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A method for extracting the parameters of a multiconductor transmission line

***Abstract**. The paper considers linear circuits, including lumped elements and uniform multiconductor transmission lines immersed in a homogenous medium. The problem of extracting the line's per–unit–length parameters at a given frequency is solved using a numerical approach. The proposed method exploits measured input and output voltage phasors of the line operating in the circuit. The core of the method is an iterative procedure for solving a system of nonlinear equations obtained after some rearrangement of the standard description of the line. A numerical example was presented to validate the proposed solution process.*

***Streszczenie.** W artykule rozważono obwody liniowe zawierające elementy skupione oraz wieloprzewodowe linie transmisyjne umieszczone w ośrodku jednorodnym. Problem ekstrakcji jednostkowych parametrów, przy zadanej częstotliwości, rozwiązano za pomocą podejścia numerycznego. Proponowana metoda wykorzystuje pomiary wartości symbolicznych (fazorów) napięcia wejściowego i wyjściowego linii pracującej w obwodzie. Rdzeniem metody jest iteracyjna procedura rozwiązywania układu równań nieliniowych. Układ ten otrzymano na drodze przekształceń standardowego opisu linii. W celu walidacji proponowanego rozwiązania przedstawiono przykład numeryczny. (**Metoda ekstrakcji parametrów wieloprzewodowej linii transmisyjnej**).*

Keywords: iterative procedure, multiconductor transmission line, nonlinear equations, numerical approach, per–unit–length parameters.
Słowa kluczowe: procedura iteracyjna, wieloprzewodowa linia transmisyjna, równania nieliniowe, podejście numeryczne, parametry jednostkowe.

Introduction

This paper deals with multiconductor transmission lines (MTL). It is focused on determining the per–unit–length (p–u–l) parameter of resistance, inductance, conductance, and capacitance for the given line. MTLs play a significant role in electrical and electronic engineering due to the need to process high–speed signals. Numerous publications are devoted to modelling, diagnosing, and analysis of MTL [1-13]. Computation of the line parameters is a crucial step for the analysis of circuits, including MTLs. It has been an active topic over the last few years, e.g., in the references [1], [3], [5-6], [11]. Many research reports refer to determining the transmission line parameters from scattering parameters in the frequency domain. A vector network analyzer (VNA) can be used to measure scattering parameters. Conversion of scattering parameters to the line parameters is described, e.g., in the references [1], [3], [6], and is implemented in RF Toolbox TM in MATLAB. This paper proposes an entirely different approach to extracting transmission line parameters. The parameters are determined numerically considering the overall circuit, including the MTL.

Let us consider (n+1)-conductor transmission line. The line is described in the frequency domain by the equations

$$\frac{\mathrm{d}\bar{\boldsymbol{V}}(z)}{\mathrm{d}z} = -\bar{\boldsymbol{Z}}\boldsymbol{I}(z) \tag{1}$$

$$\frac{\mathrm{d}\bar{\boldsymbol{I}}(z)}{\mathrm{d}z} = -\bar{\boldsymbol{Y}}\boldsymbol{V}(z) \tag{2}$$

where z is the position in the line, $\bar{\boldsymbol{V}}(z)=\left[\bar{V}_1(z)\ldots\bar{V}_n(z)\right]^{\mathrm{T}}$, $\bar{\boldsymbol{I}}(z)=\left[\bar{I}_1(z)\ldots\bar{I}_n(z)\right]^{\mathrm{T}}$ are n-dimension vectors consisting of phasors of the voltages between the conductors $1,\ldots,n$ and the reference conductor 0 and phasors of the currents flowing through the conductors $1,\ldots,n$, at the position z. $\bar{\boldsymbol{Z}} = \boldsymbol{R} + \mathrm{j}\omega\boldsymbol{L}$, $\bar{\boldsymbol{Y}} = \boldsymbol{G} + \mathrm{j}\omega\boldsymbol{C}$ where j is the imaginary unity, $\boldsymbol{R}$, $\boldsymbol{L}$, $\boldsymbol{G}$, and $\boldsymbol{C}$ are symmetrical $n\times n$ matrices of the per–unit–length parameters having the form

$$\boldsymbol{R} = \begin{bmatrix} r_1+r_0 & r_0 & \cdots & r_0 \\ r_0 & r_2+r_0 & \cdots & r_0 \\ & \vdots & & \\ r_0 & r_0 & \cdots & r_n+r_0 \end{bmatrix},$$

$$\boldsymbol{L} = \begin{bmatrix} l_{11} & l_{12} & \cdots & l_{1n} \\ l_{12} & l_{22} & \cdots & l_{2n} \\ & & \vdots & \\ l_{1n} & l_{2n} & \cdots & l_{nn} \end{bmatrix},$$

$$\boldsymbol{G} = \begin{bmatrix} \tilde{g}_{11} & -g_{12} & \cdots & -g_{1n} \\ -g_{12} & \tilde{g}_{22} & \cdots & -g_{2n} \\ & \vdots & & \\ -g_{1n} & -g_{2n} & \cdots & \tilde{g}_{nn} \end{bmatrix},$$

$$\tilde{g}_{11} = g_{11}+g_{12}+\ldots+g_{1n},\ \tilde{g}_{22} = g_{12}+g_{22}+\ldots+g_{2n},\ \ldots,\ \tilde{g}_{nn} = g_{1n}+g_{2n}+\ldots+g_{nn},$$

$$\boldsymbol{C} = \begin{bmatrix} \tilde{c}_{11} & -c_{12} & \cdots & -c_{1n} \\ -c_{12} & \tilde{c}_{22} & \cdots & -c_{2n} \\ & \vdots & & \\ -c_{1n} & -c_{2n} & \cdots & \tilde{c}_{nn} \end{bmatrix},$$

$$\tilde{c}_{11} = c_{11}+c_{12}+\ldots+c_{1n},\ \tilde{c}_{22} = c_{12}+c_{22}+\ldots+c_{2n},\ \ldots,\ \tilde{c}_{nn} = c_{1n}+c_{2n}+\ldots+c_{nn}.$$

The problem discussed in this paper is as follows: Let us consider a linear circuit, shown in Fig. 1, consisting of an MTL as well as lumped resistors, inductors, and capacitors whose values are known and driven by AC voltage sources. We wish to extract parameters of the transmission line, at given frequency f, on the basis of measured input and output voltage phasors of the line: $\bar{V}_1(0),\ldots,\bar{V}_n(0),\ldots,$

$\overline{V}_1(l), ..., \overline{V}_n(l)$. The problem is solved using a numerical method under the following four assumptions.

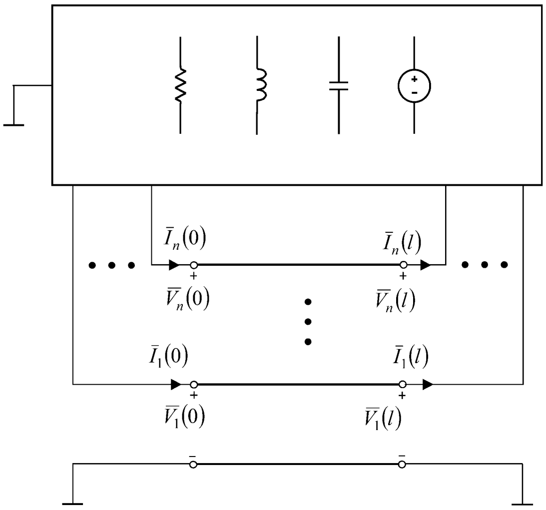

Fig.1. A circuit containing an (*n*+1)-conductor transmission line

1. The line, uniform and immersed in a homogenous medium, is characterized by the permeability $\mu = \mu_0$ and permittivity $\varepsilon = \varepsilon_0\varepsilon_r$. Permittivity ε_r and conductivity σ of the line are unknown.
2. The matrices $\boldsymbol{L}$, $\boldsymbol{G}$, and $\boldsymbol{C}$ meet the equations [4]

(3) $$\boldsymbol{LC} = \boldsymbol{CL} = \mu_0\varepsilon_0\varepsilon_r\mathbf{1}$$

(4) $$\boldsymbol{LG} = \boldsymbol{GL} = \mu_0\sigma\mathbf{1}\ .$$

3. Resistance of the reference conductor is equal to zero, i.e. $r_0 = 0$.
4. The number of conductors of the multiconductor transmission line, including the reference conductor, does not exceed five.

Preliminaries

To derive equations involving the output voltages and currents (at $z = l$) and the input voltages and currents (at $z = 0$) in the frequency domain, we apply the standard approach, as in the reference [4]. By combining (1) and (2) we obtain

(5) $$\frac{\mathrm{d}^2\overline{\boldsymbol{I}}(z)}{\mathrm{d}z^2} = \overline{\boldsymbol{YZ}}\overline{\boldsymbol{I}}(z)\ .$$

Matrix equation (5) represents a system of n coupled individual equations. To find the solution of this equation, it is decoupled using a similarity transformation. Let us assume that matrix $\overline{\boldsymbol{YZ}}$ has distinct eigenvalues labelled $\overline{\gamma}_i^2$ ($i = 1, ..., n$). Then, the eigenvectors corresponding to these eigenvalues are linearly independent. They are used to create columns of a modal matrix $\overline{\boldsymbol{T}}_I$ such that

(6) $$\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{YZ}}\overline{\boldsymbol{T}}_I = \overline{\boldsymbol{\gamma}}^2$$

where $\overline{\boldsymbol{\gamma}}^2 = \mathrm{diag}\left(\overline{\gamma}_1^2, ..., \overline{\gamma}_n^2\right)$. Hence, $\overline{\boldsymbol{YZ}} = \overline{\boldsymbol{T}}_I\overline{\boldsymbol{\gamma}}^2\overline{\boldsymbol{T}}_I^{-1}$ and (5) becomes

(7) $$\frac{\mathrm{d}^2\overline{\boldsymbol{I}}(z)}{\mathrm{d}z^2} = \overline{\boldsymbol{T}}_I\overline{\boldsymbol{\gamma}}^2\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{I}}(z)\ .$$

After simple manipulations we obtain

(8) $$\frac{\mathrm{d}^2\overline{\boldsymbol{I}}_m(z)}{\mathrm{d}z^2} = \overline{\boldsymbol{\gamma}}^2\overline{\boldsymbol{I}}_m(z)\ ,$$

where

(9) $$\overline{\boldsymbol{I}}_m(z) = \overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{I}}(z)$$

is a vector of mode current phasors of the line. Since $\overline{\boldsymbol{\gamma}}^2$ is a diagonal matrix a solution of equation (8) has the form

(10) $$\overline{\boldsymbol{I}}_m(z) = \mathrm{e}^{-\overline{\boldsymbol{\gamma}}z}\overline{\boldsymbol{A}} + \mathrm{e}^{\overline{\boldsymbol{\gamma}}z}\overline{\boldsymbol{B}}$$

where $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ are n-dimensional vectors consisting of complex constants, whereas $\mathrm{e}^{-\overline{\boldsymbol{\gamma}}z} = \mathrm{diag}\left(\mathrm{e}^{-\overline{\gamma}_1 z}, ..., \mathrm{e}^{-\overline{\gamma}_n z}\right)$, $\mathrm{e}^{\overline{\boldsymbol{\gamma}}z} = \mathrm{diag}\left(\mathrm{e}^{\overline{\gamma}_1 z}, ..., \mathrm{e}^{\overline{\gamma}_n z}\right)$. Combining (9) and (10) gives

(11) $$\overline{\boldsymbol{I}}(z) = \overline{\boldsymbol{T}}_I\left(\mathrm{e}^{-\overline{\boldsymbol{\gamma}}z}\overline{\boldsymbol{A}} + \mathrm{e}^{\overline{\boldsymbol{\gamma}}z}\overline{\boldsymbol{B}}\right)\ .$$

Hence, we obtain

(12) $$\frac{\mathrm{d}\overline{\boldsymbol{I}}(z)}{\mathrm{d}z} = \overline{\boldsymbol{T}}_I\left(-\overline{\boldsymbol{\gamma}}\mathrm{e}^{-\overline{\boldsymbol{\gamma}}z}\overline{\boldsymbol{A}} + \overline{\boldsymbol{\gamma}}\mathrm{e}^{\overline{\boldsymbol{\gamma}}z}\overline{\boldsymbol{B}}\right)\ .$$

From (2) and (12) we find the equation

(13) $$\overline{\boldsymbol{V}}(z) = \overline{\boldsymbol{Z}}_C\overline{\boldsymbol{T}}_I\left(\mathrm{e}^{-\overline{\boldsymbol{\gamma}}z}\overline{\boldsymbol{A}} - \mathrm{e}^{\overline{\boldsymbol{\gamma}}z}\overline{\boldsymbol{B}}\right),$$

where $\overline{\boldsymbol{Z}}_C = \overline{\boldsymbol{Y}}^{-1}\overline{\boldsymbol{T}}_I\overline{\boldsymbol{\gamma}}\overline{\boldsymbol{T}}_I^{-1}$ is called a characteristic impedance matrix. We now express $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ in terms of the boundary conditions $\overline{V}(0)$ and $\overline{\boldsymbol{I}}(0)$. Using (11) and (13) we write

(14) $$\overline{\boldsymbol{I}}(0) = \overline{\boldsymbol{T}}_I\left(\overline{\boldsymbol{A}} + \overline{\boldsymbol{B}}\right),\quad \overline{\boldsymbol{V}}(0) = \overline{\boldsymbol{Z}}_C\overline{\boldsymbol{T}}_I\left(\overline{\boldsymbol{A}} - \overline{\boldsymbol{B}}\right)\ .$$

Solving the equations (14) for $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ and substituting into (13) and (11) yields

(15) $$\overline{\boldsymbol{V}}(z) = \overline{\boldsymbol{Z}}_C\overline{\boldsymbol{T}}_I\left(\cosh\overline{\boldsymbol{\gamma}}z\cdot\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{Z}}_C^{-1}\overline{\boldsymbol{V}}(0) - \sinh\overline{\boldsymbol{\gamma}}z\cdot\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{I}}(0)\right)$$

(16) $$\overline{\boldsymbol{I}}(z) = \overline{\boldsymbol{T}}_I\left(-\sinh\overline{\boldsymbol{\gamma}}z\cdot\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{Z}}_C^{-1}\overline{\boldsymbol{V}}(0) + \cosh\overline{\boldsymbol{\gamma}}z\cdot\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{I}}(0)\right)$$

where $\cosh\overline{\boldsymbol{\gamma}}z = \mathrm{diag}\left(\cosh\overline{\gamma}_1 z, ..., \cosh\overline{\gamma}_n z\right)$, $\sinh\overline{\boldsymbol{\gamma}}z = \mathrm{diag}\left(\sinh\overline{\gamma}_1 z, ..., \sinh\overline{\gamma}_n z\right)$. For $z = l$ equations (15) and (16) lead to

(17) $$\overline{\boldsymbol{Z}}_C\overline{\boldsymbol{T}}_I\left(\cosh\overline{\boldsymbol{\gamma}}l\cdot\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{Z}}_C^{-1}\overline{\boldsymbol{V}}(0) - \sinh\overline{\boldsymbol{\gamma}}l\cdot\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{I}}(0)\right) - \overline{\boldsymbol{V}}(l) = \mathbf{0}$$

(18) $$\overline{\boldsymbol{T}}_I\left(-\sinh\overline{\boldsymbol{\gamma}}l\cdot\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{Z}}_C^{-1}\overline{\boldsymbol{V}}(0) + \cosh\overline{\boldsymbol{\gamma}}l\cdot\overline{\boldsymbol{T}}_I^{-1}\overline{\boldsymbol{I}}(0)\right) - \overline{\boldsymbol{I}}(l) = \mathbf{0}\ .$$

Extracting multiconductor transmission line parameters

Consider the linear circuit shown in Fig. 1, driven by AC voltage sources at frequency f. The transmission line will be replaced by input and output AC voltage sources (see Fig. 2) specified by phasors $\overline{V}_1(0), ..., \overline{V}_n(0)$, $\overline{V}_1(l), ..., \overline{V}_n(l)$ measured in the circuit. Having these voltages the circuit depicted in Fig. 2 is analysed in the frequency domain using e.g. the modified node approach, finding current phasors $\overline{I}_1(0), ..., \overline{I}_n(0)$, $\overline{I}_1(l), ..., \overline{I}_n(l)$. In consequence vectors $\overline{\boldsymbol{V}}(0)$, $\overline{\boldsymbol{V}}(l)$, $\overline{\boldsymbol{I}}(0)$, and $\overline{\boldsymbol{I}}(l)$ which appear in (17) and (18) are given, whereas the matrices $\overline{\boldsymbol{Z}}_C$, $\overline{\boldsymbol{T}}_I$, $\bar{\gamma}$ are functions of unknown line parameters. Equations (3) and (4) imply that matrices $\boldsymbol{L}$ and $\boldsymbol{R}$ as well as constants ε_r and σ enable to find all the p–u–l parameters of the line. Since matrix $\boldsymbol{L}$ is symmetric it is entirely specified by its diagonal elements and the elements located above the main diagonal. The number of these elements is $n+\frac{n^2-n}{2}$. The number of nonzero elements of the diagonal matrix $\boldsymbol{R}$ is n whereas the number of the required constants (ε_r and σ) is two. Hence, the total number of the parameters and constants necessary to identify $(n+1)$-conductor transmission line is $m=2n+\frac{n(n-1)}{2}+2$. They are labeled $p_1, ..., p_m$. Table 1 presents the relations between n and m. In addition the last row of this table gives the corresponding values of $4n$ what will be exploited in the sequel.

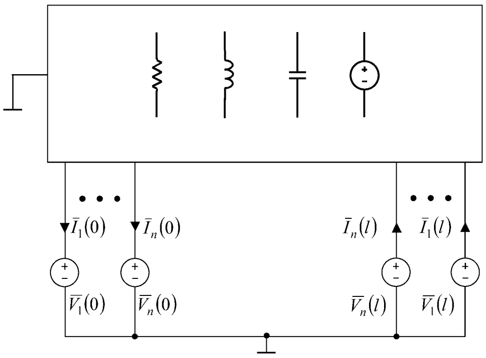

Fig. 2. Model of the circuit shown in Fig. 1

Table 1. Relations between the numbers *n* and *m*

n	2	3	4	5
m	7	11	16	22
4*n*	8	12	16	20

Values of the parameters $p_1, ..., p_m$ vary immensely, e.g. some of them can be equal 100 whereas others 10^{-9}. In consequence round off errors and deterioration of the accuracy arise while running the computation process described in the sequel. Therefore the parameters will be scaled, to reduce the differences between them, according to the equations $p_i = s_i x_i$, $i=1,...,m$, where x_i are auxiliary parameters forming vector $\boldsymbol{x}=[x_1 \ldots x_m]^{\mathrm{T}}$, whereas $s_1, ..., s_m$ are scaling factors. Thus, equations (17) and (18) can be written as

$$\begin{aligned}&\overline{\boldsymbol{Z}}_C(\boldsymbol{x})\overline{\boldsymbol{T}}_I(\boldsymbol{x})\big(\cosh\bar{\boldsymbol{\gamma}}(\boldsymbol{x})l\cdot\overline{\boldsymbol{T}}_I^{-1}(\boldsymbol{x})\overline{\boldsymbol{Z}}_C^{-1}(\boldsymbol{x})\overline{\boldsymbol{V}}(0)\\&-\sinh\bar{\boldsymbol{\gamma}}(\boldsymbol{x})l\cdot\overline{\boldsymbol{T}}_I^{-1}(\boldsymbol{x})\bar{\boldsymbol{I}}(0)\big)-\overline{\boldsymbol{V}}(l)=\boldsymbol{0}\end{aligned} \tag{19}$$

$$\begin{aligned}&\overline{\boldsymbol{T}}_I(\boldsymbol{x})\big(-\sinh\bar{\boldsymbol{\gamma}}(\boldsymbol{x})l\cdot\overline{\boldsymbol{T}}_I^{-1}(\boldsymbol{x})\overline{\boldsymbol{Z}}_C^{-1}(\boldsymbol{x})\overline{\boldsymbol{V}}(0)+\\&\cosh\bar{\boldsymbol{\gamma}}(\boldsymbol{x})l\cdot\overline{\boldsymbol{T}}_I^{-1}(\boldsymbol{x})\bar{\boldsymbol{I}}(0)\big)-\bar{\boldsymbol{I}}(l)=\boldsymbol{0}.\end{aligned} \tag{20}$$

Although the functions $\bar{\boldsymbol{\gamma}}(\boldsymbol{x})$, $\overline{\boldsymbol{T}}_I(\boldsymbol{x})$, and $\overline{\boldsymbol{Z}}_C(\boldsymbol{x})$ do not have explicit analytical form, they can be calculated for given component values of vector $\boldsymbol{x}$ using the formulas derived in previous section. Thus (19) and (20) represent a system of $2n$ individual equations in m unknown parameters and they can be presented in compact form

$$\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{0} \tag{21}$$

where $\boldsymbol{x}=[x_1\cdots x_m]^{\mathrm{T}}$, $\boldsymbol{f}(\boldsymbol{x})=[f_1(\boldsymbol{x})\cdots f_{2n}(\boldsymbol{x})]^{\mathrm{T}}$, $\boldsymbol{0}=[0\cdots 0]^{\mathrm{T}}$. Generally, system of nonlinear equations can be solved using some optimization methods, simplicial algorithm, or homotopy approach. In this paper equation (21) will be solved using an iterative method described in the sequel. Let $\boldsymbol{x}^{(k)}$ be the vector $\boldsymbol{x}$ at k-th iteration. To calculate $\boldsymbol{x}^{(k+1)}$ at $(k+1)$-st iteration the function $\boldsymbol{f}(\boldsymbol{x})$ is approximated by the first two terms of a Taylor series expansion

$$\boldsymbol{g}(\boldsymbol{x})=\boldsymbol{f}\left(\boldsymbol{x}^{(k)}\right)+\boldsymbol{J}\left(\boldsymbol{x}^{(k)}\right)\left(\boldsymbol{x}-\boldsymbol{x}^{(k)}\right) \tag{22}$$

where $\boldsymbol{J}\left(\boldsymbol{x}^{(k)}\right)$ is the $2n\times m$ Jacoby matrix at $\boldsymbol{x}=\boldsymbol{x}^{(k)}$. Let $\boldsymbol{x}^{(k+1)}$ meet the equation $\boldsymbol{g}\left(\boldsymbol{x}^{(k+1)}\right)=\boldsymbol{0}$. Then combining this equation and equation (22) we write

$$\boldsymbol{J}\left(\boldsymbol{x}^{(k)}\right)\left(\boldsymbol{x}^{k+1}-\boldsymbol{x}^{(k)}\right)=-\boldsymbol{f}\left(\boldsymbol{x}^{(k)}\right), \tag{23}$$

where $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{J}(\boldsymbol{x})$ are complex and vector $\boldsymbol{x}$ is real. Equation (23) can be written in the form

$$\boldsymbol{B}\left(\boldsymbol{x}^{(k)}\right)\left(\boldsymbol{x}^{k+1}-\boldsymbol{x}^{(k)}\right)=\boldsymbol{b}\left(\boldsymbol{x}^{(k)}\right) \tag{24}$$

where $\boldsymbol{B}\left(\boldsymbol{x}^{(k)}\right)=\begin{bmatrix}\mathrm{Re}\left(\boldsymbol{J}\left(\boldsymbol{x}^{(k)}\right)\right)\\ \mathrm{Im}\left(\boldsymbol{J}\left(\boldsymbol{x}^{(k)}\right)\right)\end{bmatrix}$, $\boldsymbol{b}\left(\boldsymbol{x}^{(k)}\right)=-\begin{bmatrix}\mathrm{Re}\left(\boldsymbol{f}\left(\boldsymbol{x}^{(k)}\right)\right)\\ \mathrm{Im}\left(\boldsymbol{f}\left(\boldsymbol{x}^{(k)}\right)\right)\end{bmatrix}$.

The equation consists of $4n$ individual real equations in m real unknowns being the components of vector $\boldsymbol{x}^{(k+1)}$. We assume that $m\le 4n$ what according to Table 1 means that the number of the line conductors $(n+1)$ does not exceed five. To solve equation (24) for $\boldsymbol{x}^{(k+1)}$ we can use the normal equation method

$$\boldsymbol{B}^{\mathrm{T}}\left(\boldsymbol{x}^{(k)}\right)\boldsymbol{B}\left(\boldsymbol{x}^{(k)}\right)\left(\boldsymbol{x}^{k+1}-\boldsymbol{x}^{(k)}\right)=\boldsymbol{B}^{\mathrm{T}}\left(\boldsymbol{x}^{(k)}\right)\boldsymbol{b}\left(\boldsymbol{x}^{(k)}\right). \tag{25}$$

Equation (25) represents system of m individual linear algebraic equations in m unknowns $x_1^{(k+1)}, x_2^{(k+1)}, ..., x_m^{(k+1)}$ at $k=0,1,2,...$. The determinant of matrix $\boldsymbol{B}^{\mathrm{T}}\left(\boldsymbol{x}^{(k)}\right)\boldsymbol{B}\left(\boldsymbol{x}^{(k)}\right)$ is greater than or equal to zero [10]. When it equals zero, the iteration method fails. If the determinant is close to zero the method may not converge. To omit this drawback we modify iteration equation (25) as in [10]

(26)

$$\left[\boldsymbol{B}^{\mathrm{T}}\left(\boldsymbol{x}^{(k)}\right)\boldsymbol{B}\left(\boldsymbol{x}^{(k)}\right)+\alpha\mathrm{e}^{-\beta k}\boldsymbol{1}\right]\left(\boldsymbol{x}^{k+1}-\boldsymbol{x}^{(k)}\right)=\boldsymbol{B}^{\mathrm{T}}\left(\boldsymbol{x}^{(k)}\right)\boldsymbol{b}\left(\boldsymbol{x}^{(k)}\right)$$

where α and β are positive constants selected on the basis of numerical experiments (see next section). As k increases the diagonal elements of the matrix $\alpha\mathrm{e}^{-\beta k}\boldsymbol{1}$ decrease and tend to zero for large value of k. In consequence equation (26) approaches equation (25).To explain in detail how the method developed in this section works we consider, without any loss of generality, a circuit including 3-conductor transmission line. The line is specified by the matrices

$\boldsymbol{L}=\begin{bmatrix} l_{11} & l_{12} \\ l_{12} & l_{22} \end{bmatrix}$, $\boldsymbol{R}=\begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$, $\boldsymbol{G}=\begin{bmatrix} \widetilde{g}_{11} & -g_{12} \\ -g_{12} & \widetilde{g}_{22} \end{bmatrix}$, $\boldsymbol{C}=\begin{bmatrix} \widetilde{c}_{11} & -c_{12} \\ -c_{12} & \widetilde{c}_{22} \end{bmatrix}$ and the constants ε_r and σ. The unknown variables in equation (21) are: $x_1 = s_1^{-1} r_1$, $x_2 = s_2^{-1} r_2$, $x_3 = s_3^{-1} l_{11}$, $x_4 = s_4^{-1} l_{22}$, $x_5 = s_5^{-1} l_{12}$, $x_6 = s_6^{-1} \varepsilon_r$, $x_7 = s_7^{-1} \sigma$. They are components of vector $\boldsymbol{x}$.

Outline of the computation process

1. Measure input and output voltage phasors of the line at given frequency.
2. Replace the transmission line by the input and output voltage sources as shown in Fig. 2 and analyse the obtained circuit to find the input and output current phasors.
3. Pick an initial guess $\boldsymbol{x}^{(0)}$ and perform the iteration process specified by formula (26). For given k - th iteration $\boldsymbol{x}^{(k)}$ matrix $\boldsymbol{B}\left(\boldsymbol{x}^{(k)}\right)$ and vector $\boldsymbol{b}\left(\boldsymbol{x}^{(k)}\right)$ (see (25)) are calculated as described in step 4.
4. We find

$$\boldsymbol{R}\left(\boldsymbol{x}^{(k)}\right)=\begin{bmatrix} s_1 x_1^{(k)} & 0 \\ 0 & s_2 x_2^{(k)} \end{bmatrix}, \quad \boldsymbol{L}\left(\boldsymbol{x}^{(k)}\right)=\begin{bmatrix} s_3 x_3^{(k)} & s_5 x_5^{(k)} \\ s_5 x_5^{(k)} & s_4 x_4^{(k)} \end{bmatrix},$$

$$\boldsymbol{C}\left(\boldsymbol{x}^{(k)}\right)=\mu_0 \varepsilon_0 s_6 x_6^{(k)} \begin{bmatrix} s_3 x_3^{(k)} & s_5 x_5^{(k)} \\ s_5 x_5^{(k)} & s_4 x_4^{(k)} \end{bmatrix}^{-1},$$

$$\boldsymbol{G}\left(\boldsymbol{x}^{(k)}\right)=\mu_0 s_7 x_7^{(k)} \begin{bmatrix} s_3 x_3^{(k)} & s_5 x_5^{(k)} \\ s_5 x_5^{(k)} & s_4 x_4^{(k)} \end{bmatrix}^{-1},$$

$$\overline{\boldsymbol{Z}}\left(\boldsymbol{x}^{(k)}\right)=\boldsymbol{R}\left(\boldsymbol{x}^{(k)}\right)+\mathrm{j}\omega \boldsymbol{L}\left(\boldsymbol{x}^{(k)}\right),$$

$$\overline{\boldsymbol{Y}}\left(\boldsymbol{x}^{(k)}\right)=\boldsymbol{G}\left(\boldsymbol{x}^{(k)}\right)+\mathrm{j}\omega \boldsymbol{C}\left(\boldsymbol{x}^{(k)}\right).$$

For matrix $\overline{\boldsymbol{Y}}\left(\boldsymbol{x}^{(k)}\right)\overline{\boldsymbol{Z}}\left(\boldsymbol{x}^{(k)}\right)$ there are calculated eigenvalues $\overline{\gamma}_1^2\left(\boldsymbol{x}^{(k)}\right), \ldots, \overline{\gamma}_n^2\left(\boldsymbol{x}^{(k)}\right)$, and matrices $\overline{\boldsymbol{T}}_I\left(\boldsymbol{x}^{(k)}\right)$, $\cosh\overline{\boldsymbol{\gamma}}\left(\boldsymbol{x}^{(k)}\right)l$, $\sinh\overline{\boldsymbol{\gamma}}\left(\boldsymbol{x}^{(k)}\right)l$ and after that $\overline{\boldsymbol{Z}}_C\left(\boldsymbol{x}^{(k)}\right)$. Next we find $\boldsymbol{f}\left(\boldsymbol{x}^{(k)}\right)$ and $\boldsymbol{J}\left(\boldsymbol{x}^{(k)}\right)$, create equation (26) and solve it for $\boldsymbol{x}^{(k+1)}$. The elements of matrix $\boldsymbol{J}\left(\boldsymbol{x}^{(k)}\right)$ are determined using the incremental method. Having $\boldsymbol{f}\left(\boldsymbol{x}^{(k)}\right)$ and $\boldsymbol{J}\left(\boldsymbol{x}^{(k)}\right)$ we find $\boldsymbol{b}\left(\boldsymbol{x}^{(k)}\right)$ and $\boldsymbol{B}\left(\boldsymbol{x}^{(k)}\right)$.

5. The iteration process is running until $\left\|\boldsymbol{x}^{(k+1)}-\boldsymbol{x}^{(k)}\right\| < \mu$ and $\left\|\boldsymbol{f}\left(\boldsymbol{x}^{(k+1)}\right)\right\| < \eta$, where μ and η are the convergence tolerances. Next we find the actual parameters using the inverse scaling $p_1 = s_1 x_1^{(k+1)}, \ldots, p_m = s_m x_m^{(k+1)}$ and using equations (3) and (4) calculate the remaining line parameters. If the above inequalities do not hold in some number of iterations N_{I} (say $N_{\mathrm{I}} = 100$) the computation process is terminated. In such case we can pick a different initial guess and repeat the procedure.

Numerical example

Proposed method was implemented in MATLAB environment. To illustrate the method a circuit including 3-conductor transmission lines shown in Fig. 3 was considered. Values of the lumped elements are indicated in the figure. The constants which appear in the iteration formula (26) are $\alpha = 0.005$, $\beta = 0.5$ and the chosen frequency is $f = 100\ \mathrm{MHz}$. The input and output voltage phasors of the line were obtained by simulation of the circuit.

Example

Consider the circuit shown in Fig. 3 where the 3-conductor transmission line has length $l = 0.4\ \mathrm{m}$ and amplitude of the power supplied voltage source is 5 V. We take into account three sets of values of the transmission line parameters summarized in Table 2. Proposed method uses the parameters $p_1 = r_1$, $p_2 = r_2$, $p_3 = l_{11}$, $p_4 = l_{22}$, $p_5 = l_{12}$, $p_6 = \varepsilon_r$, $p_7 = \sigma$.

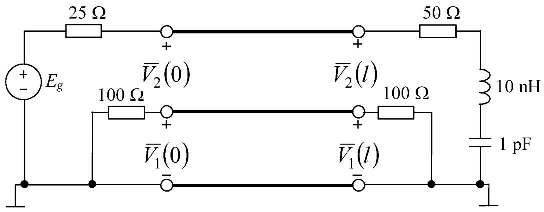

Fig.3. Circuit including 3-conductor TL

Table 2. Parameters of the line of Fig. 3

Transmission line parameters	True values of the line parameters		
	Case 1	Case 2	Case 3
r_1 [Ω/m]	0.8	50	5
r_2 [Ω/m]	0.6	50	5
l_{11} [H/m]	4.2·10^{-7}	3.8·10^{-7}	7·10^{-7}
l_{22} [H/m]	4·10^{-7}	3.6·10^{-7}	7.6·10^{-7}
l_{12} [H/m]	3.1·10^{-8}	1.1·10^{-8}	3.1·10^{-8}
ε_r	2.2	3.6	4.6
σ [S/m]	0.026	0.05	0.13
g_{11} [S/m]	0.0722	0.1604	0.2243
g_{22} [S/m]	0.0761	0.1696	0.2058
g_{12} [S/m]	0.0061	0.0051	0.0095
c_{11} [F/m]	5.40·10^{-11}	1.02·10^{-10}	7.02·10^{-11}
c_{22} [F/m]	5.69·10^{-11}	1.08·10^{-10}	6.44·10^{-11}
c_{12} [F/m]	4.54·10^{-12}	3.22·10^{-12}	2.98·10^{-12}

The scaling factors are: $s_1 = 1$, $s_2 = 1$, $s_3 = 10^{-7}$, $s_4 = 10^{-7}$, $s_5 = 10^{-7}$, $s_6 = 1$, $s_7 = 10^{-2}$. In each of the three cases we applied the method nine times starting with nine initial guesses. Thus, we performed 27 numerical experiments. Any of the initial guesses is created by picking first some initial values of the line parameters and next scaling them using the scaling factors defined above. The initial p–u–l parameters of resistances and inductances and constants ε_r and σ according to the pattern $\left\{r^0\ r^0\ l^0\ l^0\ 0.1l^0\ \varepsilon_r^{\ 0}\ \sigma^0\right\}$, where $1 \le r^0 \le 10$, $10^{-8} \le l^0 \le 10^{-6}$, $1 \le \varepsilon_r^{\ 0} \le 5$, $0.01 \le \sigma^0 \le 0.1$ and next, after scaling, the initial guess is obtained. Every time the computation process is convergent and gives, after rescaling, the line parameters which are the same as the actual ones presented in Table 2. The number of the iterations is less than 30. All the 27 numerical experiments with different initial guesses gave correct transmission line parameters.

We investigate, in the simulation way, the influence of the measuring instrument accuracy on the results provided by the method using the circuit shown in Fig. 3. We consider various accuracies of the voltage amplitude measurement and the phase measurement. Since any phasor is specified by its amplitude and phase, any measurement is characterized by a pair of amplitude and phase accuracies. We considered 10 pairs of the measurement accuracies. Sixty experiments were carried out. The voltage phasors were obtained by correcting the simulated values according to the assumed accuracy. The method found the solutions in 85% experiments and failed in 15% experiments. In all the 51 cases the p–u–l parameters of inductance and resistance r_2 as well as constants ε and σ given by the method are close to the true values. Relative errors of the p–u–l parameters of inductance do not exceed 5.5%, of resistance r_2 do not exceed 7.7 % while the relative errors of constants ε_r and σ do not exceed 0.4%. Only the p–u–l resistance r_1 calculated by the method in 20 out of 51 cases it is incorrect. Assuming measurement accuracies possible in the laboratory e.g. pair 10^{-4} V, $\left(10^{-1}\right)^{\text{o}}$ results in the identification of all (except one) parameters with acceptable accuracy.

Extension of the method

The method may be extended in straightforward manner to a broader class of circuits including *M* multiconductor transmission lines (see Fig. 4). Conceptually the approach is the same as discussed in the previous sections but the number of nonlinear equations increases and the method requires more computation power.

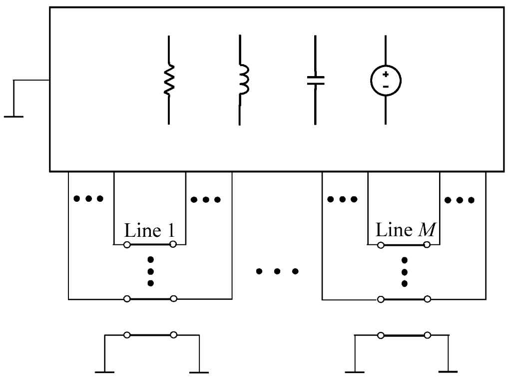

Fig.4. Circuit including M multiconductor transmission lines

Conclusion

The paper is devoted to extracting the p–u–l parameters of uniform MTLs immersed in homogenous medium. This is crucial step for the analysis of circuits including MTLs. As opposed to the known methods this paper offers a numerical iterative method for calculating the p–u–l parameters. The method is preceded by a simulation test in the frequency domain. While running the test input and output voltage phasors are simulated in the circuit including MTLs. Numerical experiments reveal that the results given by the method are very close to the true values.

The iteration process is very fast and does not require great computer power. Convergence of the process depends on the initial guess. It is selected in two steps. If the method does not converge in a preset number of iterations the computation process should be terminated and repeated using another initial guess. The method has been verified on several examples of three-, four-, and five-conductor TLs and a system containing three three-conductor TLs. In 90% of cases, the correct line parameter values were obtained. Reducing the measurement accuracy (by simulation) makes some elements of the resistance matrix estimated less accurately or the values of these elements are incorrect. This effect was not observed for other p-u-l parameters.

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