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Economic Load Dispatch Problem Solution Based on Linear Variational Inequalities-dynamic Neural Network

***Abstract**. This paper introduces a solution of the economic load dispatch (ELD) problem using linear variational inequalities (LVI) dynamic neural network (LVI-DNN). This technique guarantees the global optimality of the solution and fast convergence, so that it can be employed in large on-line power systems where variations in load are quite frequent. The new algorithm is applied and tested to an example from the literature and the solution is then compared with that obtained by some other techniques to prove the superiority and effectiveness of the proposed algorithm.*

***Streszczenie.** W artykule przedstawiono rozwiązanie problemu ekonomicznego rozłożenia obciążenia (ELD) z wykorzystaniem dynamicznej sieci neuronowej LVI-DNN z wykorzystaniem liniowych nierówności wariacyjnych (LVI). Technika ta gwarantuje globalną optymalność rozwiązania i szybką konwergencję, dzięki czemu może być stosowana w dużych systemach elektroenergetycznych on-line, gdzie wahania obciążenia są dość częste. Nowy algorytm jest stosowany i testowany na przykładzie z literatury, a następnie rozwiązanie jest porównywane z rozwiązaniem uzyskanym innymi technikami, aby wykazać wyższość i skuteczność proponowanego algorytmu. (**Rozwiązanie problemu dyspozytorskiego obciążenia ekonomicznego w oparciu o liniowe nierówności wariacyjne – dynamiczna sieć neuronowa**)*

Keywords: economic load dispatch (ELD), linear Variational inequalities (LVI), dynamic neural network (DNN), power generation, cost optimization..

Słowa kluczowe: ekonomiczne rozmieszczenie obciążenia (ELD), liniowe nierówności wariacyjne (LVI), dynamiczna sieć neuronowa (DNN), wytwarzanie energii, optymalizacja kosztów.

Introduction

Economic load dispatch (ELD) is considered as one of the most important steps to obtain a complete generation scheduling solution. ELD aims to schedule the online generators outputs with the predicted load demands over a certain period of time in order to operate an electric power system most economically within its security limits [1, 2]. By solving the ELD problem, the total generation required is allocated among the available online thermal generating units over a certain period of time. To solve the ELD problem, it is assumed that a thermal unit commitment has been previously determined. Traditionally, to solve the ELD problem, a Lagrangian augmented function is first formulated, and the optimal conditions are obtained by partial derivation of this function [1-3]. In traditional methods, calculation of the penalty factor as well as the incremental loss is always the key point in the solution algorithm. The problem can be solved using the lambda-iteration method, Newton-Raphson method and gradient method algorithms [1]. However, most of the previous work in this field is not able to provide an optimal solution and usually get stuck at a local minimum point.

Several artificial intelligence-based methods, such as simulated annealing, genetic algorithms, evolutionary programming and particle swarm optimization have been used to solve the ELD problem [4-5]. Such techniques use probabilistic rules to update their candidates' positions in the search space. Anyway, these algorithms do not always guarantee discovering the global optimal solution in a finite time but they can only find a feasible solution in short time. Various artificial neural networks based methods have been proposed for the ELD problem [6-12]. Neural networks based on supervised learning algorithms, such as MLP and RBF networks, need to be trained by data obtained from a conventional ELD solution. Furthermore, application of back-propagation type algorithms converges very slowly and suffer from local minima problem.

In the last years, various dynamic neural network models (DNN) have been developed for solving quadratic programming problems. According to their design method, these DNNs can be categorized as penalty-based DNN [13], two-layer Lagrangian DNN [14], primal-dual DNN [15], and one-layer dual DNN [16]. The projection DNN was proposed for solving general convex programming problems [17-18], which is globally convergent to exact optimal solutions of convex quadratic programming problems [17-18]. The essence of DNN optimization lies in its dynamic nature for optimization and the availability of electronic implementation. Unlike other parallel algorithms, DNNs can be implemented physically by dedicated hardware such as application-specific integrated circuits where the optimization procedure execution is truly parallel and distributed.

In this paper, the LVI-based dynamic neural network is investigated for the solution of the ELD problem. This method guaranties a fast convergence to the exact optimal ELD solution. The problem mapping is very transparent and direct, as compared to Hopfield NN. The dynamic neural network ELD solver is more suitable for hardware implementation where the solution procedures are parallel and distributed and ELD optimization problems can be solved in real time. An example from the literature is solved by the proposed method and the solution is compared with some other methods to prove the validity and superiority of the proposed technique.

ELD Problem

The ELD problem is to find the optimal combination of power generation that minimizes the total fuel cost while satisfying the total demand and power system constraints. The total fuel cost function of ELD problem is defined as follows:

$$FC(P) = \sum_{i=1}^{n} (a_i P_i^2 + b_i P_i + c_i) \tag{1}$$

where FC (\$/h is the total fuel cost), P_i (MW) is the power generation of unit i, a_i, b_i, c_i are the fuel cost coefficients for unit i, n is the number of generating units.

The economic dispatch problem is optimized subject to:

1. Power balance constraint: The total power generated must supply total load demand and transmission losses, i.e.,.

(2) $\sum_{i=1}^{n} P_i = P_L + P_D$

where P_D (MW) is the total load demand and P_L (MW) is the total transmission losses computed using the B-coefficients formula:

(3) $P_L = P^T B_0 P$

where $P^T = \begin{bmatrix} P_1 & P_2 & ... & P_n \end{bmatrix}$ and $B_0 \in \Re^{n \times n}$ is coefficients matrix.

2. Unit capacity constraint: The power P_i, generated by the i th unit, constrained between minimum and maximum limits of generation, i.e.,

(4) $P_{i\min} \le P_i \le P_{i\max}$

The ELD minimization problem can be formulated as a time-varying quadratic program (QP) subject to linear equality and bound constraints as follows:

(5) $\min FC(P) = \frac{1}{2} P^T A P + B^T P + C$

(6) $\text{s.t:} \quad EP - d = 0$

(7) $P_{\min} \le P \le P_{\max}$

where

$$A = 2\begin{bmatrix} a_1 & 0 & ... & 0 \\ 0 & a_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & ... & 0 & a_n \end{bmatrix},$$

$B^T = \begin{bmatrix} b_1 & b_2 & ... & b_n \end{bmatrix}$,

$C^T = \begin{bmatrix} c_1 & c_2 & ... & c_n \end{bmatrix}$,

$P_{\min}^T = \begin{bmatrix} P_{1\min} & P_{2\min} & ... & P_{n\min} \end{bmatrix}$,

$P_{\max}^T = \begin{bmatrix} P_{1\max} & P_{2\max} & ... & P_{n\max} \end{bmatrix}$,

$E = \begin{bmatrix} 1 & 1 & ... & 1 \end{bmatrix}$ and $d = P_L + P_D$.

Since the objective function (5) is strictly convex (due to $A > 0$ i.e., definite positive matrix) and the feasible region of linear constraints (6)-(7) is a closed convex set, if not empty, it follows from [17-18] that the constrained minimizer P^* to the quadratic program (5)-(7) is unique and satisfies the Karush-Kuhn-Tucker (KKT) optimality conditions.

LVI-DNN based ELD solution

In this section, a primal-dual dynamical QP solver is presented based on linear variational inequalities [17-18]. By the duality theory [19], for the primal problem (5)-(7), its dual problem can be derived with the aid of dual decision variables. The dual decision variable is often defined as the Lagrangian multiplier for each constraint like (6) and (7). However, to reduce the QP-solver complexity, we only need to define the corresponding dual decision variable y for equality constraint (6). Thus, the primal-dual decision vector z and its bounds are defined as

(8) $z = \begin{bmatrix} P \\ y \end{bmatrix}, z_{\min} = \begin{bmatrix} P_{\min} \\ 0 \end{bmatrix}, z_{\max} = \begin{bmatrix} P_{\max} \\ y_{\max} \end{bmatrix}$

where in hardware implementation/simulation, $y_{\max}$ is sufficiently large constant to represent $+\infty$. The convex set Ω made by z is then:

(9) $\Omega = \left\{ z \in R^{n+1} \mid z_{\min} \le z \le z_{\max} \right\}$

By defining coefficient matrix M and vector q as:

(10) $M = \begin{bmatrix} A & -E^T \\ E & 0 \end{bmatrix}, q = \begin{bmatrix} B \\ -d \end{bmatrix}$

we have the following equivalence result.

Theorem. Quadratic program (5)-(7) is equivalent to the following LVI problem, i.e., to find a vector $z^* \in \Omega$, such that:

(11) $(z - z^*)^T (Mz^* - q) \ge 0, \forall z \in \Omega$

The proof can be found in [17-18].

It is known that LVI (11) is equivalent to the following system of piecewise-linear equations:

$\Phi\left(z - (Mz + q)\right) - z = 0$ (12)

were $\Phi(.)$ is the Ω-projection operator defined as $\Phi(z) = \begin{bmatrix} \phi_1(z) & \phi_2(z) & ... & \phi_{n+1}(z) \end{bmatrix}^T$ with:

(12) $\phi_i(z) = \begin{cases} z_{i\min}, & z_i < z_{i\min} \\ z_i, & z_{i\min} \le z_i \le z_{i\max} \\ z_{i\max}, & z_i > z_{i\max} \end{cases}, i = 1,2,..,n+1$

Finally, from neural network design techniques [15-18], it follows that the LVI-based primal-dual neural network, being the QP solver for (5)-(7), can use the following dynamic equation:

(13) $\Lambda^{-1} \dot{z} = \left(I_{n+1} + M^T\right)\left[\Phi\left(z - (Mz + q)\right) - z\right]$

where $\Lambda = diag\left\{\alpha_1, \alpha_2, .., \alpha_{n+1}\right\}$ and $\alpha_i > 0$ are capacitive parameters used to scale the network convergence.

Furthermore, (14) can be rewritten in a more compact form as:

(15) $\dot{z} = -Wz + W\Phi(Vz - q)$

where $W = \Lambda\left(I_{n+1} + M^T\right), V = I_{n+1} - M$.

The convergence proprieties of the LVI-DNN (15) stated by the following theorem.

Theorem 2 (LVI-DNN convergence): With the existence of at least one optimal solution to the QP (5)-(7), starting from any initial state $z(0)$, the state vector $z(t)$ of the LVI-DNN (15) is convergent to an equilibrium point z^*, of which the first n elements constitute the optimal solution P^* to the ELD problem (5)-(7). Moreover, if there exists a constant $\gamma > 0$ such that $\left\|\Phi(Vz - q) - z\right\|_2^2 \le \gamma \left\|z^* - z\right\|_2^2$, then the exponential convergence can be achieved with a convergence rate proportional to $\min\{\alpha_i\}\gamma$.

Proof. Can be generalized from [17-18] and the references therein by using Lyapunov function candidate $L = \left\|z^* - z\right\|_2^2$ and projection-related inequalities.

Expressed in the i th-neuron form, LVI-DNN (15) can be further written as

(16) $\dot{z}_i = \sum_{j=1}^{n+1} w_{ij} \left(\phi\left(\sum_{k=1}^{n+1} v_{ik} z_k - q_i \right) - z_i \right), i = 1,2,..,n+1$

where w_{ij} denotes the ij th entry of weighting matrix W, and v_{ik} denotes the ik th entry of weighting matrix V.

Simulation results

The LVI-DNN performances are tested on two widely used power system benchmarks.

1. 3-unit power system. The cost coefficient data along with power generating limits for the 3-unit power system are listed in Table 1. The transmission losses are computed using $B_0 = diag\{0.00003, 0.00009, 0.00012\}$.

Table 1. 3-unit power system parameters.

N°	a_i	b_i	c_i	P_{min} (MW)	P_{max} (MW)
1	0.00156	7.92	561	150	600
2	0.00194	7.85	310	100	400
3	0.00482	7.97	78	50	200

The block diagram realization of LVI-DNN (15) using Matlab\Simulink is given in Fig. 1, where saturation functions, gain matrices and integrators are used to implement, respectively, action functions, weighting coefficients and differential equations integration.
The three generators are initialized to their mean power values, $y(0) = 1$, $\alpha_i = 0.1, i = 1,2,3$ and $\alpha_4 = 0.04$.

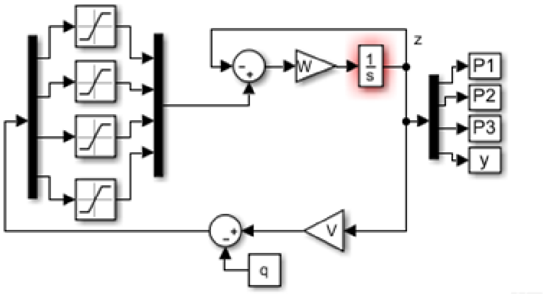

Fig. 1. Simulink block diagram of 3-unit ELD LVI-DNN solver.

The LVI-DNN is compared to conventional optimization method (CM) and to Hopfield neural networks based techniques, such as, Standard Hopfield Neural Network (SHN) and three improved Hopfield neural network approaches (AHN, IHN and PHB (see [7]). The comparison is made for 340 (MW) and 850 (MW) power demands. The LVI-DNN dynamics, for 340 (MW) and 850 (MW) power demands, are shown on fig. 1 and fig. 2, respectively. These figures illustrate the convergence of the 3 generators powers toward their optimal values, the evolution of total generated power to satisfy equality constraint (2) and cost evolution. It clear that LVI-DNN converges to the optimal solution more rapidly for 850 (MW) power demand. This fact is predicable, since in the 850 (MW) power demand case the optimal ELD solution is closer to initial powers values.

Tables 2 and 3 summarize the LVI-DNN results along with the results of previously cited methods taken from [7]. Compared to the best results, For 340 (MW) load demand, LVI-DNN achieves a loss reduction by 0.1355 (MW), a cost reduction by 0.9 ($/h) or equivalently 7884 ($/year), and a -0.0085 (MW) power generation error compared to 0.012 (MW) for the previous best result. For 850 (MW) load demand, LVI-DNN achieves a loss reduction by 0.26 (MW), a cost reduction by 2.7 ($/h), or equivalently 23652 ($/year), and a null power generation error.

Table 2. 3-unit ELD comparative results for 340 (MW) load demand.

Methods	P_L (MW)	Generated powers (MW)			Cost ($/h)
		P_1	P_2	P_3	
CM	2.762	152.18	140.57	50.00	**3742.9**
SHN	**2.754**	170.35	104.18	68.211	3748.5
AHN	2.762	159.64	133.02	50.092	3743.1
IHN	2.762	152.52	139.85	50.381	3742.9
PHN	2.77	152.23	140.54	50.00	3743.0
LVI-DNN	**2.6185**	**162.31**	**130.30**	**50.00**	**3742.0**

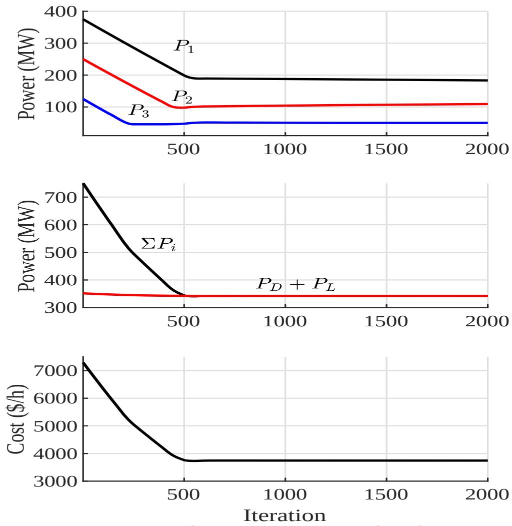

Fig. 2 LVI-DNN evolution for 3-unit ELD, 340 (MW).

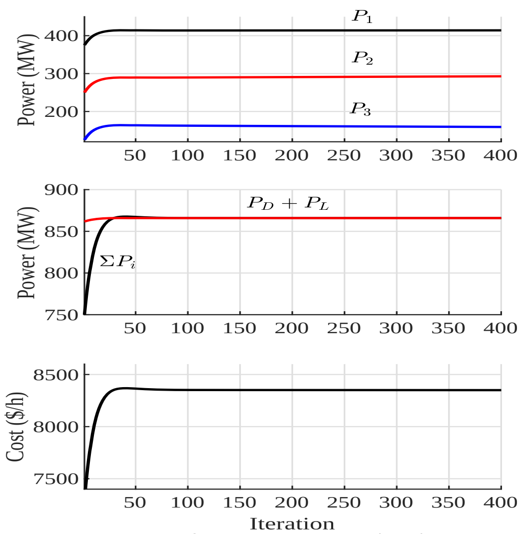

Fig. 3. LVI-DNN evolution for 3-unit ELD, 850 (MW).

Table 3. 3-unit ELD comparative results for 850 (MW) load demand.

Methods	P_L (MW)	Generated powers			Cost ($/h)
		P_1	P_2	P_3	
CM	17.14	401.22	341.08	124.84	8351.4
SHN	**17.12**	373.73	310.27	183.12	8370.6
AHN	17.14	383.79	331.98	151.362	8355.4
IHN	17.14	401.67	340.66	124.81	8351.4
PHN	17.14	401.66	340.66	124.82	**8351.4**
LVI-	**16.86**	**408.16**	**332.49**	**126.21**	**8348.7**

2. 13-unit power system. The cost coefficient data along with power generating limits for the 3-unit power system are listed in table 4. The transmission losses are computed using B_0 coefficients listed in (17). The LVI-DNN dynamics evolution for 975 (MW) and 2575 (MW) load demands are shown in fig. 4 and fig. 5, respectively.
Table 5 shows the LVI-DNN results along with the results of conventional method (CM) and RBF neural network based method (PNM) taken from [8]. For the three load demands, LVI-DNN provides loss reductions of 4.5983, 15.18 and 34.421 (MW), respectively. The production cost is reduced by 37.9 ($/h), 145.92 ($/h) and 307.85 ($/h), for the three load demands respectively. The power generation errors are very low compared to the two other methods results.

Table 4. 13-unit power system parameters.

N°	a_i	b_i	c_i	P_{min}	P_{max}
1-6	0.00324	7.74	240.0	60.0	180.0
7,8	0.00284	8.60	126.0	40.0	120.0
9,10	0.00284	8.60	126.0	55.0	120.0
11	0.00028	8.10	550.0	0.0	680.0
12	0.00056	8.10	309.0	0.0	360.0
13	0.00056	8.10	307.0	0.0	360.0

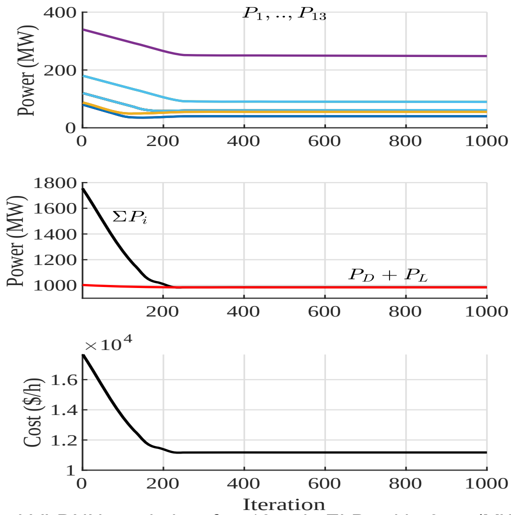

Fig. 4. LVI-DNN evolution for 13-unit ELD with 975 (MW) load demar

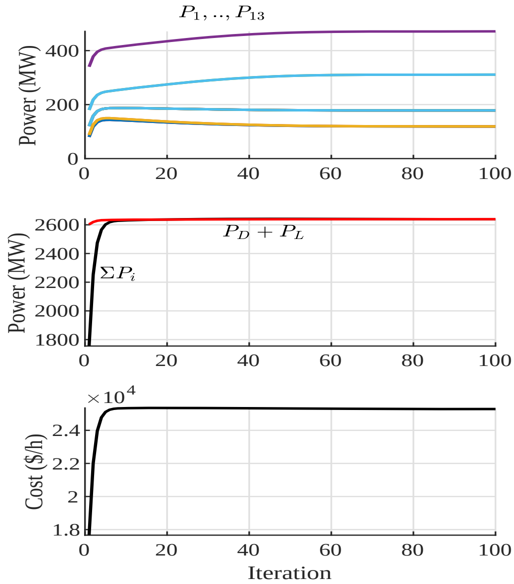

Fig. 5. LVI-DNN evolution for 13-unit ELD with 2575 (MW) load demand.

Table 5. 13-unit ELD comparative results.

P_D (MW)	Methods	$\sum_i P_i$ (MW)	P_L (MW)	Cost ($/h)
975	CM	988.40	13.4009	11201.90
	PNM	988.41	13.4011	11201.95
	LVI-DNN	**983.8**	**8.8026**	**11164.00**
1925	CM	1985.2	60.1509	19506.92
	PNM	1985.2	60.1509	19506.92
	LVI-DNN	**1970.00**	**44.9710**	**19361.00**
2575	CM	2685.7	110.6924	25517.99
	PNM	2685.3	110.6711	25514.85
	LVI-DNN	**2651.3**	**76.2500**	**25207.00**

Conclusion

This paper presents a new approach for the economic operation of power systems. It uses a dynamical neural network model to determine the optimal scheduling of the power plants. Numerical results show that highly optimal solutions can be obtained by the proposed method. The LVI-DNN algorithm is very fast and requires small computing resources. Further, LVI-DNN scals very well of large power systems and has the potential for hardware implementation using analog electronics devices.

$$B_0 = 10^{-4}\begin{bmatrix} 0.5 & 0.1 & 0.05 & 0.05 & 0.1 & 0.05 & 0.15 & 0.05 & 0.05 & 0.1 & 0.05 & 0.05 & 0.15 \\ 0.1 & 0.4 & 0.1 & 0.05 & 0.15 & 0.05 & 0.05 & 0 & 0.1 & 0.05 & 0.05 & 0 & 0.05 \\ 0.05 & 0.1 & 0.3 & 0.05 & 0.05 & 0 & 0.1 & 0 & 0.05 & 0 & 0.05 & 0 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.44 & 0.15 & 0.1 & 0 & 0 & 0.05 & 0 & 0.05 & 0 & 0 \\ 0.1 & 0.15 & 0.05 & 0.15 & 0.4 & 0.15 & 0.1 & 0.05 & 0 & 0 & 0.05 & 0 & 0 \\ 0.05 & 0.05 & 0 & 0.1 & 0.15 & 0.4 & 0.1 & 0.05 & 0 & 0.05 & 0 & 0 & 0 \\ 0.15 & 0.05 & 0.1 & 0 & 0.1 & 0.1 & 0.3 & 0.15 & 0.1 & 0 & 0.05 & 0 & 0 \\ 0.05 & 0 & 0 & 0 & 0.05 & 0.05 & 0.15 & 0.4 & 0.1 & 0.1 & 0 & 0.05 & 0 \\ 0.05 & 0.1 & 0.05 & 0.05 & 0 & 0 & 0.1 & 0.1 & 0.3 & 0.1 & 0.05 & 0.05 & 0 \\ 0.1 & 0.05 & 0 & 0 & 0 & 0.05 & 0 & 0.1 & 0.1 & 0.5 & 0.15 & 0.1 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0 & 0.05 & 0 & 0.05 & 0.15 & 0.4 & 0.15 & 0.05 \\ 0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0.05 & 0.1 & 0.15 & 0.5 & 0.1 \\ 0.15 & 0.05 & 0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0.05 & 0.1 & 0.7 \end{bmatrix}$$

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