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Continuous Power Flow Analysis in the Trigonometric Circle

***Abstract**. This paper delves into Continuous Power Flow (CPF) Analysis within the framework of the Trigonometric Circle, presenting a novel approach to visualize power system behavior. It introduces the idea of representing CPF program outcomes on the trigonometric circle, also known as the unit circle in trigonometry, offering a unique perspective on the evolving relationships between voltage magnitudes and angles as the continuation parameter changes. By utilizing this circle, the paper provides a clear depiction of system behavior, enabling the identification of critical points such as voltage stability limits and potential instability as the trajectory nears the circle's boundary. The paper concludes by suggesting the display of CPF results, particularly Vi = f(λi) and δi = f(λi), on the trigonometric circle, demonstrating its practical application in a 2-node network. This innovative visualization method enhances the understanding and analysis of power systems under various conditions.*

***Streszczenie.** W artykule zagłębiono się w analizę ciągłego przepływu mocy (CPF) w ramach koła trygonometrycznego, prezentując nowatorskie podejście do wizualizacji zachowania systemu elektroenergetycznego. Wprowadza ideę przedstawiania wyników programu CPF na okręgu trygonometrycznym, znanym również jako okrąg jednostkowy w trygonometrii, oferując unikalną perspektywę na ewoluujące relacje między wielkościami napięcia i kątami w miarę zmiany parametru kontynuacji. Wykorzystując ten okrąg, artykuł zapewnia przejrzysty obraz zachowania systemu, umożliwiając identyfikację punktów krytycznych, takich jak granice stabilności napięcia i potencjalna niestabilność, gdy trajektoria zbliża się do granicy okręgu. Artykuł kończy się propozycją przedstawienia wyników CPF, w szczególności Vi = f(λi) i δi = f(λi), na okręgu trygonometrycznym, demonstrując jego praktyczne zastosowanie w sieci 2-węzłowej. Ta innowacyjna metoda wizualizacji ułatwia zrozumienie i analizę systemów elektroenergetycznych w różnych warunkach. (**Ciągła analiza przepływu mocy w okręgu trygonometrycznym**)*

Keywords: Continuation Power Flow, Trigonometric Circle, Number of Iterations, Step Size factor, Voltage stability
Słowa kluczowe: Kontynuacja przepływu mocy, okrąg trygonometryczny, liczba iteracji, współczynnik wielkości kroku, stabilność napięcia

Introduction

In this study, we investigate the potential of utilizing the trigonometric circle to illustrate the outcomes of Continuation Power Flow (CPF) analysis. We explore how this graphical method improves comprehension of power system behavior as the continuation parameter fluctuates. By representing voltage angles and magnitudes on the trigonometric circle, our goal is to offer a lucid and intuitive visualization of voltage stability evolution.

The research presented herein [1] delves into CPF Analysis within the framework of the Trigonometric Circle, introducing an innovative visualization approach for understanding power system behavior. It advocates for the presentation of CPF results on the trigonometric circle, thereby enhancing analysis, particularly for a 2-node network scenario.

In reviewing the cited articles ([1]-[26]), several common themes and connections emerge, providing a comprehensive overview of related works in the field of Continuation Power Flow (CPF) analysis, voltage stability assessment, and power system visualization. By analyzing these references collectively, we can identify existing technical gaps and how the author's work aims to address them. [1] presents CPF results on the trigonometric circle, providing a graphical representation for voltage stability assessment. Articles [2] and [3] explore static CPF analysis. [2] examines CPF tangent vector evolution, aiding in voltage stability assessment by comparing results to mathematically derived tangents. Meanwhile, [3] focuses on correction step convergence in CPF iterations, introducing a divergence indicator for improved stability and efficiency.

Articles [4], [5], and [6] discuss CPF methodologies, prediction techniques, and parameterization methods. They explore predictors, correction steps, and parameterization techniques for effective CPF analysis. The author's work aims to enhance CPF analysis by proposing a novel visualization method within the Trigonometric Circle framework, which can aid in identifying critical points and potential voltage stability concerns.

In articles [7] and [8], the use of visualization tools such as the trigonometric circle in CPF analysis and the comparison between the Ratio and Unit Circle Methods underscore the importance of graphical representations and methodological considerations in understanding and solving power system problems efficiently.

Articles [9]-[18] delve into various voltage stability enhancement and assessment methods, including the integration of Static Var Compensators (SVCs), exploration of Flexible Alternating Current Transmission Systems (FACTS) equipment, and analysis of the impact of Unified Power Flow Controllers (UPFCs) on voltage stability. In line with this body of research, the author's work offers a novel visualization approach within the Continuation Power Flow (CPF) analysis framework, thereby contributing to the advancement of voltage stability assessment and analysis.

Articles [19]-[23] delve into instability detection techniques. They discuss factors contributing to voltage instability, voltage stability criteria, and methods for detecting loadability limits and system collapse. Additionally, they explore approaches to CPF analysis, reliability-centered maintenance planning, and power system stability assessment. While existing studies offer valuable insights into voltage stability and system reliability, there is a recognized need for more tailored visualization methods and integration approaches, particularly for CPF outcomes within the Trigonometric Circle framework. The proposed visualization approach within the CPF analysis framework, utilizing the Trigonometric Circle, presents CPF results clearly and intuitively, thereby enhancing voltage stability assessment and deepening understanding of power system behavior.

In this paper, we introduce a novel utilization of CPF Analysis within the Trigonometric Circle framework for power system visualization [1]. By representing CPF program outcomes on the unit circle in trigonometry, this approach offers a clear view of how voltage magnitudes and angles evolve with changes in the continuation parameter. This unique visualization method effectively illustrates system behavior, highlighting critical points such as voltage stability limits and potential instabilities near the circle's boundary. Through the depiction of Vi = f(λi) and δi = f(λi), our proposed approach enhances the understanding and analysis of power systems, as demonstrated in a 2-node network application [1].

In this study [2], the focus lies on analyzing the evolution of the tangent vector during CPF iterations. The research utilizes a 17-node network, which is modeled using MATLAB. The study encompasses formulating the node admittance matrix and generating equations for the network. Results obtained from CPF tangent vector calculations are compared with mathematically derived tangents, demonstrating agreement. Additionally, a chart is presented to visualize the PV curve and tangent vector evolution, effectively indicating stability limits. This chart serves as an additional voltage stability indicator, complementing the PV curve and enhancing system assessment.

In [3], the study focuses on analyzing the convergence behavior of correction steps in CPF iterations and proposes the development of a divergence indicator. The research introduces the concept of a "convergence area" to forecast convergence and the requirement for corrective measures. This innovative concept demonstrates effectiveness, especially during the crucial stage of CPF analysis, thereby enhancing system stability and efficiency.

In [4], the paper provides a concise overview of CPF methods, covering prediction, parameterization, correction, and step size determination. It explores linear and nonlinear predictors such as tangent, secant, and higher-order methods, highlighting the significance of arclength parameterization for improving prediction accuracy and reducing correction step iterations.

In [5], the CPF method integrates a predictor-corrector scheme, employing locally parameterized continuation techniques. This fusion enables the method to adeptly trace power flow solution paths, especially around turning points.

In [6], a method is introduced to trace power flow solutions from base load to steady-state voltage stability limit, known as CPF. This method stays well-conditioned near the critical point, preventing divergence even with single-precision computation. Intermediate results are used to generate a voltage stability index, identifying vulnerable system areas. The paper provides examples analyzing voltage stability under different load increase scenarios.

The use of the trigonometric circle provides a clear visualization of voltage magnitudes and angles throughout Continuation Power Flow (CPF) analysis. Each CPF result is plotted on the circle, with the x-coordinate representing the cosine of the angle and the y-coordinate representing the sine, allowing for a dynamic representation of angle variations. As the continuation parameter changes, these points form a trajectory on the circle, revealing the evolution of voltage angles over time. This approach facilitates the identification of critical points and provides valuable insights into potential voltage stability issues [7].

In [8], comparing the Ratio and Unit Circle Methods highlights their distinct approaches. The Ratio Method relies on element ratios for solutions, whereas the Unit Circle Method employs geometric interpretation of complex numbers. While the Ratio Method is effective for algebraic cases, the Unit Circle Method excels in phase and frequency analyses, with method selection depending on the problem context.

In [9], the study addresses the increasing demand for electrical power, which pushes power systems to operate near their critical limits, posing risks to the economy and the environment. The research focuses on enhancing voltage stability and load-ability margin by integrating a Static Var Compensator (SVC) through CPF analysis on an IEEE-14 bus system. Through this approach, weak buses are identified, and SVC implementation strategically improves system stability, resulting in an increased load-ability margin from 0.56 to 0.597.

In [10], the paper tackles issues with the conventional PV/PQ switching method used in Power Flow (PF) and CPF calculations, which can result in non-physical solutions and inaccurate load margin calculations. It presents two smooth functions and an active-set scheme, with the Type-I model providing faster convergence for physically feasible power flow solutions. The Type-II model improves CPF accuracy and speed. This approach, based on smooth-model-based active-set assistance, effectively delivers physical solutions and accurate load margin calculations.

The paper in [11] demonstrates that for each new fault location, a new impedance matrix can be obtained without the need for recalculating the matrix inversion. This can be achieved through a simple extension of the initial nodal impedance matrix, which is initially calculated for the input model of the network. The paper derives the necessary formulas for such an extension and presents a flowchart of the computational method.

In [12], the paper examines voltage instability and collapse in power systems and investigates the use of FACTS equipment to enhance stability, efficiency, reliability, and environmental benefits. However, determining the optimal placement of these devices remains challenging. The study compares Multi-Criteria Decision Making (MCDM) techniques for locating a Static Var Compensator (SVC) to improve voltage stability, employing PSAT in MATLAB with the IEEE 14-bus test system.

In [13], the paper investigates the impact of integrating the Unified Power Flow Controller (UPFC) into CPF analysis on voltage stability. The authors propose a UPFC load flow model and conduct simulations using the Power System Analysis Toolbox (PSAT) in MATLAB. They analyze how UPFC affects power system behavior, particularly loadability and system stability limits.

In [14], the paper presents the Adaptive Multi-Step Levenberg-Marquardt (AMSLM) method to address voltage instability in ill-conditioned power systems. The study introduces an AMSLM-based CPF method for assessing voltage stability in these challenging systems.

In [15], CPF is used to determine static voltage stability limits in power systems, ensuring stable operations. Simulation results show that the extended CPF method proposed provides more accurate assessments of static voltage stability, particularly in systems with high renewable power generation (RPG) penetration. The study also indicates that the voltage support capability of each node increases with the proportion coefficient.

In [16], detecting loadability limits is signaled by a decrease in Lambda. Voltage stability relies on boosting reactive power at a node to elevate node voltages. However, instability arises if this action lowers voltages in at least one node after raising reactive power, indicating a breach of the maximum power limit and a subsequent decrease in consumed power, a hallmark of voltage instability.

In [17], the paper highlights the reduction of voltage levels within the high-voltage grid as a factor associated with voltage instability. Additionally, the failure of large generating units is identified as another contributor to instability.

In [18], the voltage stability criterion dictates that in a given operational state of the electrical network, the voltage level at each node should rise as the injected reactive power increases at that node. Conversely, if increasing the injected reactive power at a node leads to a voltage decrease in at least one node within the grid, the grid is deemed voltage unstable.

In [19], the CPF problem is described as a power flow challenge with static and non-linear load flow equations, requiring iterative solutions. The specified values include loads (PL, QL) and active feeds (PG), while reactive feeds (QG) are considered potentially free variables, especially at PV nodes. The primary state variables are voltage magnitudes (V) and voltage angles (δ) for each network node.

In [20], a method is proposed to forecast cable section outage frequency, taking into account the increase in repair-joints. The approach entails analyzing joint number time series using aging models for cables and joints. When applied to real medium voltage cable network data, the method demonstrates potential for reducing outage frequency through maintenance measures.

In [21], a new three-stage outage and maintenance model is presented for reliability-centered preventive maintenance planning. Departing from traditional Markov theory limitations, this model enables individual outage and maintenance rates for each stage of system degradation. Its application provides detailed insights into aging dynamics, outages, and repair and maintenance activities, enhancing system reliability planning.

In [22], the paper tackles complexities in modern power systems, focusing on the load factor Lambda (λ) and its impact on stability. It also explores power balance's role in stability maintenance during CPF analysis. With renewable sources' integration causing active load variations, understanding and managing load fluctuations are crucial for grid reliability.

In [23], a simplified version of the Kessel/Glawitsch method is introduced, using elements from the admittance matrix instead of the nodal impedance matrix. This modification removes the requirement for matrix inversion and accounts for power flows from branches connected to the load node for equivalent load calculation. The proposed index, based on this refinement, shows superior early warning capabilities in power system stability assessment.

As detailed in [24], the power flow problem begins with a single-line diagram of the power system, providing essential input data for computer-based solutions, including bus, transmission, and transformer data. At each bus labeled "k," the paper emphasizes four key variables: Voltage magnitude (Vk), Phase angle (δk), Real power (Pk), and Reactive power (Qk). These variables collectively contribute to a comprehensive analysis of the power system's performance and behavior.

In [25], utilizing the complete voltage profile is crucial for assessing the proximity to system collapse. Practical measures for system safeguarding can be implemented based on insights from established PV and QV curves. Achieving this comprehensive voltage profile involves employing successive power flow solutions or continuation methods.

In [26], a schematic diagram outlining security assessment and the determination of security limits includes key elements: computing voltage stability limits, assessing voltage stability margin, employing the Singular Value Method, and predicting the maximum loading point.

This predictive step can leverage insights from a preceding load flow solution.

In [27], the authors explore load flow analysis in local microgrids with storage. They use the Matpower toolbox to analyze various operational scenarios, emphasizing the balancing of active and reactive power. Future studies may focus on optimizing power flow and diversifying energy sources.

In [28], the authors review the impact of grid-connected photovoltaic (PV) systems on voltage stability. They discuss analysis techniques and highlight the need for further research considering factors like PV system design and meteorological conditions to effectively address voltage instability.

In [29], the authors investigate iterative linear methods for solving power flow problems in power systems. They evaluate different techniques and suggest the Bi-conjugate Gradient Stabilized (BiCGStab) method as suitable for faster solving of large systems.

Finally, in [30], the author analyzes legislative support for prosumer energy within the electrical safety framework. They examine how the development of renewable energy sources impacts grid operations and aim to identify adjustments needed to encourage investment while ensuring energy network security.

The remainder of the paper is structured as follows: The Methods section introduces an overview of CPF and its significance. Within this section, we explore two key aspects: one focusing on CPF Analysis and the other delving into the Utilization of the Trigonometric Circle in CPF Analysis. The Results section presents the findings of our study, which are divided into two parts: one covering the Calculation of CPF in a Network with 2 Nodes, and the other detailing the Results on the Trigonometric Circle for a 2-Node Network. Finally, the paper concludes with the Conclusions section.

Methods: continuous power flow analysis and utilization of the trigonometric circle in CPF analysis.

Continuation power flow (CPF) analysis

CPF is a technique employed to analyze power system behavior under varying parameter conditions, aiding in the understanding of voltage stability and overall system performance. The CPF procedure entails iterative adjustments of active and reactive loads, akin to load flow calculations for determining voltage magnitudes and angles. Initially, the lambda tangent vector (dLambda) guides the curve's positions, transitioning from positive to zero at the critical point and then to negative. The component with the largest tangent vector serves as the initial continuation parameter, recalculated at each step. As the stability limit approaches, voltage changes take precedence, leading to a shift in the continuation parameter between Lambda and voltage magnitude (V) based on their respective rates of change. Once dλ turns negative beyond the critical point, the continuation parameter switches back from V to λ when its absolute value exceeds one [1].

Formulation of "Power Flow" equations.

After incorporating the lambda parameter, the nonlinear equation system undergoes transformation, as illustrated in fig 2. The initial step in the predictor phase involves computing the tangent vector component.

(1) $$F(\_v, \delta, \lambda) = 0$$

(2) $$tk = \pm 1$$

(3) $$\begin{bmatrix} \underline{F\delta}, \underline{FV}, \underline{F\lambda} \\ ek \end{bmatrix} \left[\underline{t} \right] = \begin{bmatrix} 0 \\ \pm 1 \end{bmatrix}$$

Solving equation (3) yields the tangent vector component [t]. The vector Ek contains all elements as 0, except for the kth element. During CPF iterations, if the continuation parameter increases, tk is assigned +1; otherwise, it's set to -1.

After the predictor step, the next phase is the corrector step. The estimated values from the predictor step need correction. The equation set, represented as F(v, δ, λ) = 0, is expanded with an additional equation to determine the status of the variable serving as the continuation parameter. This augmentation involves the Jacobian matrix (Jaug).

(4) $$\begin{bmatrix} F(v,\delta,\lambda) \\ . \\ X_k - it_{ip},k \end{bmatrix} = [0]$$

(5) $$X_k = it_p,k$$

Xk represents the variable, Lambda or V, selected as the continuation parameter.

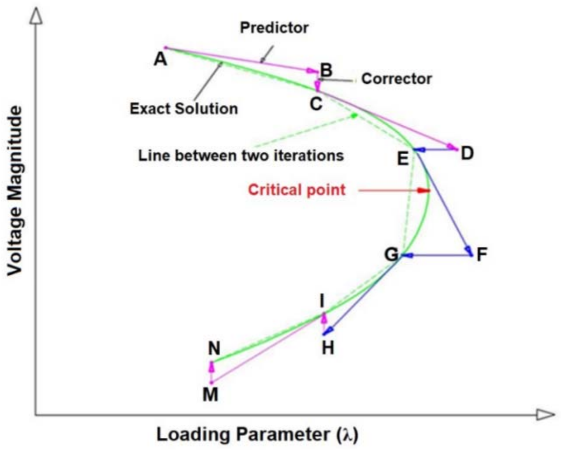

Fig. 1. PV curve illustrating the predictor-corrector scheme in power flow analysis [1].

itp,k- signifies the estimated value of X_k following the predictor step. Here, i corresponds to the CPF iteration number. The solution relies on the Newton-Raphson method [2], [3].

Utilization the Trigonometric Circle in CPF Analysis

The trigonometric circle, with a radius of 1 unit, is a fundamental concept in trigonometry and is centered at the origin of a coordinate system. Points on the trigonometric circle are represented by coordinates (x, y), where x corresponds to the cosine of the angle, and y corresponds to the sine of the angle. The angle is measured counterclockwise from the positive x-axis. The equation of the trigonometric circle is x^2 + y^2 = 1, which describes the relationship between coordinates (x, y) on the circle. Angles on the trigonometric circle can be measured in degrees (°) or radians (rad), with a full revolution corresponding to 2π radians or 360°.

The circle's visual representation aids in understanding complex relationships between voltage magnitudes and angles. Integrating CPF results with the trigonometric circle offers a unique approach to visualize voltage stability and power system behavior.

In the context of CPF analysis, the trigonometric circle provides an innovative platform to present the obtained results. Voltage angles are directly linked to angles on the trigonometric circle, while voltage magnitudes correspond to coordinates along the circle's circumference. By plotting the calculated voltage angles and magnitudes on the trigonometric circle, a visual representation of the power system's behavior is created. This approach facilitates the identification of critical voltage stability points, allowing for the quick interpretation of the impact of parameter variations.

Moreover, the trigonometric circle aids in comprehending the effects of changing parameters on voltage stability in a more intuitive manner.

The trigonometric circle provides a graphical representation that helps convey the changes in voltage magnitudes and angles as the continuation parameter is varied.

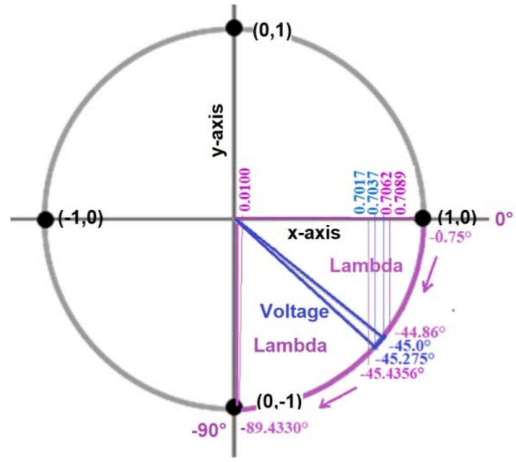

Fig. 2 Trigonometric Circle in CPF Analysis

Here's how the results of CPF can be presented on the trigonometric circle:

For each step of the CPF analysis, you can represent the voltage magnitudes and angles of buses on the trigonometric circle. The x-coordinate of a point on the circle corresponds to the cosine of the angle, and the y-coordinate corresponds to the sine of the angle. This allows for visualizing how the voltage angles change as the continuation parameter varies.

As the continuation parameter is gradually changed, the operating point of the network moves along a path on the trigonometric circle. Plotting these points sequentially on the circle creates a trajectory that shows the evolution of voltage angles as the parameter changes. This trajectory can help identify critical points where voltage stability might be compromised [1], [**7**].

Results and discussion: Analysis of CPF in a Network and Presentation of Results using Trigonometric Circle. Calculation of CPF in a Network with 2 Nodes

Critical points, such as voltage collapse or limit violations, can be easily identified on the trigonometric circle. Overlaying multiple trajectories on the same trigonometric circle can facilitate the comparison of different cases or scenarios. Overall, using the trigonometric circle to present CPF results provides an intuitive and insightful way to understand how voltage angles evolve in a power system as the operating conditions change. It allows for quickly identifying voltage stability concerns and making informed decisions to ensure the secure and reliable operation of the network.

CPF analysis is applied to a network with 2 nodes, investigating the effects of parameter variations on voltage stability.

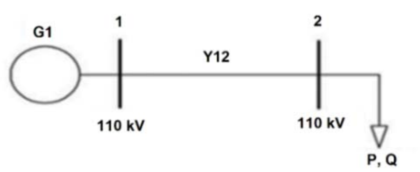

Fig. 3. 2-Node Network

Predictor and Corrector for λ Continuation Parameter

Including voltage angle δ2, voltage magnitude V2, and load factor λ.

δ_2=0.01; V_2=1.004 λ=0.3;

$$Jaug=\begin{vmatrix} \frac{\partial f1}{\partial \delta 2} & \frac{\partial f1}{\partial V2} & PoK \\ \frac{\partial f2}{\partial \delta 2} & \frac{\partial f2}{\partial V2} & QoK \\ 0 & 0 & 1 \end{vmatrix} \quad d\lambda=1 \quad ek=[0\ 0\ 1]$$

In the vector ek, element k is not equal to zero. Specifically, when k equals 3, it corresponds to the parameter dλ, representing the tangent vector component.

$$t=inv\,(Jaug)*\begin{vmatrix}0\\0\\1\end{vmatrix} \Rightarrow t=\begin{vmatrix}-0.0100\\0.0001\\1.0000\end{vmatrix}$$

For the increment factor, τ, the value is set to 0.3

$$it_{1p}=ito+\tau{*}t \quad \Rightarrow \quad it_{1p}=\begin{vmatrix}0.010\\1.004\\0.300\end{vmatrix}$$

Corrector Step for the λ Continuation Parameter:

δ_2=0.01; V_2=1.004 λ=0.3; → CONST

$$Jaug=\begin{vmatrix} \frac{\partial f1}{\partial \delta 2} & \frac{\partial f1}{\partial V2} & PoK \\ \frac{\partial f2}{\partial \delta 2} & \frac{\partial f2}{\partial V2} & QoK \\ 0 & 0 & 1 \end{vmatrix} \quad d\lambda=1 \quad ek=[0\ 0\ 1]$$

As long as the λ-continuation parameter is accepted, during corrector steps, the λ column and the ek vector are eliminated.

$$\boldsymbol{Jo}=\begin{vmatrix} \frac{\partial f1}{\partial \delta 2} & \frac{\partial f1}{\partial V2} \\ \frac{\partial f2}{\partial \delta 2} & \frac{\partial f2}{\partial V2} \end{vmatrix} \quad \boldsymbol{Jo}=\begin{vmatrix}10.0392 & 0.1300\\ 0.1306 & 10.0809\end{vmatrix}$$

The Jacobi matrix without the λ-column and **ek**-vector is denoted as **Jo**

$$Dfp=Po\,(1+K{*}\lambda)+V_2{*}Y_{21}{*}\cos(\delta_2-\theta_{21})+V_2^{\,2}{*}Y_{22}{*}\cos(\theta_{22})$$

$$Dfq=Po\,(1+K{*}\lambda)+V_2{*}Y_{21}{*}\sin(\delta_2-\theta_{21})+V_2^{\,2}{*}Y_{22}{*}\sin(\theta_{22})$$

$$Ddk=-inv(Jo)\begin{bmatrix}Dfp\\Dfq\end{bmatrix} \Rightarrow Ddk=\begin{bmatrix}-0.0229\\-0.0037\end{bmatrix}$$

$\delta_2=\delta_2+D\delta_2=0.01+(-0.0229)=-0.0129$

$V_2=V_2+DV_2=1.004+(-0.0037)=1.0003$

For δ2=-0.0129 and V2=1.0003, another corrector step is executed, and the correction steps are reiterated until Ddk≅0.

The final solution of the 1st iteration is:

$$\Rightarrow \quad it_1=\begin{vmatrix}-0.0130\\0.9999\\0.3000\end{vmatrix}$$

Predictor: λ - Continuation Parameter, Corrector: V-Continuation parameters

In the 15th iteration, the condition to switch the continuation parameter from λ to V is met. The voltage angle (δ_2), voltage magnitude (V_2), and load factor are as follows: δ_2 = -0.7829, V_2 = 0.7089, and λ = 48.9996.

The final solution of the 15th iteration is:

$$\Rightarrow \mathrm{it}_{15}=\begin{vmatrix}-0.7867\\0.7062\\49.000\end{vmatrix}$$

The schematic representation of the analyzed 2-node network is depicted. Additionally, the functions V = f(λ) and δ = f(λ) are graphically mapped.

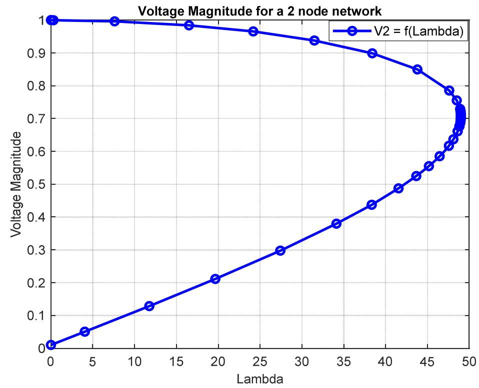

Fig. 4. Voltage Magnitude -dependent load V= f(λ)

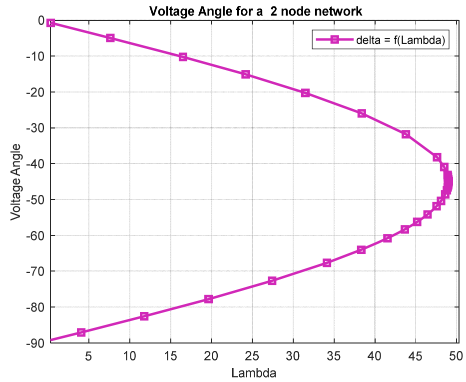

Fig. 5. Voltage angle-dependent load δ = f(λ)

Table 1. Results of Voltage Angle and Voltage Magnitude Calculations for a 2-Node Network using CPF Program:

Iteration	δ_2 (per unit)	V_2 (per unit)	δ_2 (degrees)	V2 (%)
1	-0.0130	0.9999	-0.75°	99.99
2	-0.0865	0.9963	-4.95°	99.63
3	-0.1788	0.9841	-10.246°	98.41
4	-0.2639	0.9654	-15.12°	96.54
5	-0.3537	0.9381	-20.27°	93.81
6	-0.4538	0.8988	-26.00°	89.88
7	-0.5553	0.8497	-31.82°	84.97
8	-0.6672	0.7856	-38.23°	78.56
9	-0.7146	0.7554	-40.95°	75.54
10	-0.7537	0.7292	-43.184°	72.92
11	-0.7630	0.7228	-43.72°	72.28
12	-0.7748	0.7146	-44.4°	71.46
13	-0.7818	0.7097	-44.8°	70.97
14	-0.7829	0.7089	-44.86°	70.89
15	-0.7867	0.7062	-45.0°	70.62
16	-0.7902	0.7037	-45.275°	70.37
17	-0.7930	0.7017	-45.4356°	70.17
18	-0.7967	0.6991	-45.6475°	69.91
19	-0.8035	0.6942	-46.0372°	69.42
20	-0.8142	0.6865	-46.6502°	68.65
21	-0.8281	0.6763	-47.4466°	67.63
22	-0.8481	0.6614	-48.5926°	66.14
23	-0.8801	0.6371	-50.4260°	63.71
24	-0.9058	0.6171	-51.8985°	61.71
25	-0.9458	0.5851	-54.1903°	58.51
26	-0.9822	0.5552	-56.2759°	55.52
27	-1.0184	0.5247	-58.3500°	52.47
28	-1.0619	0.4872	-60.8424°	48.72
29	-1.1180	0.4374	-64.0567°	43.74
30	-1.1813	0.3797	-67.6835°	37.97
31	-1.2686	0.2977	-72.6854°	29.77
32	-1.3580	0.2112	-77.8077°	21.12
33	-1.4418	0.1286	-82.6091°	12.86
34	-1.5203	0.0505	-87.1068°	5.05
35	-1.5609	0.0100	-89.4330°	1.00

In Fig. 4, the voltage magnitude is depicted as a function of load Lambda (λ) for a node network, whereas Fig. 5 illustrates the voltage angle as a function of load lambda (λ)

Table 1 displays the results of voltage angle and voltage magnitude calculations for a 2-node network using the CPF program. In Table 1, the red color value V_2=0.7062 represents the critical voltage value in the network. Beyond this threshold, in the subsequent iteration, the voltage value will fall within the lower section of the PV curve.

Presentation of Results on the Trigonometric Circle for a 2-Node Network

Presentation of the functions $V_i = f(\lambda_i)$ and $\delta_i = f(\lambda_i)$ calculated by the CPF program within a trigonometric circle is also feasible. Figure 6 illustrates the trigonometric circle computed using the CPF program for a 2-node network across iterations 1, 5, 10, 15, 17, 30, and 35. The x-coordinate represents the voltage magnitudes of the load node within the interval [0,1], while the voltage angles are referenced to the designated node. From iteration 1 to iteration 14, the load parameter is utilized as a continuous parameter. These iterations are depicted in purple in Fig. 6. In iteration 15, the transition from the continuous parameter lambda (λ) to voltage (V) occurs. The values for voltage magnitude and voltage angle at iteration 1 are: $V_2 = 0.9999$, $\delta_2 = -0.75$.

In iteration 16, continuation becomes a parameter.

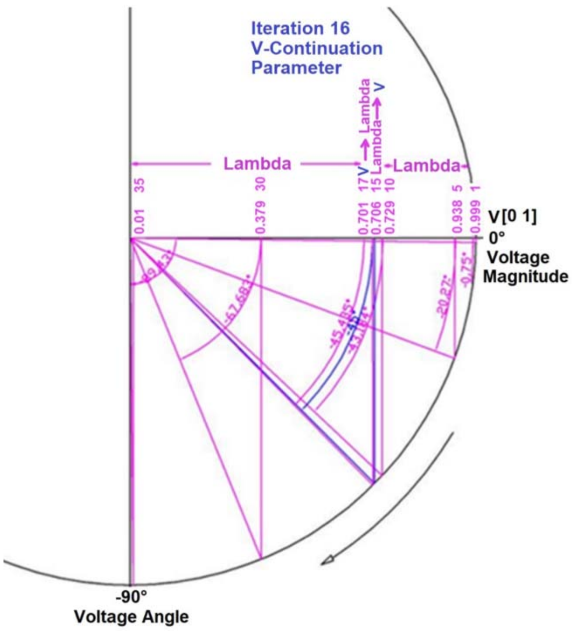

Fig. 6. 2-node network shown in the trigonometric circle

Table 2. Voltage Magnitudes and Voltage Angles for Iterations 1, 5, 10, 15, 17, 30 and 35 for the 2-Node Network.

	δ_2 (°)	V (p.u.)
1. Iteration	-0.75	0.9999
5. Iteration	-20.27	0.9381
10. Iteration	-43.184	0.7292
15. Iteration	-45.00	0.7062
17. Iteration	-45.4356	0.7017
30. Iteration	-67.6835	0.3797
35. Iteration	-89.433	0.0100

The iteration is shown in blue color. In the iteration 17 the continuation parameter is reversed to *λ*. The display is again in purple color. The values of the voltage magnitudes and voltage angles for the iterations 1, 5, 10, 15, 17, 30 and 35 are given in Table 2.

Conclusions

In this paper, the novel application of utilizing a trigonometric circle to visualize the results Vi = f(λi) and δi = f(λi) obtained from the CPF program for a 2-node network analysis is presented. The exploration of this graphical representation in power system analysis unveils its potential benefits in understanding power system behavior and voltage stability. By integrating CPF results with the trigonometric circle, an intuitive approach is offered that simplifies the complex relationship between voltage magnitudes and angles.

This intuitive visualization aids engineers and researchers in grasping the effects of parameter variations on voltage stability. Plotting voltage angles and magnitudes on the trigonometric circle enables swift identification of critical points and potential voltage instability scenarios.

Moreover, this approach provides a clear depiction of voltage stability limits and the ramifications of parameter changes on system behavior, thereby facilitating informed decisions in system planning, operation, and optimization. The presented approach not only demonstrates its immediate utility but also holds promise for broader applications in power system analysis and visualization. It has the potential to enhance the efficiency of power system operations and decision-making processes by offering a more intuitive and insightful representation of system behavior. In conclusion, the integration of the trigonometric circle with CPF analysis provides a valuable tool for advancing power system analysis and understanding, thereby contributing to the efficiency, reliability, and planning of power systems. Further research and application of this approach could lead to the development of more advanced techniques in power system analysis and optimization.

DECLARATION OF COMPETING INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

I am the sole author of this paper.

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