**Yahia JAZYAH**[1]

Faculty of Computing Studies, Arab Open University, Kuwait (1)
yahia@aou.edu.kw
ORCID: 1. 0000-0001-9874-1306

doi:10.15199/48.2024.08.34

# Characterization and Modelling of Carrier Frequency Offset in OFDM Systems under Additive White Gaussian Noise

***Abstract**. Carrier Frequency Offset (CFO) is a critical factor affecting the synchronization accuracy in Orthogonal Frequency Division Multiplexing (OFDM), and understanding its effects and developing effective mitigation strategies are crucial for achieving robust communication systems. This research investigates the model and impact of subcarrier variance on the normalized CFO density, varying length of Cyclic Prefix (CP), and the Maximum Likelihood (ML) estimation of time and frequency offset in OFDM system in the presence of Additive White Gaussian Noise (AWGN).*

***Streszczenie.** Przesunięcie częstotliwości nośnej (CFO) jest krytycznym czynnikiem wpływającym na dokładność synchronizacji w multipleksowaniu z ortogonalnym podziałem częstotliwości (OFDM), a zrozumienie jego skutków i opracowanie skutecznych strategii łagodzenia mają kluczowe znaczenie dla uzyskania solidnych systemów komunikacyjnych. W niniejszym badaniu badano model wpływu wariancji podnośnej na znormalizowaną gęstość CFO oraz zmienną długość estymacji prefiksu cyklicznego (CP) i maksymalnej wiarygodności (ML) przesunięcia czasu i częstotliwości w systemie OFDM w obecności addytywnego białego szumu Gaussa (* AWGN). **(Charakterystyka i modelowanie przesunięcia częstotliwości nośnej w systemach OFDM w warunkach addytywnego białego szumu gaussowskiego**)

**Keywords:** AWGN, CFO, Maximum Likelihood, OFDM.
**Słowa kluczowe:** AWGN, CFO, Maksymalne prawdopodobieństwo, OFDM.

## Introduction

OFDM is a modulation scheme that has gained widespread adoption in modern communication systems due to its efficiency in addressing challenges related to multipath fading, spectrum utilization, and high data rate transmission.

In principle, OFDM divides the available spectrum into multiple orthogonal subcarriers, each carrying a unique data stream. These subcarriers are closely spaced and are orthogonal to each other. The key advantage of OFDM is the orthogonality between subcarriers, which allows for simultaneous transmission of data streams without interference.

At the transmitter, data symbols are modulated onto the subcarriers, and an Inverse Fast Fourier Transform (IFFT) is applied to convert them from the frequency domain to the time domain. Whereas at the receiver, the received signal undergoes a Fast Fourier Transform (FFT) to recover the original data symbols.

OFDM has several applications in modern communication system, including Wireless Local Area Networks (WLAN) standards, such as IEEE 802.11a, 802.11g, 802.11n, and 802.11ac. It provides high data rates, robustness against multipath fading, and improved spectral efficiency. Digital audio and video broadcasting standards, such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB-T, DVB-S, DVB-C), and the emerging Next-Gen TV (ATSC 3.0) standard. Cellular Communications such as Long-Term Evolution (LTE) and its successor, 5G New Radio (NR). Broadband access including Digital Subscriber Line (DSL), where it is used in standards like Asymmetric DSL (ADSL) and Very High-Speed DSL (VDSL). Powerline Communication (PLC). And in Wireless Metropolitan Area Networks (WMAN), Wireless Personal Area Networks (WPAN) standards, and Worldwide Interoperability for Microwave Access (WiMAX).

OFDM provides several advantages. It is highly resistant to multipath fading, making it suitable for wireless communication in environments with reflections and signal scattering, it enables efficient use of available spectrum by packing multiple subcarriers closely together, providing high spectral efficiency, it allows for flexible resource allocation, which enables efficient communication in both time and frequency domains, and it minimizes Interference between subcarriers due to their orthogonality, and so it enhances the system's robustness against various forms of interference.

Cyclic Prefix (CP) is a guard interval appended to each OFDM symbol. It consists of a copy of the end portion of the symbol and is used to combat multipath interference. While Signal to Noise Ratio (SNR) is a key parameter in determining the quality of communication in any system. OFDM, CP, and SNR are interconnected aspects of communication systems. The use of CP in OFDM systems helps combat the effects of multipath propagation, and higher SNR contributes to the robustness and overall performance of the system. The choice of CP length and system parameters depends on the characteristics of the communication channel and the desired trade-offs between efficiency and robustness.

This work studies the effect of the number of subcarriers on the CFO, the effect of SNR and CP on ML estimation of time and frequency offset in OFDM systems. And modelling those effects to help in the development of robust synchronization mechanisms.

The remaining of this article is summarized as follow: part 2 presents OFDM performance, part 3 presents the CFO, part 4 presents literature review, part 5 presents the simulation results and discussion, and part 6 is the conclusion.

## Synchronization for Optimal OFDM Performance

Accurate synchronization [1] is of paramount importance for the optimal performance of OFDM systems. OFDM relies on precise timing and frequency synchronization to maintain orthogonality between subcarriers and to mitigate the impact of channel impairments.

The fundamental principle of OFDM is the orthogonality between subcarriers. Accurate frequency synchronization ensures that the subcarriers remain orthogonal, minimizing inter-carrier interference (ICI) and allowing simultaneous transmission of multiple data streams. OFDM symbols are transmitted with a Cyclic Prefix (CP) to combat multipath fading. Precise timing synchronization is critical to preserve time orthogonality and avoid inter-symbol interference (ISI).

In wireless communication, signals often encounter reflections and delays due to multipath propagation.

Accurate synchronization helps in mitigating the effects of multipath fading and frequency offset and doppler shifts.

OFDM provides efficient spectrum utilization by minimizing guard intervals and reducing CP overhead.

**Carrier Frequency Offset (CFO)**

CFO is a significant synchronization impairment that can have a profound impact on the performance of communication systems, particularly those employing OFDM.

CFO refers to the difference between the carrier frequency of the received signal and the carrier frequency expected at the receiver. It results from imperfections in the local oscillators at the transmitter and receiver. In OFDM, CFO causes a phase rotation on individual subcarriers, disrupting the orthogonality between them. This can lead to ICI and degradation of the system's performance.

What causes CFO is the imperfections in the oscillators at the transmitter and receiver; which arises from manufacturing tolerances, temperature variations, or aging effects. Doppler shifts, for example at high-speed vehicular communication. And frequency drifts over time or random walks

CFO has several impacts on OFDM Systems. It introduces a phase rotation on each subcarrier, causing the received signal to be rotated with respect to the expected signal, the phase rotation results in interference between adjacent subcarriers, leading to ICI, this degrades the orthogonality between subcarriers, impacting the system's ability to transmit and receive data simultaneously. And causes spreading of the symbols across multiple subcarriers, making it challenging to correctly demodulate and decode the transmitted information. Which make it important to mitigate CFO impact to prevent ICI, improve symbol recovery, and enhance system performance.

**a) CFO Estimation and Compensation Techniques**

CFO estimation and compensation techniques [1] [2] [3] are essential for maintaining the performance of communication systems, particularly in OFDM systems. The CFO ($f_{CFO}$) in hertz is given by Equation 1.

(1) $$f_{CFO} = \frac{\Delta\phi}{2\pi T}$$

where: T - symbol period, Δϕ - phase difference between the received signal and the local oscillator at the receiver.

ML estimation is one of the techniques used in signal processing to estimate parameters based on the likelihood function. In the context of OFDM systems, ML estimation can be applied to estimate the timing offset and CFO. Here's a brief overview of ML estimation for timing and CFO in OFDM:

CFO estimation involves determining the frequency offset between the transmitted and received signals. ML estimation for CFO typically involves searching for the frequency offset that maximizes the likelihood function.

The ML estimate of the CFO can be expressed using equation 2.

(2) $$f_{CFO} = argmax f_{CFO}\ Likelihood\ Function$$

The likelihood function is often based on the correlation between the received signal and the expected signal at different frequency offsets.

Another technique is correlation-based time offset estimation. The basic idea is to correlate the received signal with its delayed version at different time offsets and find the offset that maximizes the correlation.

The correlation function R(τ) for time offset τ is given by equation 3.

(3) $$\mathrm{R}(\tau) = \sum_{n=0}^{N-1} r(n).x(n-\tau)$$

where: r(n) - received signal, x(n) - known transmitted signal, τ - time offset, N - length of the correlation.

This involves searching for the time offset that maximizes the absolute value of the correlation function.

Pilot-based time offset estimation is another method where pilot symbols with known values are often inserted into the transmitted signal. The receiver can correlate the received pilot symbols with their expected values to estimate the time offset. See equation 4.

(4) $$\mathrm{T}\tau = \mathrm{argmax}_{\tau} \sum_{Pilot\ locations} r(n).x(n-\tau)$$

In OFDM systems, pilot symbols with predefined values, are often inserted into the transmitted signal that are used at the receiver to estimate the frequency offset.

The basic idea is to correlate the received pilot symbols with their expected values at different frequency offsets and find the offset that maximizes the correlation. The correlation function for frequency offset $f_{CFO}$ is given by equation 5.

(5) $$\mathrm{R}(\mathrm{f}_{\mathrm{CFO}}) = \sum_{Pilot\ locations} r(n).x(n).e^{-2\pi \mathrm{f}_{\mathrm{CFO}} n}$$

The frequency offset $f_{CFO}$ can be estimated using equation 6.

(6) $$f_{CFO} = argmax f_{CFO}\ |\mathrm{R}(f_{CFO})\ |$$

CFO estimation and compensation techniques are essential for maintaining the performance of communication systems, particularly in OFDM systems. Table 1 summarizes the common CFO estimation and compensation techniques.

**Literature Review**

This section provides insights into the challenges, advancements, and various techniques proposed to mitigate the impact of CFO.

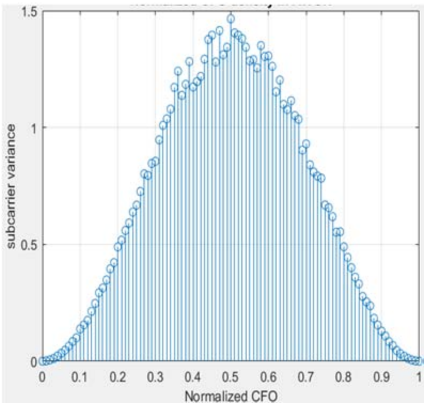
Salari and Chan in [4] proposed sparse Bayesian learning (SBL) scheme to iteratively estimate the CFO, channel impulse response (CIR), and variance of the noise jointly, using the EM algorithm, the proposed scheme outperforms the existing methods. Dorokhin in [5] investigated the Index Modulation-assisted Spread Spectrum (ISS-OFDM) synchronization issues. Primarily, the impact of sampling clock (SCO) and CFO was studied analytically. Van Linh et. al. in [6] proposed a pilot-based CFO estimation and compensation algorithm that addressed the CFO issue in OFDM wireless communication systems. Ribeiro and Gameiro in [7] propose an algorithm that explores the time-domain (TD) properties of transmitted symbols which is independent of the channel estimation and decision and presenting a very low computational load. Yan et. al. in [8] proposes a low-complexity residual carrier frequency offset compensation (RCFOC) approach based on spectrum symmetry for coherent optical OFDM systems. Numerical results verifies that high accuracy RCFO estimation is achieved by with optimal parameters of moderate values. Kalbat et. al. in [9] considered and evaluated the performance evaluation of precoded OFDM (P-OFDM) in the presence of CFO in terms of the signal-to-interference plus-noise-ratio (SINR) and BER, simulation results showed that P-OFDM is substantially more sensitive to CFO compared with conventional OFDM. Kumarapandian and Reena in [10] showed that CP can be exploited to mitigate or remove

Table 1. CFO estimation and compensation techniques

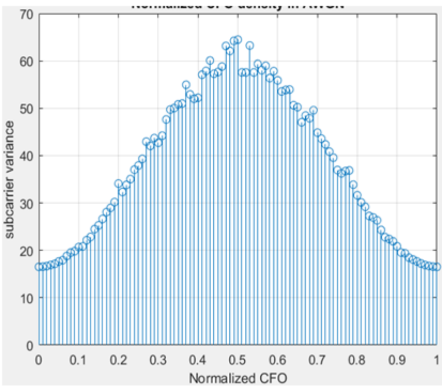
| Algorithm | Example | Principle | Estimation | Compensation |
|---|---|---|---|---|
| Pilot-Based | Pilot Symbols | pilot symbols, are inserted into the transmitted signal. | The receiver correlates the received signal with the known pilot symbols to estimate the CFO. | The estimated CFO is used to compensate for the carrier frequency offset during signal demodulation |
| | Data-Aided | Combines data symbols with known pilot symbols | Utilizes both data and pilot symbols to improve the accuracy of CFO estimation | The estimated CFO is applied to compensate for the frequency offset during demodulation. |
| Time-Domain CFO | CP Exploitation | Exploits the CP in OFDM symbols. | Correlates the CP of the received signal | Adjusts the received signal based on the estimated CFO |
| | Schmidl-Cox | Utilizes the autocorrelation properties of the CP | Applies a sliding correlator | Adjusts the received signal based on the estimated CFO |
| Frequency-Domain CFO | Maximum Likelihood (ML) | Finds the CFO value that maximizes the likelihood of the received signal. | Iteratively searches for the CFO that maximizes the likelihood function | Applies the estimated CFO to correct the frequency offset. |
| | Subcarrier Interpolation | Interpolates the frequency response of subcarriers | Analyses the frequency domain characteristics of subcarriers. | Adjusts the received signal based on the estimated CFO |
| Blind CFO | Cyclic Feature Detection | Explores cyclostationary properties of the received signal. | Utilizes cyclic features to estimate the CFO without known pilot symbols | Adjusts the received signal based on the estimated CFO. |
| | Second-Order Statistics | Analyses the second-order statistics of the received signal. | Analyses the second-order statistics of the received signal. | Adjusts the received signal based on the estimated CFO |
| Joint CFO and Channel Estimation | Time-Frequency Domain | Simultaneously estimates CFO and channel parameters. | Jointly analyses the time and frequency characteristics of the received signal | Adjusts for both CFO and channel effects. |
| Iterative Algorithms | Expectation-Maximization (EM) | An iterative optimization algorithm | Iteratively refines estimates by maximizing the expected log-likelihood. | Adjusts the received signal based on refined estimates |

completely the CFO. Kaur and Kumar in [11] investigated the offset effect on conventional and Discrete Wavelet Transform (DWT) OFDM under different communication channels (AWGN and Rayleigh). Jiang et. al. in [12] investigated the effects of CFO on Offset spatial modulation (OSM) OFDM systems over multipath Rayleigh fading channel. Aziz at. al. in [13] analyzed CFO and its effect on OFDM and describe techniques (time and frequency) to estimate CFO.

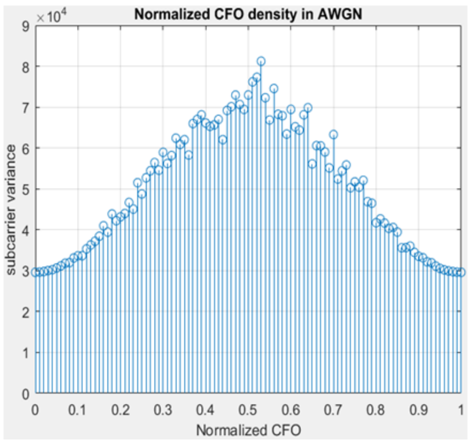
a)




b)




c)




d)

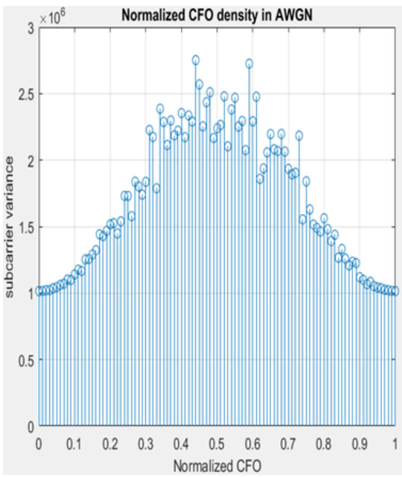



Fig. 1. Normalized CFO density in AWGN when used data subcarriers (a) 1024, (b) 512, (c) 128, and (d) 64

Table 2. Effect of Used Data Subcarriers on Min., Max., and SNR of CFO

| used data subcarriers | Max. subcarrier variance | Min. subcarrier variance | SNR (dB) |
|---|---|---|---|
| 1024 | 1.5 | 0 | 9.932666 |
| 512 | 65 | 17 | 6.922366 |
| 256 | 2300 | 750 | 3.912066 |
| 128 | 82000 | 30000 | 0.901766 |
| 64 | 2550000 | 1000000 | -2.10853 |

**Simulation Results and Discussion**

This section study the relation between CFO and data subcarriers, SNR, and CP. Matlab tool is used to simulate and test the relation between CFO and other metrics.

Fig. 1. represents Normalized CFO density in AWGN when used data subcarriers are 1024, 512, 128, and 64 respectively, with CP is 16, and SNR is 10dB, the figure shows that decreasing the data subcarriers number, decreasing the subcarrier variance, this phenomenon is related to the spreading of the available energy across the subcarriers.

In OFDM, data is transmitted over multiple subcarriers, and each one is operating at a different frequency. The total available transmitted power is distributed across these subcarriers.

The power per subcarrier is inversely proportional to the number of subcarriers. As the number of data subcarriers decrease, the power per subcarrier increases, concentrating more energy on each individual subcarrier.

The variance of a signal is related to its power. If total power is fixed and distribute across fewer subcarriers, each subcarrier carries more power, and resulting in lower variance based on equation 7.

(7) $$\sigma^2 = E[(x-\mu)^2]$$

where: σ2 is the variance, E is the expected value, μ is the mean of the random variable.

The CFO density of the k-th subcarrier fΔfk(x)) is given in equation 8:

(8) $$f_{\Delta\mathrm{fk}}(\mathrm{x}) = \frac{1}{\sqrt{2\pi\sigma_\mathrm{k}^2}} * e - \frac{x^2}{2\sigma_\mathrm{k}^2}$$

where: x is the CFO value on the k-th subcarrier.

Table 2 presents the Effect of used data subcarriers on Min., Max. and SNR of CFO.

Fig. 2 represents the previous data visually.

Fig. 3 studies the effect of SNR on ML estimation of time and frequency offset in OFDM systems with CP =128, SNR = 15dB, and used data subcarriers is 1024.

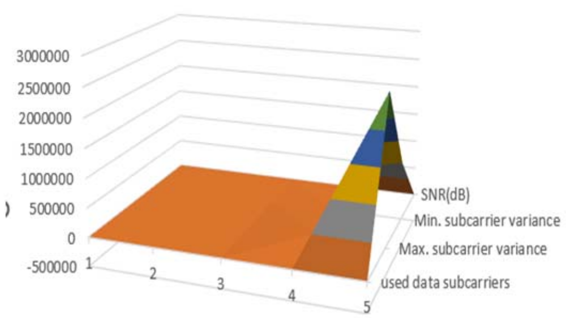



Fig. 2. Effect of used data subcarriers on Min., Max. and SNR of CFO

a)
b)
c)

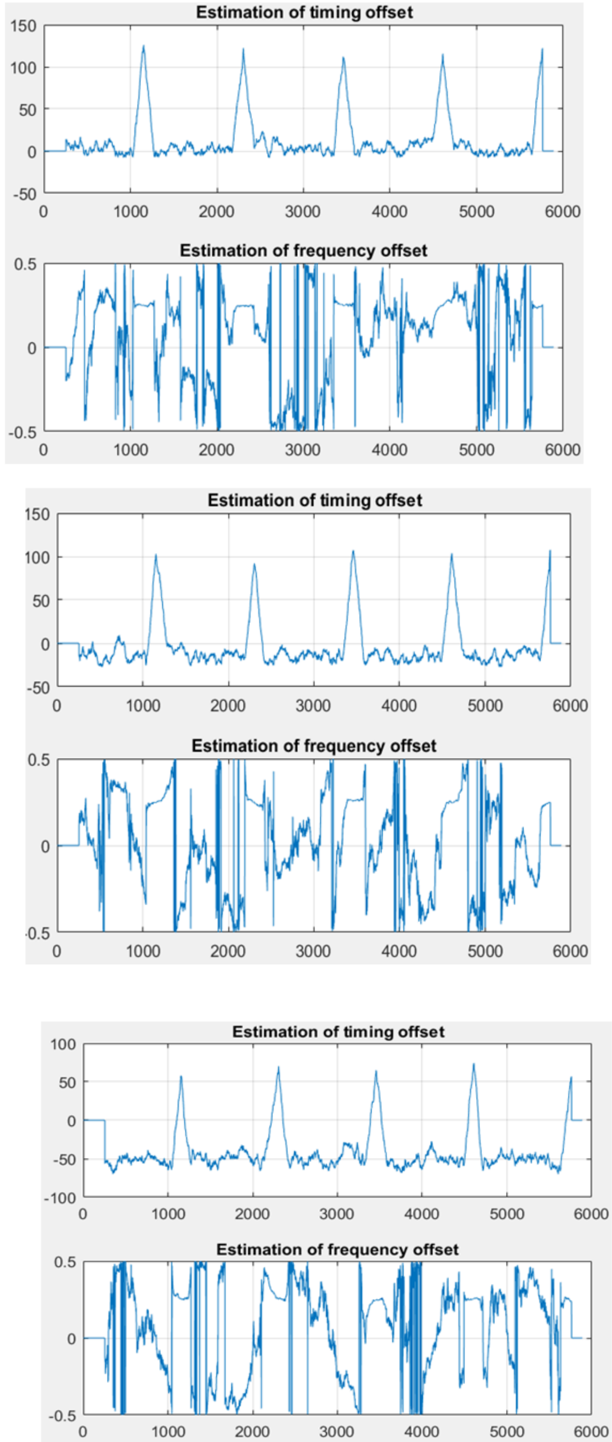



Fig. 3. ML estimation of time and frequency offset in OFDM systems, when SNR in dB (a) 15, (b) 10, (c) 6, and (d) 3

SNR influences the performance of ML estimation of time and frequency offsets in OFDM, where higher SNR leads to more accurate ML estimation. Because when SNR increases, the signal becomes more distinguishable from the noise, making it easier to identify the correct time and frequency offsets. Which results in less robust of ML estimation in the presence of high noise levels.

Higher SNR results in faster convergence of ML estimation algorithms, and requires fewer iterations to reach accurate estimates. And increases the likelihood of making correct decisions.

By and large, performance is depending on the specific requirements of the communication system, it is a trade-off between achieving higher data rates and maintaining reliable synchronization.

The Cramer-Rao Lower Bound (CRLB) is used to analyze the effect of SNR on ML estimation of time and frequency offsets in OFDM systems as a theoretical measure. The CRLB provides a lower limit on the variance of any unbiased estimator, indicating the best achievable precision for parameter estimation. When SNR increases, the noise in the system decreases relative to the signal, leading to a decrease in the CRLB. Lower CRLB values indicate that the ML estimator can achieve lower variances, implying better estimation precision. Achieving CRLB requires an unbiased estimator and an accurate ML model.

CRLB for the variance of an unbiased estimator is given by equation 9.

(9) $$\sigma^2(\theta_{\mathrm{ML}}) >= \frac{1}{r(\theta)}$$

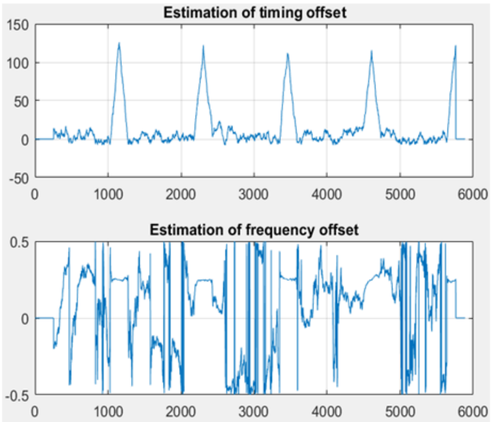
where: θ is a parameter which could represent time or frequency offset, θML is ML estimator, r(θ) is the Fisher Information represented in equation 10.

(10) $$r(\theta) = -\mathrm{E}(\,\partial^2 \ln\left(\frac{f(x|\theta)}{\partial\theta^2}\right)$$

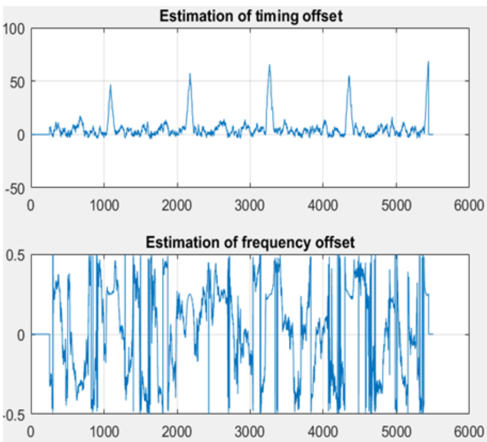
where: f(x|θ) is the probability density function (pdf) of the observations x given the parameter θ, ∂ is the partial derivative.

Fig. 4 studies the effect of CP on ML estimation of time and frequency offset in OFDM systems, with SNR = 15dB, and used data subcarriers 1024.

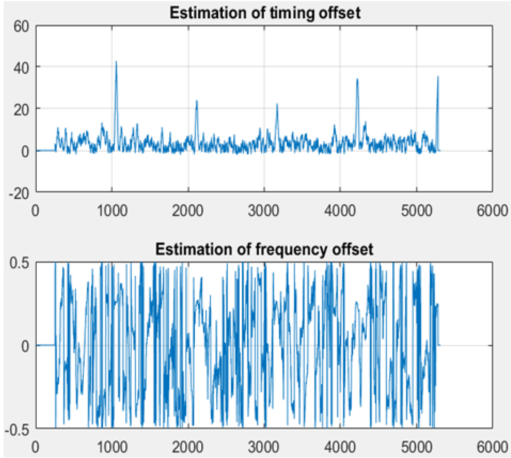
a)




b)




c)




d)

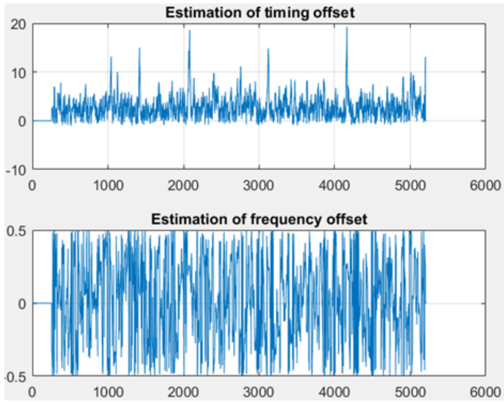



Fig. 4. ML estimation of time and frequency offset in OFDM systems, when CP (a) 128, (b) 64, (c) 32, and (d) 16

Longer cyclic prefix provides more protection against intersymbol interference and improves the robustness of ML estimation in the presence of multipath channels. Whereas, longer cyclic prefix reduces the efficiency of spectrum utilization.

The length of the CP is calculated by equation 11.

(11) $$\mathrm{L}(w) = \prod_{n=1}^{N+L}\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) e -^{|r(n)-s(n)^2|/2\sigma^2}$$

(12) $$w = \begin{cases} \tau, \; time\ offset\ estimation \\ f_{offset}, \; frequency\ offset\ estimation \end{cases}$$

where: L is the length of the CP, N is the number of subcarriers, S(n) is the transmitted signal.

CP in OFDM systems helps maintain the orthogonality between subcarriers and provides a known reference for CFO estimation. This is crucial for mitigating the impact of CFO and ensuring accurate demodulation in communication systems employing OFDM

## Conclusion

Understanding and modelling the impact of Normalized CFO density in AWGN for different subcarriers, varying length of CP are essential steps in the design and optimization of OFDM systems. This knowledge guides the development of robust synchronization mechanisms and contributes to the overall reliability and efficiency of communication systems in the presence of frequency offsets. While understanding the impact of Normalized CFO density in AWGN across different subcarriers is crucial for the design and optimization of OFDM systems.

## REFERENCES


[1] H. Zhou, A. V. Malipatil and Y. -F. Huang, "Synchronization Issues in OFDM Systems," *APCCAS 2006 - 2006 IEEE Asia Pacific Conference on Circuits and Systems*, Singapore, 2006, pp. 988-991, doi: 10.1109/APCCAS.2006.342228.

[2] D. V. Linh and V. V. Yem, "Carrier Frequency Offset Estimation and Compensation Technique for Massive MIMO-OFDM Wireless Communication System," *2022 International Conference on Electrical, Computer and Energy Technologies (ICECET)*, Prague, Czech Republic, 2022, pp. 1-5, doi: 10.1109/ICECET55527.2022.9872995.

[3] D. Van Linh, N. T. Anh, H. T. Phuong Thao and V. Van Yem, "Carrier Frequency Offset Estimation and Compensation Technique Based on Pilot Tones for OFDM Wireless Communication Systems," *2023 15th International Conference on Knowledge and Systems Engineering (KSE)*, Hanoi, Vietnam, 2023, pp. 1-5, doi: 10.1109/KSE59128.2023.10299480.

[4] S. Salari and F. Chan, "Joint CFO and Channel Estimation in OFDM Systems Using Sparse Bayesian Learning," in *IEEE Communications Letters*, vol. 25, no. 1, pp. 166-170, Jan. 2021, doi: 10.1109/LCOMM.2020.3024817.

[5] S. Dorokhin, "Joint SCO and CFO estimation algorithm for ISS-OFDM systems," *2020 International Conference Engineering and Telecommunication (En&T)*, Dolgoprudny, Russia, 2020, pp. 1-5, doi: 10.1109/EnT50437.2020.9431303.

[6] D. Van Linh, N. T. Anh, H. T. Phuong Thao and V. Van Yem, "Carrier Frequency Offset Estimation and Compensation Technique Based on Pilot Tones for OFDM Wireless Communication Systems," *2023 15th International Conference on Knowledge and Systems Engineering (KSE)*, Hanoi, Vietnam, 2023, pp. 1-5, doi: 10.1109/KSE59128.2023.10299480.

[7] C. Ribeiro and A. Gameiro, "Mitigating CFO in OFDM systems by exploring the symbol structure," *2008 IEEE International Symposium on Wireless Communication Systems*, Reykjavik, Iceland, 2008, pp. 42-46, doi: 10.1109/ISWCS.2008.4726014.

[8] Q. Yan, X. Hong and X. Hong, "Low-Complexity Residual Carrier Frequency Offset Mitigation Based on Spectrum Symmetry for CO-OFDM Systems," in *Journal of Lightwave Technology*, vol. 36, no. 22, pp. 5210-5217, 15 Nov.15, 2018, doi: 10.1109/JLT.2018.2870907.

[9] F. Kalbat, A. Al-Dweik, B. Sharif and G. K. Karagiannidis, "Performance Analysis of Precoded Wireless OFDM With Carrier Frequency Offset," in *IEEE Systems Journal*, vol. 14, no. 2, pp. 2237-2248, June 2020, doi: 10.1109/JSYST.2019.2922098.

[10] S. Kumarapandian and M. P. Reena, "Performance analysis of CFO mitigation algorithms in asynchronous Cooperative OFDM communication system," *International Conference on Sustainable Energy and Intelligent Systems (SEISCON 2011)*, Chennai, 2011, pp. 632-637, doi: 10.1049/cp.2011.0438.

[11] N. Kaur and N. Kumar "On Effects of Frequency Offset on Simulink Model of OFDM," *Wireless Pers Commun.* **115**, 2349–2361 (2020). https://doi.org/10.1007/s11277-020-07685-5

[12] T. Jiang, Y. Xiao, L. Dan, X. Ji, W. Tang and Y. Zhao, "Performance Analysis of OSM-OFDM Systems with Carrier Frequency Offset," *2020 IEEE 20th International Conference on Communication Technology (ICCT)*, Nanning, China, 2020, pp. 780-784, doi: 10.1109/ICCT50939.2020.9295759.

[13] W. Aziz, E. Ahmed, G. Abbas, S. Saleem, Q. Islam, "Performance analysis of carrier frequency offset (CFO) in OFDM using MATLAB," *Journal of Engineering (JOE)* 1, no. 1 (2012): 5-10.