during the establishment of a renewable energy-driven power system is the fluctuation in energy generation from wind and solar sources across time. The inherent instability of energy production from these sources arises due to the dynamic nature of weather conditions [1, 2]. Wind power stands as a notable renewable energy resource on our planet. Contemporary research in this domain is oriented toward reducing the overall expenses of Wind Energy Conversion Systems (WECS) while simultaneously enhancing the power output quality. In line with this objective, multiple endeavors have been undertaken to eliminate the necessity for costly procurement and upkeep of mechanical sensors [3]. Wind energy, as a renewable source, has attracted a great deal of attention since it does not produce atmospheric emissions. However, generating electricity from wind can be challenging due to various reasons, including operation and maintenance costs. Recently, reducing costs, including control and condition monitoring, and improving reliability, has become essential in wind turbine operational strategies. This is because these factors have a long-term impact on the productivity and reliability of wind site operations. Therefore, to minimize costs and maximize the efficiency of wind turbines, experts have introduced advanced control and monitoring techniques, including condition monitoring and control optimization. The successful implementation of such strategies can lead to a significant reduction in costs and a sustainable future for generations to come [4, 5].

Nowadays, variable speed wind turbines are becoming more popular than fixed speed wind turbines, because they are more energy-saving, high-quality and more cost-effective, and this is a topic that many scientists and engineers are interested in [6].

Many complex engineering systems are equipped with multiple actuators that can influence their static and dynamic behavior. For multivariable (MIMO) systems, the goal is to achieve the desired behavior of multiple output variables by simultaneously processing multiple inputs [7].

Wind turbine control methods started with PI controllers, which imposed many limitations and system operating and maintenance costs. The advancement of modern control theory enables the utilization of linear state space models for wind turbines, allowing the implementation of control systems more powerful than a PID controller does. These modern control methods have also reduced maintenance and operating cost by the use of Kalman filters for state estimation which has reduced the number of sensors needed to implement full state feedback controllers [8]. The robust Linear Quadratic Gaussian (LQG) control design is one of the choices for many Multi-Input Multi-Output (MIMO) systems; is an optimal solution whose name is separation theorem because it assumes a linear system, a quadratic cost function and Gaussian noise [9]. Linear Quadratic Gaussian (LQG) control aims to minimize white noise disturbances. It has the advantages of being easy to understand and simple to design [10, 11]. Designing an effective control system requires an accurate model. Under certain practical conditions, the dynamics of the controlled installation may not be exactly modeled, and there may be system disturbances and measurement noises in the installation [12].

Controller (LQG/LTR); Linear Quadratic Gaussian with Loop Transfer Recovery (LTR) can provide good performance and guaranteed stability against such noise. The synthesis (LQG/LTR) of the controller makes it possible to obtain by simulation the curves of the singular values, the curves of gain, to make the analysis of the robustness in stability and the robustness in performance of the system.

System Description

The system studied is a horizontal axis wind turbine with three blades of length R, fixed on a drive shaft, which connected to a gain multiplier G. This multiplier drives an electric generator. The basic form of a wind turbine system consists either of a wind turbine coupled to an electrical generator directly or via a gearbox. Despite its simplicity, the wind turbine presents an interesting problem from the point of view of control theory, because it is a nonlinear system, undergoes cycle Disorders caused by operating phenomena, and It is based on a random variable characterized by sudden changes. Thus, the quality of a wind turbine controller is determined by its ability to handle typical uncertainties, external random signals and periodic disturbances [13].

Wind turbines generate electricity through harnessing the wind's energy to propel an electrical generator. As the wind flows across the turbine's blades, it creates an aerodynamic force that results in rotation. The spinning blades subsequently drive a shaft housed within the nacelle, connecting to a gearbox. This gearbox elevates the rotational speed to a level suitable for the generator, which utilizes magnetic fields to transform the rotational energy into electrical power. The generated power is then directed to a transformer, responsible for adapting the generator's output to the required voltage for the overall power collection system [14]. The fundamental constituents comprising a representative illustration of a typical wind turbine generator depicted in Figure 1.

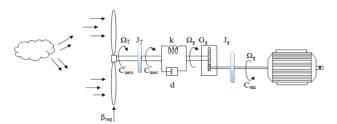


Fig 1. Wind Turbine Mechanical Model [15]

Depending on the wind speed, we can consider the operation of wind turbines with several regions. The first is in the presence of low wind speed, where the wind turbine must increase the amount of energy it extracts from the wind, while in high wind speed, also called high wind region, the amount of electrical power should be limited to the rated value of the generator to avoid overheating.

System characteristics are in Appendix 1.\

System Modeling

A. Aerodynamic Model

The power of the air mass, which crosses the surface equivalent to the active surface S of the wind turbine, is given by the following equation [4, 16, 17].

(1)
$$P_V = \frac{1}{2} \rho S V_v^3$$

Where: $\rho = 1,22 \text{ kg/m3}$; S Is the circular area swept by the turbine blades; Vv Is the wind speed.

The following formula calculates the maximum power of

a wind turbine based on the Betz limit.

$$\begin{cases} P_{aero} = \frac{16}{27} \, P_V = 0.59 \, P_V \\ P_{aero} = \frac{1}{2} \, \rho \, S \, V_v^3 c_p(\lambda,\beta) \end{cases}$$
 With a theoretical maximum power coefficient

$$(3) C_p^{opt} = \frac{2 P_{aero}}{\rho \, S \, V_p^3}$$

And the power coefficient of the turbine equals (4) $C_P = \frac{2 P_{tur}}{\rho S V_v^3}$

$$(4) C_P = \frac{2 P_{tur}}{0 S V_0^3}$$

λ the ratio of the linear speed at the tip of the blades and the wind speed calculated by:

$$\lambda = \frac{R \Omega_{tur}}{V_{tur}}$$

Where: Ω_{tur} and R is the angular velocity of the turbine and blade length.

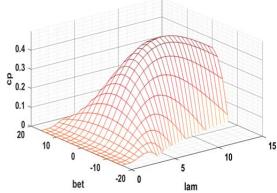


Fig 2. Power coefficient $Cp(\lambda, \beta)$

B. Model of the mechanical part

(6)
$$J_T \frac{d\Omega_T}{dt} = C_{aero} - C_{em}$$

$$(7) J_{a-ls} \frac{d\Omega_{g-ls}}{dt} = C_{mec} - G_a C_{em}$$

B. Model of the mechanical part
$$(6) \qquad J_{T} \frac{d\Omega_{T}}{dt} = C_{aero} - C_{em}$$

$$(7) \qquad J_{g-ls} \frac{d\Omega_{g-ls}}{dt} = C_{mec} - G_{g}C_{em}$$

$$(8) \qquad \frac{d\Omega_{mec}}{dt} = K(\Omega_{T} - \Omega_{g-ls}) + d\left(\frac{d\Omega_{T}}{dt} - \frac{d\Omega_{g-ls}}{dt}\right)$$

$$(9) \qquad \Omega_{g-ls} = \frac{\Omega_{g}}{\Omega_{g}}$$

$$(10) \qquad J_{g-ls} = G_{g}^{2}J_{g}$$

(9)
$$\Omega_{g-ls} = \frac{\Omega_g}{\Omega_g}$$

$$(10) I_{a-ls} = G_a^2 I_a$$

Where:

 Ω_{g-ls}, J_{g-ls} being the speed of rotation and the inertia of the generator brought back to the low speed shaft respectively. The blade angle can modeled by the following differential equation

$$(11) \qquad \dot{\beta} = 1/\tau_{\mathcal{B}} \left(\beta_{ref} - \beta\right)$$

(11) $\dot{\beta}=1/\tau_{\beta}~(\beta_{ref}-\beta)$ Where β_{ref} is the reference signal for the blade pitch angle β and $\tau\beta$ is the time constant of the pitch actuator. Typically, β ranges from 0, to 450, and varies at a maximum rate of ±10. The generated electric power is

$$(12) P = G_g C_{em} \Omega_{g-ls}$$

The aerodynamic torque C_{aero} exhibits nonlinearity in wind turbine system instance. Therefore, in contrast to Ω_T and β , we must adopt a technique of linearizing the aero torque [15].

(13)
$$\Delta C_{aero} = a \Delta \Omega_T + b \Delta \beta$$

(14)
$$a = \frac{\partial C_{aero}(\Omega_T(t), \beta(t))}{\partial \Omega_T} \mid_{opt}$$

(13)
$$\Delta C_{aero} = a \Delta \Omega_T + b \Delta \beta$$

$$(14) \qquad a = \frac{\partial C_{aero}(\Omega_T(t), \beta(t))}{\partial \Omega_T} \Big|_{opt}$$

$$(15) \qquad a = \frac{1}{2} \rho \pi R_T^3 \frac{V_{opt}^2}{\Omega_{T-nom}} \Big[\frac{\partial C_p(\lambda, \beta)}{\partial \lambda} - \frac{C_{p-nom}}{\lambda_{nom}} \Big]$$

$$(16) \qquad b = \frac{\partial C_{aero}}{\partial \beta} = \frac{\partial}{\partial \beta} \Big(\frac{1}{2} \rho \pi R_T^3 C_p(\lambda, \beta) \Big)$$

$$(17) \qquad b = \frac{1}{2} \rho \pi R_T^2 \frac{V_{opt}^3}{\Omega_{T-nom}} \Big[\frac{\partial C_p(\lambda, \beta)}{\partial \beta_{opt}} \Big]$$
We will consider a pominal operating points.

(16)
$$b = \frac{\partial c_{aero}}{\partial \beta} = \frac{\partial}{\partial \beta} \left(\frac{1}{2} \rho \pi R_T^3 C_p(\lambda, \beta)\right)$$

(17)
$$b = \frac{1}{2} \rho \pi R_T^2 \frac{V_{opt}^3}{\Omega_{T-nom}} \left[\frac{\partial C_p(\lambda, \beta)}{\partial \beta_{opt}} \right]$$

We will consider a nominal operating point we find the

following equation. (18)
$$J\frac{d\Delta\Omega_T}{dt} = J(\frac{d\Omega}{dt} - \frac{d\Omega_{nom}}{dt})$$

(19)
$$J \frac{d\Delta\Omega_T}{dt} = \Delta C_{aero} - G_g \Delta C_{em}$$

For a 2 Mw wind turbine we find (20) $C_p(\lambda,\beta) = 0.18 \left(\frac{90}{0.4+0.5\lambda} - 6.8 - 0.115\beta^2\right) e^{\frac{-8}{0.4+0.5\lambda}+0.16}$

State Space Representation

A wind turbine system is highly non-linear, consisting of two systems coupled via a speed multiplier with a transmission system [15, 18, 19].

The linearization of the flexible model around an operating point leads to the following equations.

point leads to the following equations.
$$\begin{cases} \Delta \dot{\Omega}_T = a \frac{\Delta \Omega_T}{J_T} + b \frac{\Delta \beta - \Delta C_{mec}}{J_T} \\ \Delta \dot{\Omega}_{g-ls} = \frac{\Delta C_{mec}}{J_{g-ls}} - \frac{G_g \Delta C_{em}}{J_{g-ls}} \\ \Delta \dot{\beta} = -\frac{1}{\tau_\beta} \Delta \beta + \frac{1}{\tau_\beta} \Delta \beta_{ref} \\ \Delta \dot{C}_{mec} = \left(K + \frac{da}{J_T}\right) \Delta \Omega_T - K \Delta \Omega_{g-ls} + \frac{db}{J_T} \Delta \beta + d(-\frac{1}{J_T} - \frac{1}{J_{g-ls}}) \Delta C_{mec} \end{cases}$$
 The state space of the wind system is described as follows:

(22)
$$\begin{cases} \dot{x} = A(t)x(t) + B(t)u(t) \\ y = C(t)x(t) + D(t)u(t) \end{cases}$$

Where $x(t) \in \Re n$ is the system state vector, $u(t) \in \Re \ell$ control vector, A, B and C are the state, control, and output matrices respectively, D the matrix direct transmission of the system.

With

(23)
$$A = \begin{bmatrix} \frac{a_i}{J_T} & 0 & \frac{b_i}{J_T} & \frac{-1}{J_T} \\ 0 & 0 & 0 & \frac{1}{J_{g-BV}} \\ 0 & 0 & -\frac{1}{\tau_B} & 0 \\ K + \frac{a_i}{J_T} & -K & \frac{db}{J_T} & (-\frac{d}{J_T} - \frac{d}{J_{g-BV}}) \end{bmatrix}$$

(24)
$$B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-G_g}{J_{g-BV}} \\ \frac{1}{\tau_B} & \frac{dG_g}{J_{g-BV}} \\ 0 & \frac{1}{J_{g-BV}} \end{bmatrix}$$

$$\begin{bmatrix} K + \frac{a_i}{J_T} & -K & \frac{db}{J_T} \\ (24) & B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-G_g}{J_{g-BV}} \\ \frac{1}{\tau_\beta} & \frac{dG_g}{J_{g-BV}} \end{bmatrix}$$

$$(25) \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\Omega_{g-ls_{nom}}} & 0 & 0 \end{bmatrix}$$

$$(26) \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & G_g \Omega_{T-nom} \end{bmatrix}$$
Where:

Where:

 $x = [\Delta\Omega_T \quad \Delta\Omega_{g-ls} \quad \Delta\beta \quad \Delta C_{mec}]^T, \ u = [\Delta\beta_{ref} \quad \Delta C_{em}]^T$ and $y = [\Omega_T \quad P]^{\mathrm{T}}$, the state vector, the input vector with two variables, and the output vector respectively.

The numerical values are given in Appendix 2.

Transfer Matrix of Wind Turbine
$$(27) \qquad G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

$$G_{11}(s) = \frac{-0.016158(s^2 + 2.556e - 06s + 126.7)}{(s + 0.04861)(s + 0.5)(s^2 + 0.008932s + 149.9)}$$

$$G_{12}(s) = \frac{-6.7273e - 05(s + 4.957e07)}{(s + 0.04861)(s + 0.5)(s^2 + 0.008932s + 149.9)}$$

$$G_{21}(s) = \frac{-4.8295e - 11(s - 4.59e09)}{(s + 0.04861)(s^2 + 0.008932s + 149.9)}$$

$$G_{22}(s) = \frac{174.55(s + 0.06241)(s^2 + 0.008932s + 149.9)}{(s + 0.04861)(s^2 + 0.008932s + 149.9)}$$
This system has four conjugate complex modes
$$\begin{cases} S = -0.0045 + 12.2447i \\ S_2 = -0.0045 - 12.2447i \\ S_3 = -0.0486 \\ S_4 = -0.5000 \end{cases}$$

The Singular Values of the Process

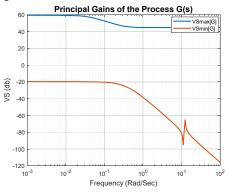


Fig 3. Wind Turbine Principal Gains

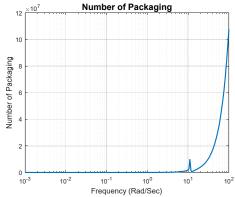


Fig 4. Conditioning number.

LQG/LTR Controller Design

LQG Controller Design

The problem of wind turbine control is particularly well suited to the LQG control (linear quadratic Gaussian) method of creating an optimal controller. In fact, depending on the various control objectives, the LQG synthesis enables one to acquire the behavior of a linear closed-loop system that is optimal for a quadratic criterion. Additionally, the stochastic characteristics of the system disturbances and measurement noises, and consequently the stochastic characteristics of the turbulent component of wind speed, are taken into account by the LQG synthesis [20-22].

(29)
$$\begin{cases} \dot{x} = Ax(t) + B(t)u(t) + w(t) \\ y = Cx(t) + Du(t) + v(t) \end{cases}$$

Where w and v the state noise and the measurement noise are centered white noises of variance

(30)
$$E\{w \ w^T\} = W \ge 0$$

(31)
$$E\{v \ v^T\} = V > 0$$

Consists of the criterion to minimize this problem and calculate as follows [22, 23].

(32)
$$J(x_0,u) = \lim_{t \to \infty} E\left(\frac{1}{tf} \int_{t0}^{tf} \left(\left(x(t) \right)^T Q x(t) + \frac{1}{t} \right)^T dt \right)$$

$$(u(t))^T Rx(t) dt$$

The KALMAN filter must be tuned by the variance matrices V and W so that the state is well reconstructed, and the LQ command must be tuned by the weighting matrices Q and R $(Q=Q^T \ge 0 \text{ et } R=R^T > 0)$ to have a "good" return of state [21, 22, 24, 25]. The adjustment of the LQG corrector is based on a separate adjustment of the KALMAN filter and the LQ control (by state feedback), according to the separation theorem. By the estimate x of the state x of the KALMAN Filter and Calculate the optimal gain of the KALMAN filter [26]

$$(33) \qquad \dot{\hat{x}} = A\hat{x} + Bu + K_f(y - C\hat{x} - Du)$$

 K_f The optimal gain of the KALMAN filter.

The minimization of the criterion is $\min_{K_f} J_G$

(34)
$$E(\mathcal{E}_{x}^{T} \mathcal{E}_{x})$$

And

$$\dot{\boldsymbol{\epsilon}}_{x} = (A - K_{f} C) \boldsymbol{\epsilon}_{x} + \left[I_{n} - K_{f} \right] \begin{bmatrix} w \\ v \end{bmatrix}$$

The vector $[w^T \ v^T]^T$ represents a Gaussian white noise centered on a spectral density $\begin{bmatrix} w & 0 \\ 0 & v \end{bmatrix}$. Moreover, this accounts for both the mean and covariance of the estimation error.

(36)
$$m(t) = E[\mathcal{E}_{x}(t)] = m(t_0) e^{(A-K_fC)(t-t_0)}$$

(37)
$$P_f(t) = [(\mathcal{E}_x(t) - m(t))(\mathcal{E}_x(t) - m(t))^T]$$

The filter is stable if $P_f(t)$ verifying the following differential equation

(38)
$$\dot{P}_{f}(t) = (A - K_{f}C)P_{f}(t) + P_{f}(t)(A - K_{f}C)^{T} + [I_{n} - K_{f}]\begin{bmatrix} w & 0 \\ 0 & v \end{bmatrix}\begin{bmatrix} I_{n}^{T} \\ -K_{f}^{T} \end{bmatrix}$$

 $\dot{P}_f(t)=0$ and $P_f(t)=P_f$ then verify the LYAPUNOV equation in steady state

(39)
$$(A - K_f C) P_f(t) + P_f(t) (A - K_f C)^T + [I_n - K_f] \begin{bmatrix} w & 0 \\ 0 & v \end{bmatrix} \begin{bmatrix} I_n^T \\ -K_f^T \end{bmatrix} = 0$$

(40)
$$(A - K_f C)P_f(t) + P_f(t)(A - K_f C)^T + W + K_f V K_f^T = 0$$

The equation (41) define the Kalman filter gain matrix:

(41)
$$K_f = P_f C^T V^{-1}$$

(42)
$$P_f A^T + A P_f - P_f C^T V^{-1} C P_f + W = 0$$

(43)
$$P_f = P_f^T > 0$$

 $u = -K\hat{x}$
(44) $K = R^{-1}B^TP$

(44)
$$\begin{cases} K = R^{-1}B^{T}P \\ PA + A^{T} - PBR^{-1}P + Q = 0 \end{cases}$$

The state representation of the LQG controller is given by:

(45)
$$\begin{bmatrix} \dot{\hat{x}} \\ u \end{bmatrix} = \begin{bmatrix} A + BK + K_fC - K_fDK & K_f \\ -K & 0 \end{bmatrix} \begin{bmatrix} \dot{\hat{x}} \\ y \end{bmatrix}$$

The Controller Transfer Matrix K(s)

(46)
$$K(s) = -K(sI - A + BK + K_fC - K_fDK)^{-1}K_f$$

(47)

W

Kalman

Filter

LQG Regulator

Fig 6. Structure of LQG control.

C. LQG/LTR Synthesis (Loop Transfer Recovery) The LQG/LTR approach synthesizes control using state feedback known as "LQ control." This entails selecting suitable Q and R weightings, considering robustness. The KALMAN filter is computed by adjusting the variance matrix, gradually aligning it with the LQ control step, which leads to its name - Loop Transfer Recovery (LTR).

Stability and Performance Specifications

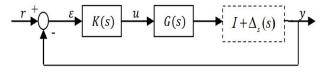


Fig 7. Structure of LQG control.

 $u,\,y,\,r$ and ϵ is the command, output, reference and error, respectively. G(s),K(s) the system and controller transfer matrix respectively $\Delta_s(s)$ is the multiplicative uncertainty transfer matrix at the output.

Considering that, the transfer matrix of the system $Gp(j\omega)$ deviates from a nominal transfer matrix $G(j\omega)$ by a $\Delta_s(s)$ quantity.

(48)
$$Gp(j\omega) = [Ip + \Delta_s(j\omega)].G(j\omega)$$

(49)
$$\Delta_s(s) = [Gp(j\omega) - G(j\omega)].G^{-1}(j\omega)$$

Then the maximum singular values $\bar{\sigma}$ written by:

(50)
$$\bar{\sigma}[\Delta_s(j\omega)] = \bar{\sigma}[[Gp(j\omega) - G(j\omega)].G^{-1}(j\omega)]$$

The following relation gives the choice of the stability specification Wt:

(51)
$$\bar{\sigma}[\Delta_s(j\omega)] \leq \bar{\sigma}[Wt(j\omega)] \quad \forall \omega$$

The following relations give the stability and performance robustness condition, respectively:

(52)
$$\bar{\sigma}[T(jw)] < \bar{\sigma}[Wt(jw)]^{-1}$$

(53)
$$\bar{\sigma}[S(jw)] \leq \bar{\sigma}[Wp(jw)]^{-1}$$

Where T(jw) the nominal is closed loop transfer matrix and S(jw) is the sensitivity matrix.

The nominal process transfer matrix defined by [27, 28]:

(54)
$$T(jw) = G(jw).K(jw).[I + G(jw).K(jw)]^{-1}$$

It represents the transfer matrix between the reference r and the output y.

The error ε . And $S(j\omega)$ is the sensitivity matrix defined by:

(55)
$$S(jw) = [I + G(jw).K(jw)]^{-1}$$

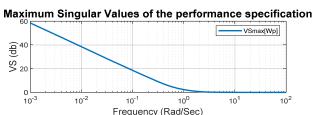
It represents the transfer matrix between the reference r and the error ϵ .

$$(56) S(jw) + T(jw) = I$$

The following relations are the matrix of the stability and performance robustness specifications, respectively:

(57)
$$Wt(s) = \begin{bmatrix} 0.665 s + 1 & 0 \\ 0 & 0.665 s + 1 \end{bmatrix}$$
(58)
$$Wp(s) = \begin{bmatrix} \frac{1.2 s + 1}{1.2 s} & 0 \\ 0 & \frac{1.2 s + 1}{1.2 s} \end{bmatrix}$$

The frequently representation of the specifications is given in the following figures



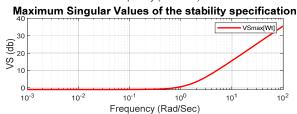


Fig 8. Maximum singular values of robustness and stability specification.

By understanding the frequency of the system at one moment, we can predict its state at the next moment and take actions to enhance reliability. Recent advances in frequency measurement methods, which are based on highly accurate frequency measuring devices, have improved accuracy. However, there are still challenges to measuring low and high frequencies accurately, the following figures show Performances condition at high frequencies and low frequencies.

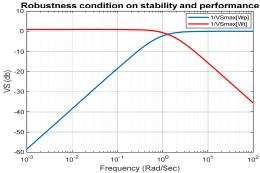


Fig 9. Robustness condition on stability and performance.

B. Performances condition in high frequency (HF)

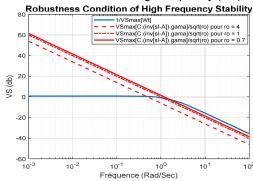


Fig 10. Robustness condition in (HF).

C. Performances condition in Low frequency (LF)

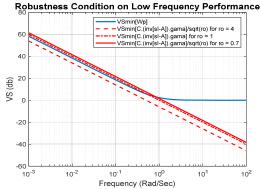


Fig 11. Robustness condition in (LF).

Figures for the case of robustness on performance at high frequency and robustness on performance at low frequency setting show that the optimal values of Γ and ρ were obtained by trial and error starting from the previous conditions.

D. Principal gains, Sensibility and robustness conditions

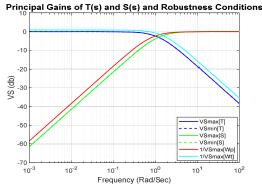
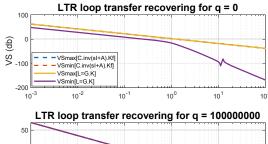


Fig 12. Principal gains, Sensibility and robustness conditions.



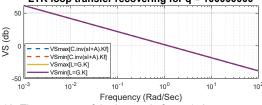
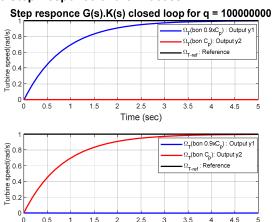


Fig 13. The recovery of the loop gain for q=1e8.

The Step Response of the Process



Time (sec)

Fig 14. Closed-loop step response.

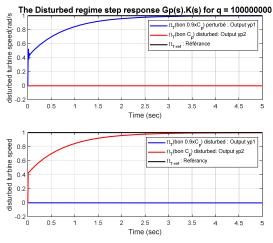


Fig 15. Disturbed regime in closed-loop step response.

Interpretations

In the provided figures, we designed a robust control system for a wind turbine that encountered turbulence in wind speed and white noise. We illustrated the resulting status response of the closed-loop control system. Our goal was to maintain effective tracking of the reference signal, and from the plots, it's evident that the LQG controller successfully achieves several condition estimator objectives under nominal values despite the influence of noise, wind speed fluctuations, and white noise. However, there is room for improvement. We need to optimize the wind turbine's

performance to capture the maximum achievable power while ensuring optimal stability. This can be achieved by integrating other available technologies such as Loop Transfer Recovery (LTR). We resolved the issue of poorly conditioned main gains by applying the LQG technique. Furthermore, we verified the condition of stability robustness. After implementing the LQG/LTR loop transfer, we can confidently assert that this method holds high feasibility in practical applications. Its potential for generalization and applicability in the field of controlling turbulent systems is remarkable.

Conclusions

In this paper, a robust linear quadratic Gaussian controller for variable speed wind turbines is proposed. The control strategy that includes case feedback an integrated procedure and an output reference model, was designed taking into account wind speed variation using LQG and LQG/LTR tuning control for a multivariate system both during nominal operation and at load applied to the system. The simulation results justify the frequent use of this method in the spontaneous field. This technique can achieve robustness-satisfied behavior at nominal and turbulent speed. A comparative study of the results obtained through the concept of the proposed control system led to the conclusion that the use of an LQG controller with an LTR loop transmission overlay; was suitable and had good and stable performance for variable speed wind applications. According to the simulation results and compared to the classical controllers, the control through the proposed LQG / LTR combination ensures high efficiency in regulating the rotational speed of the wind turbine by controlling the blades (pitch control), which allows the wind system to operate in a safe interval and ensures maximum reliability. With very acceptable power and performance.

Appendix 1

TABLE I Parameters for the mechanical system model

Symbol	name	value	unit
P_{nom}	Rated turbine power	2	MW
Ω_{T-nom}	Rated turbine speed	18	rpm
R_T	Blade length	40	m
J_T	Rotor inertia	$4.9*10^6$	$N.m.s^2$
$J_{g ext{-}ls}$	Inertia of the generator brought	$0.9*10^6$	$N.m.s^2$
	back to the low speed shaft		
λ_{opt}	Optimal gear ratio	9	rad
G_{g}	Multiplier Gain	92.6	/
d	Friction coefficient	$3.5*10^{5}$	$N.m^{-1}.s$
k	Coefficient of elasticity	$114*10^{6}$	$N.m^{-1}$
V_{ν}	Wind speed	11.65	m/s
ρ	Air density	1.225	$Kg.m^3$
$ au_{eta}$	cost of time	0.2197	S
$C_{p\text{-}opt}$	Optimal power coefficient	0.4475	/

Appendix 2

Numeric Values for state space Model

$$\begin{split} \mathbf{A} = & \begin{bmatrix} -0.05754 & 0 & -0.03232 & -2.041e7 \\ 0 & 0 & 0 & 0.0001029 \\ 0 & 0 & -0.5 & 0 \\ 1.14e8 & -1.231e6 & -0.07433 & -3.025e6 \end{bmatrix} \\ \mathbf{B} = & \begin{bmatrix} 0 & 0 \\ 0 & 0.8822 \\ 0.5 & 0 \\ 0 & 23.66e - 5 \end{bmatrix} \\ \mathbf{C} = & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 17.59 & 0 & 0 \end{bmatrix} \\ \mathbf{D} = & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 174.5 \end{bmatrix} \end{split}$$

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