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INS/GPS KF Integration Performance Improvement based on Accurate Inertial Sensors Stochastic Error Modelling

***Abstract**. Inertial Navigation Systems (INS) provides precise data for short time-period, but their accuracy diminishes over time, especially with low-cost sensors. To maintain acceptable accuracy, sensor error components must be accurately calibrated/modelled. Different methods have been used to characterize the inertial sensors stochastic errors, including the Autocorrelation function (ACF), Allan variance (AV) and the Generalized Method of Wavelet Moments (GMWM). This paper focuses on the analysis of Micro-Electromechanical Systems (MEMS)-based inertial sensor errors under various conditions. The inertial sensor stochastic error processes are estimated using both the AV and the GMWM techniques. Based on the comparison between both stochastic analysis tools, the GMWM was selected and a GMWM-based model selection criteria is utilized to rank candidate error models. An extended 39-states integrated GNSS/INS navigation algorithm (based on the chosen error model) is proposed and compared with a standard 15- states integrated GNSS/INS navigation algorithm (based on 1st Gauss-Markov process for modelling the stochastic errors). The study analyses various stochastic error models using real data of Inertial Navigation System (INS) and Global Positioning System (GPS) with intended GPS signal outage periods. Results reveal enhanced position accuracy with the proposed algorithm and superior performance with GMWM-based error model over standard ACF-based one.*

Abstrakcyjny.** Inercyjne systemy nawigacji (INS) dostarczają dokładnych danych przez krótki okres czasu, ale ich dokładność maleje z czasem, szczególnie w przypadku tanich czujników. Aby zachować akceptowalną dokładność, składowe błędu czujnika muszą być dokładnie skalibrowane/modelowane. Do charakteryzowania błędów stochastycznych czujników inercyjnych zastosowano różne metody, w tym funkcję autokorelacji (ACF), wariancję Allana (AV) i uogólnioną metodę momentów falkowych (GMWM). W artykule skupiono się na analizie błędów czujników inercyjnych opartych na systemach mikroelektromechanicznych (MEMS) w różnych warunkach. Procesy błędów stochastycznych czujnika inercyjnego są szacowane przy użyciu technik AV i GMWM. Na podstawie porównania obu narzędzi analizy stochastycznej wybrano GMWM, a kryteria wyboru modelu oparte na GMWM zastosowano do uszeregowania modeli potencjalnych błędów. Zaproponowano rozszerzony, 39-stanowy zintegrowany algorytm nawigacji GNSS/INS (oparty na wybranym modelu błędów) i porównano go ze standardowym 15-stanowym zintegrowanym algorytmem nawigacji GNSS/INS (opartym na pierwszym procesie Gaussa-Markowa do modelowania błędów stochastycznych). W pracy przeanalizowano różne modele błędów stochastycznych wykorzystując rzeczywiste dane z systemu nawigacji inercyjnej (INS) i globalnego systemu pozycjonowania (GPS) z przewidywanymi okresami zaniku sygnału GPS. Wyniki ujawniają zwiększoną dokładność pozycjonowania dzięki proponowanemu algorytmowi i lepszą wydajność dzięki modelowi błędów opartemu na GMWM w porównaniu ze standardowym modelem opartym na ACF. (**Poprawa wydajności integracji INS/GPS KF w oparciu o dokładne modelowanie błędów stochastycznych z czujników inercyjnych)

Keywords: Inertial Navigation Systems (INS), Global Positioning System (GPS), Autocorrelation function (ACF), Allan Variance (AV), Generalized Method of Wavelet Moments (GMWM), Inertial Sensors Errors, Confidence Interval (CI), Wavelet Information Criterion (WIC).
Słowa kluczowe: Inercyjne systemy nawigacji (INS), globalnego systemu pozycjonowania (GPS), Funkcja autokorelacji (ACF),.

Introduction

Navigation is a method to determine the velocity and position, sometimes include the attitude, of a moving target with apriore known reference [1]. There are two methods to get a navigation solution: position fixing, with The Global Positioning System (GPS) being a typical example, and dead reckoning (DR), with Inertial navigation system (INS) being a typical example. The GPS is an accurate navigation system that is known to be for different outdoor applications. GPS may be exposed to signal outages, jamming, multipath effects, or any other sources of interference [2]. On the other hand, the INS provides high frequency precise short-term navigation information [3]. However, their accuracy degrades rapidly with time [1]. The INS/GPS integration provides an accurate complementary system with better performance, in comparison of standalone system and overcomes their individual drawbacks [4].

Generally, for INS/GPS integration, GPS is usually responsible for providing velocity and position information (with relatively high accuracy and low data rate). On the other hand, INS provides the position, velocity and attitude (PVA) information (with higher data rate). GPS updates prevents INS from drifting and INS provides continuous navigation solution even with GPS signal outages. Recently, Micro Electromechanical Systems (MEMS)-based inertial sensors have been developed to provide a low-cost navigation solution that could be used in INS/GPS integration system [5]. MEMS-based inertial sensors are known with their light weight, low-cost, small size and low power consumption modules, but rarely achieve the required navigation solution accuracy [6] because of the unwanted deterministic and stochastic errors that added to the signal measured by the sensor [7]. Based on that, laboratory calibration is essential to eliminate the deterministic error's part (such as systematic bias offset and scale factor error). The stochastic error's part (such as white noise and random walk) requires special noise characterization techniques for modelling and estimation. The two error types considerably influence the PVA of the moving platform estimated by the module which affects the accuracy of the navigation solution, especially when INS works as a standalone system during GPS signal outages.

Review of Previous Work

An Inertial Measurement Unit (IMU) generally outputs 3D accelerations and angular rates, which further be integrated to achieve representative PVA information [1]. IMU outputs normally includes different types of error sources, deterministic and stochastic errors [8]. These errors will be accumulated by the integration process in the navigation algorithm resulting in a significant drift in the navigational states [9]. So, various stochastic error modelling methods have been used to reduce the inertial sensors error terms, some of the methods are frequency domain and the others are time domain [10], [11]. For instance, Autocorrelation function (ACF) method was used in [12] to examine the correlation time of consumer grade IMU used to determine an attitude information. Another correlation method, an autoregressive moving-average process, was used in [13] which associates the autocovariance of a difference equation to the coefficients. However, a major limitation of the correlation technique is typically not the ideal option to handle high range of dynamics or higher order random processes [14].

Furthermore, Power Spectral Density (PSD), known as a frequency-domain stochastic analysis technique, was used to model the inertial sensor errors is the in [15]. ACF is, in fact, the Fourier Transform (FT) of the ACF. But still a major restriction of this technique is associated with the low frequency part of the PSD log-log plot which however transmits some data with high uncertainty [16]. This drawback affects the accuracy of recognizing the parameters of low frequency noise greatly while examining the noise of inertial sensor output for long-term datasets. There are various time-domain methods that have been investigated. The most widely used one is the Allan Variance (AV) which was essentially presented to study the stability of the frequency for oscillators [14]. AV is known to be straightforward, simple to compute and to understand. However, a major limitation of such technique is the incidence of ambiguity when represents a real AV log-log plot as a sum of more than one random process. Another estimation method, called the Generalized Method of Wavelet Moments (GMWM) that was presented in [17]. GMWM has some advantages that could offer an efficiency and flexibility in computations and considered as user-friendly and statistically tool to select and estimate from a broad range of stochastic error models [18]. From the Generalized Method of Moments (GMM) estimators, the GMWM utilizes the relationship between the Wavelet Variance (WV) and the parameters of the stochastic process, then the parameters could be estimated by reducing the distance between the implied and empirical WV to estimate the time series stochastic error processes [17], [19]. Beside the limitations of each technique, all the previous research works have a common restriction while examining the environmental conditions, such as temperature variation [20] and platform dynamics [21]. Bothe aforementioned environmental conditions affects the sensor's stochastic error parameters values, especially for long data sets, which, consequently, affects the output of the inertial sensors. However, the investigation of the effect of the environmental conditions on the stochastic error modelling process is beyond the scope of this paper.

Aim and Scope of Study

The goal of the paper is to enhance the accuracy of the navigation solution of INS/GPS integrated systems for low-cost MEMS-based IMUs using stochastic error models through the following steps:

- Laboratory stationary data collection process using Spatial Advanced Navigation IMU was executed.
- Three different noise analysis approaches are utilized to expose, examine and reveal the inertial sensor stochastic error part:
  1. ACF as a commonly used approach (where its results will be used in the standard 15-states integrated navigational algorithm)
  2. AV and GMWM as more complicated techniques with additional mathematical calculation steps
- A fair comparison will be applied to examine the advantages and constraints of both AV and GMWM approaches in terms of the ability of identifying inertial sensors errors parameters. The best results from one of them will be used in the proposed extended integrated navigational algorithm after performing a fitting test.
- Finally, a proposed extended navigation algorithm (based on the best model chosen from the better approach) is introduced. Then a fair comparison is conducted between the performance of both extended and standard widely used navigation algorithms (based on one 1st order Gauss Markov process per sensor that is obtained from the ACF technique) during GPS signal outage periods.

Methodology

Random Processes Models for Inertial Sensor Errors:

The main inertial sensors stochastic error processes are defined as follow (see Table 1 for more details regarding associated (PSD)):

- Gauss Markov (GM):

This error process considered to have an exponentially correlation time [22]. It has an acceptable accuracy while fitting various physical processes with simple math operations.

- Quantization Noise (QN):

This type of error is generated while representing an analog signal in digital form. The difference between the amplitudes of the sampled points and the bit resolution of the Analog to Digital converter is referred to as quantization error [14].

- White Noise (WN):

This error is also known as Angular Random Walk (ARW) for gyroscopes and Velocity Random Walk (VRW) for accelerometers and is considered as a high-frequency noise type that has very short correlation time compared with the sample period [14].

- Bias Instability (BI):

This error considered as a low-frequency noise type and generated due to the use of the electronics components in MEMS-based inertial sensors. It causes fluctuations in the bias in the measurements [23].

- Random Walk (RW):

This error considered as an exponentially low-frequency noise type that has long correlation time [24]. This is known as Rate Random Walk (RRW) for gyroscopes and Acceleration Random Walk (AccRW) for accelerometers.

- Drift Ramp (DR):

This error considered as a very low-frequency noise type and is known as Drift Rate Ramp (DRR) for gyroscopes and Drift Acceleration Ramp (DAccR) for accelerometers.

Table 1. Random Processes with Associated PSDs

Stochastic Processes	Coefficient	PSD
Gauss Markov	$\boldsymbol{\beta, \sigma_{GM}^2}$	$S_{GM}(f) = \frac{2\sigma_{GM}^2\beta}{(2\pi f)^2+\beta^2}$
Quantization Noise	**Q**	$S_{QN}(f) = 4\,\frac{Q^2}{T_s} sin^2(2fT_s)$
White Noise	**W**	$S_{WN}(f) = W^2$
Bias Instability	**B**	$S_{BI}(f) = \begin{cases} \frac{B^2}{2\pi f} & if\ f \le f_0 \\ 0 & if\ f > f_0 \end{cases}$
Random Walk	**K**	$S_{RW}(f) = \left(\frac{K}{2\pi f}\right)^2$
Drift Ramp	**D**	$S_{DR}(f) = \frac{D}{(2\pi f)^3}$

Stochastic Noise Characterization Techniques [25]:

Autocorrelation function (ACF):

Autocorrelation is the correlation between two values in a time series. ACF, sometimes known as a serial correlation in the discrete time case, reveals how the correlation between any two values of the signal changes as their separation changes [26]. It is a time-domain representation method that clarifies the change of the data's phase and the correlation with the original data [27]. The ACF assess the correlation between observations in a time series for a set of lags. ACF is used to identify which lags have significant correlations, understanding the patterns and the properties of the time series and then using this information to model the time series stochastic data [28]. In this paper, ACF is used to characterizing the inertial sensor stochastic errors (as an easy and standard stochastic approach) by determining the 1st order GM process parameters, the time constant reciprocal (β) and the GM standard deviation

(σ_{GM}), for the IMU signal. The ACF of a 1st order GM process is shown in Fig. 1.
The ACF is defined mathematically as follow:

(1) $$\rho_x(\tau) = \frac{Cov(X_t - X_{t+\tau})}{Var(X_t)} = \frac{\mathbb{E}[X_t, X_{t+\tau}] - \mathbb{E}^2[X_t]}{\mathbb{E}[X^2] - \mathbb{E}^2[X_t]}$$

Where $\rho_x(\tau)$ is the ACF of X, τ is the lag/shift time, $Cov(X_t - X_{t+\tau})$ is the covariance between the two functions, $Var(X_t)$ is the variance of X_t and $\mathbb{E}[.]$ Is the expectations.

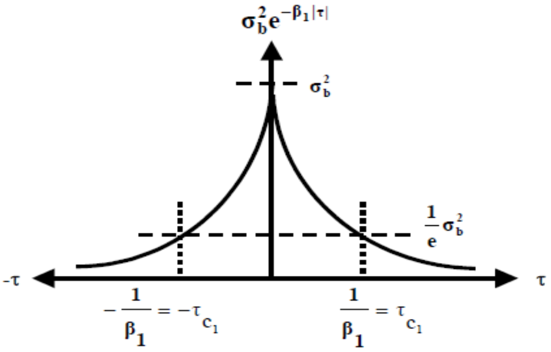

Fig. 1. The ACF of a 1st Gauss Markov process [29].

Allan Variance (AV):

The Allan variance is a time domain tool of representing the Root Mean Square (RMS) random-drift errors as a function of averaging times [14]. It is straightforward to understand, compute and interpret. The AV technique could be used to characterize various inertial sensor stochastic error processes in different datasets. The AV mathematical processes are demonstrated in [30], [31].

The mathematical equation formula of the standard AV is:

(2) $$\sigma^2(T) = \frac{1}{2(N-2n)} \sum_{k=1}^{N-2n} [\bar{\Omega}_{next}(T) - \bar{\Omega}_k(T)]^2$$

Where $\bar{\Omega}_k(T)$ is the mean value of the k cluster, $\bar{\Omega}_{next}(T)$ is the mean value of the sub-sequent cluster, T is the length of the cluster, N is the entire length of the data points, n is the number of data points in each cluster $(n < N/2)$.

The relationship between the standard AV $\sigma^2(T)$ and the Power Spectral Density of the stochastic processes is:

(3) $$\sigma^2(T) = 4 \int_0^\infty S_\Omega(f) \frac{\sin^4(\pi f T)}{(\pi f T)^2} df$$

Where $S_\Omega(f)$ is the PSD of the stochastic process Ω(T).

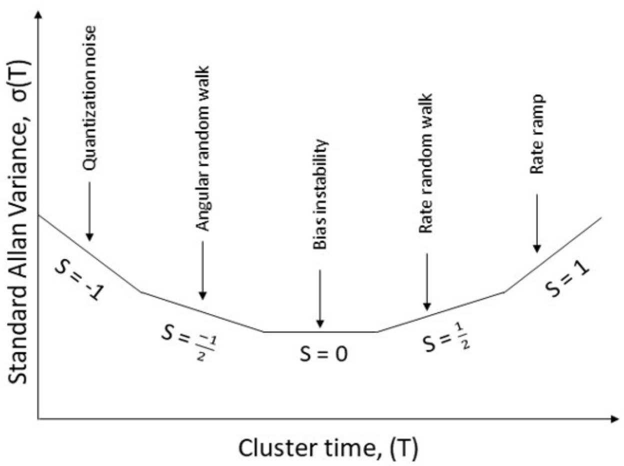

Fig. 2. Standard deviation AV log-log plot with the basic noise parameters [32].

The AV algorithm is applied to the static data collected in laboratory condition to obtain the characteristic curve to reveal the types and values of the stochastic errors that concentrating in the data. The collecting data should be free of outliers to get the correct model that represent the signal. So, a contamination test is needed to implement to the data before applying the AV algorithm. The AV approach cannot offer this task counter to the GMWM which will be highlighted in the next section.

Generalized Method of Wavelet Moments (GMWM):

GMWM is considered as an efficient and flexible tool to determine and select the best model that represents the signal under investigation. GMWM is an optimal estimator technique based on the WV and the Generalized Method of Moments (GMM) [17]. The GMWM estimates the parameters of the stochastic processes by decreasing the distance between the implied WV and the empirical one. The flowchart of the GMWM approach is explained in Fig. 3. A contamination test could be applied to the collected datasets by computing the robust WV and compared with the classical one to check the datasets that contain any contaminations (or outliers) according to the CI and as shown in Fig. 4 and 5 [17]. The next step is the estimation of stochastic error parameters values of all the suggested models. Then, the Wavelet Information Criterion (WIC) approach is used to ranking and selecting the best model that represent the signal [19]. The WIC approach includes the objective function term, show if the suggested model fitting the signal, and the optimism term, show the complex degree of the suggested model. The large the number of the stochastic processes in the selected model, the small the value of the objective function term and the large the value of the optimism term.

The direct PSD integration is the wavelet coefficients variances. This equation forms a relationship between the PSD and WV as follow:

(4) $$v_j(\Theta) = \int_{-1/2}^{1/2} S_{W_j}(f)\, df = \int_{-1/2}^{1/2} |H_j(f)|^2\, S_{F_\Theta}(f)\, df.$$

Where $|H_j(f)|$ is the transfer function of the filter $h_{j,l}$, F_Θ is the estimated model that consists of the stochastic processes, $|.|$ denotes the modulus operator and S_{F_Θ} is the PSD of the estimated model F_Θ.

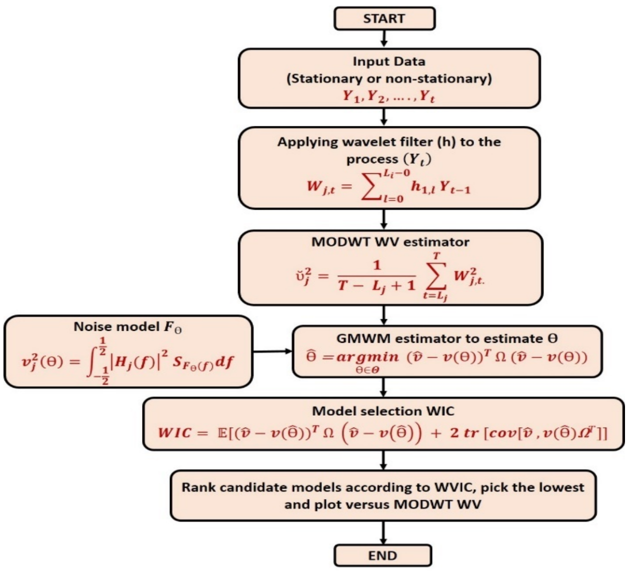

Fig. 3. The flowchart of the GMWM approach [32].

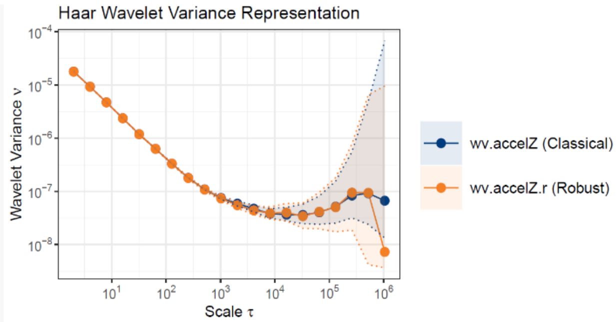

Fig. 4. Classical Vs robust WV of accel Z.

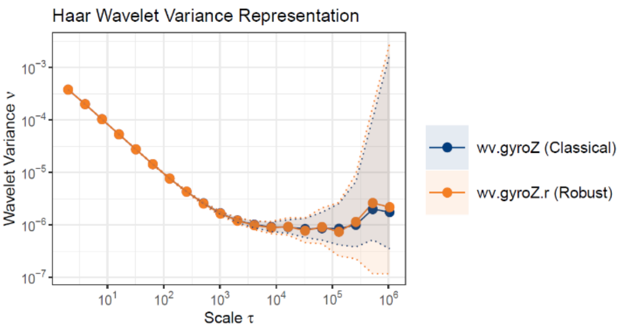

Fig. 5. Classical Vs robust WV of gyro Z.

The GMWM estimator is used for decreasing the distance between the implied WV and the empirical one to evaluate the stochastic processes parameters as follow:

(5) $$\hat{\Theta} = \text{argmin}_{\theta\in\Theta}\left(\hat{V} - V(\Theta)\right)^T \Omega \left(\hat{V} - V(\Theta)\right)$$

Where Ω is a positive definite weighting matrix, selected by a specific manner, to create the GMWM estimator more practical and efficient, Θ is the vector of the time series model parameter [33].
The full mathematical and calculation steps for the GMWM approach is explained in [33].

Inertial Data Collection and Analysis

In this paper, inertial sensor data collected from Spatial Advanced Navigation module was used for the stochastic error modelling using 3 different techniques (ACF, AV and GMWM). The Spatial module is relevant small (i.e., a cube of 30 mm in height with 40.6 mm length and 24 mm width) and light in weight, 37 grams. The initial bias for gyro is about $0.2^{\circ}/s$ and for accelerometer is about 20 ug. Two different methods are used to collecting the datasets, the static and dynamic in-field modes. First, multiple datasets were collected in static conditions using the same module and at the same environmental conditions (including room temperature and data length). The purpose of repeating the same test is to ensure that every individual sensor has the same performance for each test. Second, the parameters obtained from the analysing of such multiple datasets using various stochastic techniques will be used as input to the integrated navigation algorithm to evaluate the performance of INS standalone-based navigational solution during intended GPS signal outage.

Static Data Collection:

In the static mode 4 hours static datasets were collected at room temperature in laboratory conditions for each inertial sensor at 100 Hz sampling rate (see Fig. 6).

The performance estimation of the inertial sensor MEMS-based IMU is applied using the ACF, AV and GMWM approaches to study the characterization of the stochastic noise and identify the parameters of the inertial sensor stochastic error processes for each individual technique. Then, a comparison between AV and GMWM methods is applied to know the best technique for representing the inertial sensor stochastic errors by study the fitting from both techniques with the original signal (see Fig. 11).

Fig. 6. Collecting static data setup in laboratory conditions.

<u>ACF Data Analysis:</u>
The ACF approach is applied to all static datasets. Taking the 2nd dataset as an example, Fig. 7 and 8 illustrate a 1st order GM process for x-gyro and x-accelerometer, respectively. Table 1 lists the coefficients of the stochastic errors after applying the ACF approach to the accelerometers and gyros of the 2nd dataset.

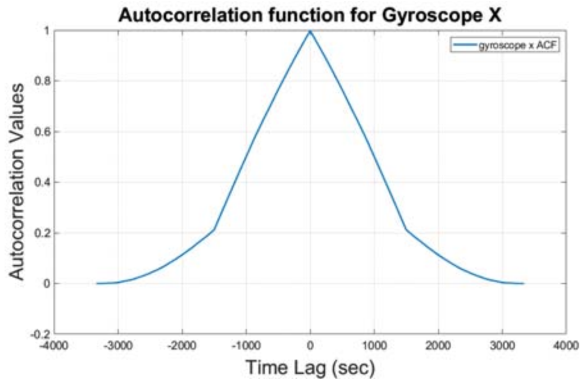

Fig. 7. A 1st order GM process for gyroscope x.

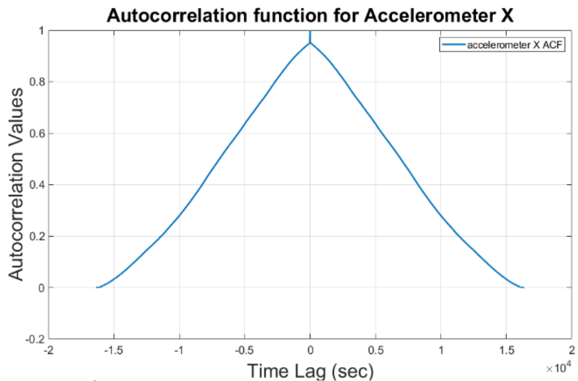

Fig. 8. A 1st order GM process for accelerometer x.

Table 2. ACF parameters for the 2nd dataset as an example using the ACF method.

	$\beta\ (sec^{-1})$	$\sigma^2_{GM}(m^2/sec^4)$
Accel X	0.000115275	1
Accel Y	0.0001579941	1
Accel Z	0.00090662653	1
	$\beta\ (sec^{-1})$	$\sigma^2_{GM}(deg^2/h^2)$
Gyro X	0.00021	1
Gyro Y	0.0002	1
Gyro Z	0.005197	1

AV Data Analysis:

The AV approach is applied to all the static datasets. Taking the 2nd dataset as an example, Fig. 9 and 10 illustrates a standard deviation AV log–log plot for the Spatial gyros and accelerometers measurements, respectively. Table 2 lists the coefficients of the stochastic errors after applying the slope fitting technique associated with the AV approach to the accelerometers and gyros data. From Table 2, for Gyros, the dominant noise process for the high-frequency term at slope –1/2 is the ARW and the dominant noise process for the low-frequency term at slope 1/2 is the RRW and, for Accelerometers, the dominant noise process for the high-frequency term at slope –1/2 is the VRW and the dominant noise process for the low-frequency term at slope 1/2 is the AccRW as shown in Fig. 11.

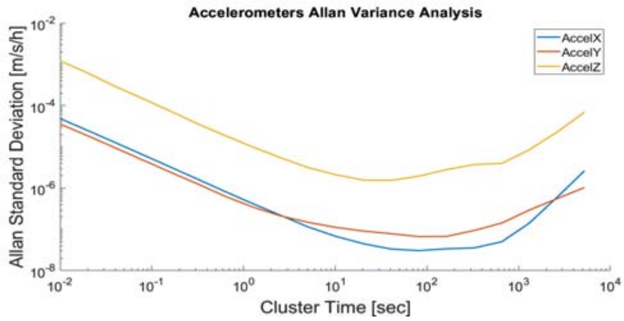

Fig. 9. Standard deviation AV analysis log-log plot for accelerometers measurements for the 2nd static dataset.

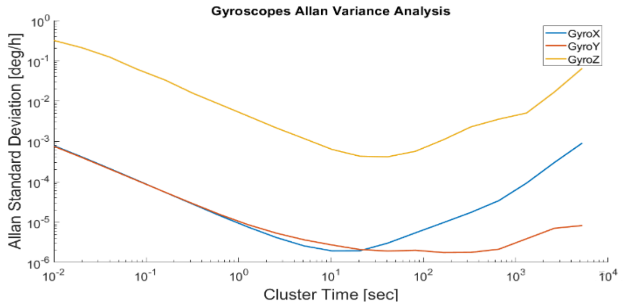

Fig. 10. Standard deviation AV analysis log-log plot for gyroscopes measurements for the 2nd static dataset.

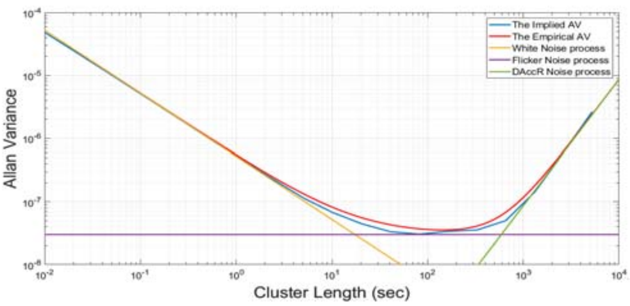

Fig. 11. Standard deviation AV log-log plot for Accel X as an example with the corresponding identified error processes.

Table 3. Stochastic error processes parameters for the 2nd static dataset as an example using the AV method.

	White Noise (m/s/√h)	Bias Instability (m/s/h)	AccRW (m/s/$h^{1.5}$)
Accel X	0.00071958	0.00026149	1.58e-05
Accel Y	0.00064866	0.00038815	2.56e-05
Accel Z	0.0035083	0.0018724	0.0001326
	White Noise	Bias Instability	RRW
	(deg/ √h)	(deg/h)	(deg/$h^{1.5}$)
Gyro X	0.0030743	0.0020929	0.0004313
Gyro Y	0.0031968	0.0019866	9.143e-05
Gyro Z	0.073062	0.030576	0.0033188

GMWM Data Analysis:

As mentioned before, GMWM is used to characterizing the stochastic error processes existing in the measurements of the static data then calculating the corresponding parameters of them [34]. First, a contamination test is applied to compare between the classical and robust WV of the static dataset to check if the data is contaminated by any outliers. As shown from Fig. 4 and 5, the two WVs are laying inside the confidence interval (CI) of each other so the data is considered to be clean. The WIC approach select the best model that represents the corresponding stochastic error processes. Some candidate models were chosen accurately to represent the inertial sensor signal structures as found in Table 3. The WIC numerical value was calculated for each model and the model 3*GM+RW+WN (indexed 23) was found to has the smallest numerical value of the WIC for gyros and accelerometers, respectively [35], [36]. Fig. 12 and 13 show that the selected model has an estimated WV with accurate fitting with the empirical WV of the original sensors data. Table 4 lists the numerical values of the corresponding parameters of each stochastic noise processes for the selected model after performing the GMWM approach. Based on the analysis performed using the AV and GMWM, it is clear that the GMWM outperforms the standard AV in terms of:

- Characterizing latent stochastic noise terms associated with inertial sensors.
- Testing inertial sensors data undertest using contamination test.

Thus, results obtained from the GMWM will be adopted for building a proposed integrated navigation algorithm.

Table 4. Suggested models for representing the inertial sensor signal for the Spatial module static dataset.

Index	**Suggested model**	**Index**	**Suggested model**
1	GM	**23**	3GM+DR+RW
2	GM+WN	**24**	3GM+WN+DR+RW
3	GM+DR	**25**	4GM
4	GM+RW	**26**	4GM+WN
5	GM+WN+DR	**27**	4GM+DR
6	GM+WN+RW	**28**	4GM+RW
7	GM+DR+RW	**29**	4GM+WN+DR
8	GM+WN+DR+RW	**30**	4GM+WN+RW
9	2GM	**31**	4GM+DR+RW
10	2GM+WN	**32**	4GM+WN+DR+RW
11	2GM+DR	**33**	5GM
12	2GM+RW	**34**	5GM+WN
13	2GM+WN+DR	**35**	5GM+DR
14	2GM+WN+RW	**36**	5GM+RW
15	2GM+DR+RW	**37**	5GM+WN+DR
16	2GM+WN+DR+RW	**38**	5GM+WN+RW
17	3GM	**39**	5GM+DR+RW
18	3GM+WN	**40**	5GM+WN+DR+RW
19	3GM+DR	**41**	WN
20	3GM+RW	**42**	DR
21	3GM+WN+DR	**43**	RW
22	3GM+WN+RW	**44**	DR+RW

Table 5. The stochastic error processes parameters for the 2nd dataset as an example using the GMWM for the best selected model.

Sensor	GM#1 β (sec^{-1})	GM#1 σ^2_{GM} (m^2/sec^4)	GM#2 β (sec^{-1})	GM#2 σ^2_{GM} (m^2/sec^4)	GM#3 β (sec^{-1})	GM#3 σ^2_{GM} (m^2/sec^4)	WN (m/s/√sec)	AccRW (m/s/$s^{1.5}$)
Accel X	5.610237e-04	1.340726e-07	1.222059e-05	1.849303e-07	4.137647e+00	3.944171e-05	1.171017e-14	2.420073e-06
Accel Y	7.170143e-04	3.563531e-08	6.714848e-04	1.247150e-07	4.085116e+00	4.931947e-05	3.117214e-12	4.119769e-07
Accel Z	4.552093e-03	1.594431e-07	3.725952e-04	1.126158e-07	3.813356e+00	3.437161e-05	6.441214e-12	2.748302e-06
Sensor	**GM#1** β (sec^{-1})	**GM#1** σ^2_{GM} (deg^2/h^2)	**GM#2** β (sec^{-1})	**GM#2** σ^2_{GM} (deg^2/h^2)	**GM#3** β (sec^{-1})	**GM#3** σ^2_{GM} (deg^2/h^2)	**WN** (deg/√sec)	**RRW** (deg/$s^{1.5}$)
Gyro X	7.564539e-03	2.072914e-06	1.494725e-04	3.735622e-06	7.715196e+00	1.094648e-03	2.080217e-10	1.132238e-04
Gyro Y	2.208402e-03	1.590331e-06	2.420189e-04	7.664207e-08	3.165993e+00	7.690319e-04	1.733841e-09	4.980642e-05
Gyro Z	5.621800e-03	5.526396e-06	1.609017e-04	4.008904e-06	3.045637e+00	7.639954e-04	6.363819e-11	2.684013e-05

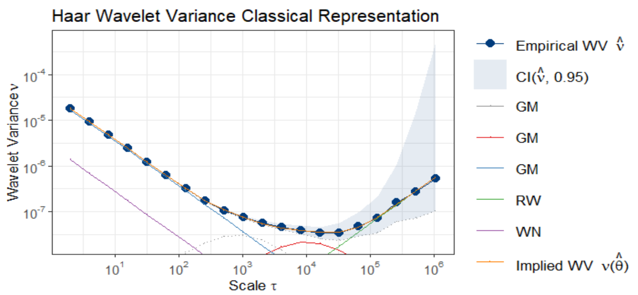

Fig. 12. The empirical vs implied WV for the selected model for Accel X as an example.

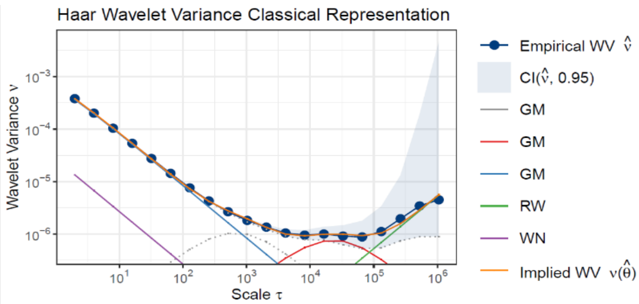

Fig. 13. The empirical vs implied WV for the selected model for Gyro X as an example.

Dynamic Data Collection:

After collecting the static data, dynamic in-field datasets are collected to compare between the models and associated parameters the parameters estimated using different stochastic approaches for the IMU undertest. The Spatial Advanced Navigation was installed on the roof of a moving vehicle with a laptop to record the datasets. A long straight line with multi turns was travelled in open sky conditions for 30 minutes dynamic with 200 Hz sampling rate for inertial sensors and 1 HZ sampling rate for GPS.

Testing and Validation using a Modified INS/GPS Navigation Algorithm

After choosing the best model from the static dataset, a modified loosely coupled INS/GPS integration navigation algorithm was proposed based on the selected best model from the GMWM. The modified navigation algorithm that consists of 39 states (see Fig. 14) based on the best model with the smallest value of WIC. The model structure has:

- Three 1st order GM processes
- A WN process
- A RW process

Thes, the model is presented as 3GM+RW+WN. The navigation solution obtained for the dynamic dataset with GPS signal outages from the modified 39-states integrated navigation algorithm is compared to the standard 15-states one, i.e., the navigation solution is evaluated separately from the two integrated navigation algorithms to compare the performance of each other. In this case, the navigation solution is evaluated from the INS only, the INS works as a stand-alone system, until the GPS signal is repurchased. Therefore, the accuracy of the navigation solution during GPS signal outages is completely relies on the quality of the INS sensor data after inertial sensor error models are implemented.

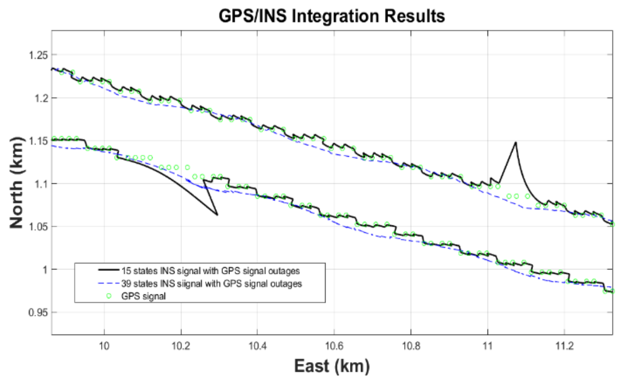

Fig. 15. The position error for the two navigation algorithms in 2D.

Tests results

Dynamic data was employed with the standard navigation algorithm that consists of 15-states based on the 1st order GM process with intended GPS outages at different periods, then the same data is used with the modified navigation algorithm that consists of 39-states based on the model selected by the GMWM (3*GM+RW+WN) with the same GPS outages at the same periods used for the standard algorithm. The position errors are computed by subtracting the INS/GPS solution that contains the GPS signal outage from the reference solution, GPS data. Then, the magnitude of the position errors during the selected GPS signal outages was computed for each algorithm, individually as found in Table 6. As shown from Fig. 15, the position errors from the modified navigation algorithm are less than that from the standard navigation algorithm. This means that the quality of the INS-based navigation solution (when the INS is used as a standalone system during GPS signal outage) is improved by using the modified 39-states integrated navigation algorithm associated with the stochastic noise parameters estimated using the GMWM.

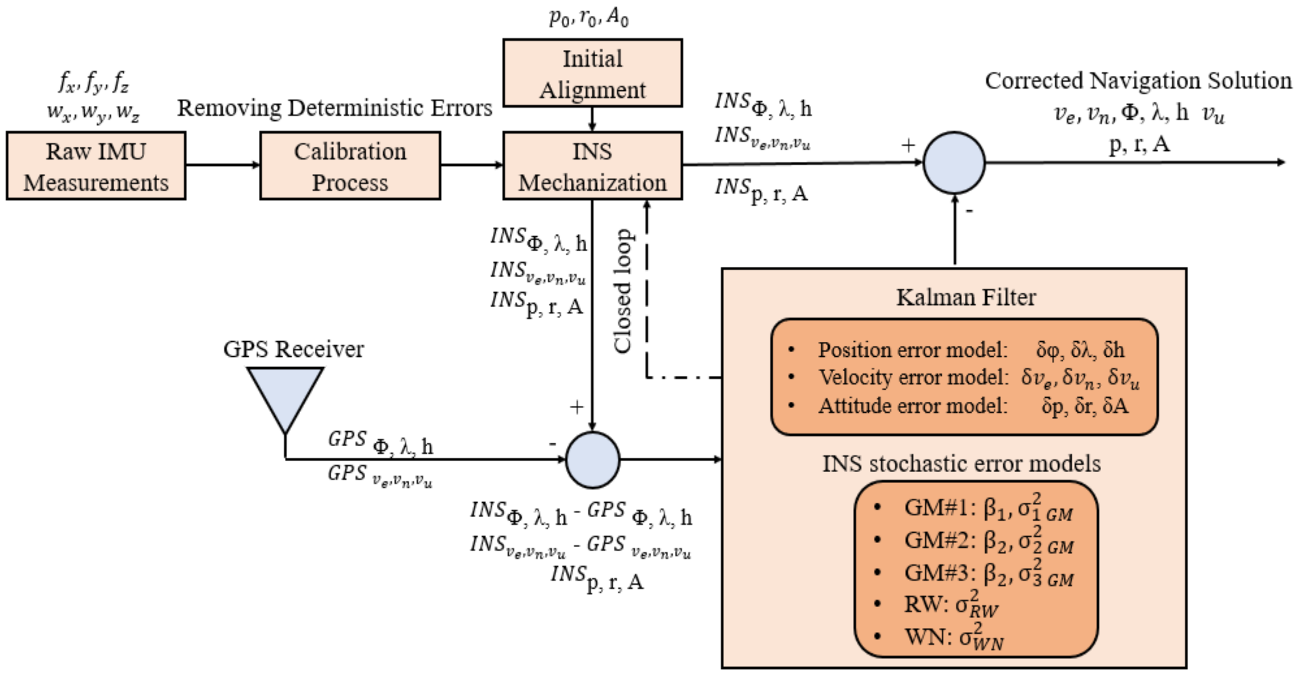

Fig. 14. Extended 39-stated loosely coupled INS/GPS integration using Kalman Filter.

Table 6. Magnitude of the position error for the two algorithms during GPS signal outage periods.

Model	Outage 1 (m)	Outage 2 (m)	Outage 3 (m)
The modified algorithm (39 states)	55	11	33
The standard algorithm (15 states)	138	19	85

Table 7. accelerometer biases and gyro drifts estimation results.

sensor	**Accelerometer Bias** (m/s/s)
Accel X	-0.0264
Accel Y	-0.0091
Accel Z	0.0133
sensor	**Gyroscope Drift** (degree/sec)
Gyro X	-0.0024
Gyro Y	0.00101
Gyro Z	0.001139

Conclusion

The target of this paper is to improve the positioning accuracy of INS/GPS integrated systems using a modified navigation algorithm. First, calibration methods are used to calculate the biases for the accelerometers and gyros, six position static test method and rate test method are used as found in Table 7. Second, static and dynamic datasets were collected under specific conditions. Then, three different tools, namely the ACF (as a standard common noise characterization approach), AV and GMWM, were used for modelling the inertial sensors stochastic errors. The GMWM presents higher efficiency in terms of the characterization of the stochastic error terms for inertial sensors measurements than the ACF and AV. The ACF and AV tools could not identify the outliers in the datasets which could lead to incorrect modelling for the stochastic error part in addition to their individual demerits. The GMWM defeated this demerit by comparing the classical WV with a robust one to check for outliers through a contamination test. The WIC value, which concessions the complexity and goodness-of-fit of each model, was calculated for forty-four models and the model structured as 3*GM+RW+WN was found to have the smallest value of WIC for accelerometers and gyros.

Based on the selected model, a modified integrated navigation algorithm was proposed that consists of 39-states. Comparison between the modified algorithm and the standard 12-states algorithm using in-field dynamic dataset was obtained during intended GPS signal outages. Results highlighted that a single 1st order GM process is not practical for modelling the stochastic error part of the inertial sensors, especially for low-cost sensors. Thus, to enhance the accuracy navigation solution, more stochastic error processes are needed for modelling the inertial sensor stochastic errors in the INS/GPS integration algorithms. It is clarified that the drift in the INS signal in the modified algorithm is reduced by significant values than the standard one. So, the modified navigation algorithm leads to more robust navigation solution.

Future work will cover:

- Using Artificial intelligence, machine learning and deep learning for analysis and modelling the stochastic error for low-cost MEMS-based IMUs.
- Studying the temperature effect on the stochastic error modelling of inertial sensors.

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