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doi:10.15199/48.2025.01.49

Analysis of dynamic states in pump drive systems incorporating a long elastic shaft

Abstract. *In the article, a mathematical model of a complex drive system was developed, consisting of an-air coil, an asynchronous motor, a long shaft with elastic clutches. This model is based on interdisciplinary variational approaches. Transient states analysis of the drive system was performed for various clutches with different parameters. The influence of the clutch selection on the dynamic states of the pump drive system was determined. The moments of elasticity, dissipation, voltage and voltage drops in the power supply network were determined. The results of the computer simulation are presented in the figures form.*

Streszczenie*. W artykule opracowano matematyczny model złożonego układu napędowego, składającego się z cewki powietrznej, silnika asynchronicznego, długiego wału z elastycznymi sprzęgłami. Model ten bazuje na interdyscyplinarnych podejściach wariacyjnych. Analizę stanów przejściowych układu napędowego przeprowadzono dla różnych sprzęgieł z różnymi parametrami. Określono wpływ wyboru sprzęgła na stany dynamiczne układu napędu pompy. Wyznaczono momenty sprężystości, rozproszenia oraz napięcie i spadki napięcia w sieci zasilającej. Wyniki symulacji komputerowej przedstawiono w postaci rysunków. (****Analiza stanów dynamicznych w układach napędowych pomp z długim elastycznym wałem****).*

Keywords: interdisciplinary modelling, drive systems, pump systems, asynchronous motors, long shaft, complex motion transmission.
Słowa kluczowe: modelowanie interdyscyplinarne, układy napędowe, napędy pomp, silniki asynchroniczne, długi wał, złożona transmisja ruchu.

Introduction

Long shaft pump driver systems are used in a wide range of application and sizes. Pump drive systems with high capacities can be used as cooling pumps and in pumping systems: seawater, groundwater, irrigation. They also find application in the chemical and petrochemical industries. Pump drive systems with a long shaft are also used in firefighting in places far from land, such as sea platforms [1]. The use of pump systems in so many services explains why they are the subject of much research. Such pump drive systems are subject of research for rotor strength and critical speed for various shaft lengths. Research on pump drive systems with long shaft include the analysis of rotor vibrations, which can cause resonance and cause enormous damage [2]. The discussed drive systems were tested to analyze the hydrodynamic properties of the pumping system, and experiments were performed on the energy characteristics and measurement of pressure fluctuations for high flow pumps [3]. The research also covers the selection, suitability and efficiency of pump systems for various substances. In vertical pump systems, the forces acting on the bearing system and the misaligned coupling were analyzed [4].

Pump drive systems have been and are the subject of many studies, but the topic of forces acting on the long elastic shaft of the drive system is not fully described. Research on pump drive systems with a long elastic shaft is aimed at analyzing and explaining the moments that act on the long shaft during operation. This is important due to the widespread use of vertical pumps with a long shaft in many areas, including in conventional power plants for drawing water from rivers and lakes In places such as power plants, it is important to ensure a continuous supply of water from rivers to ensure the safe operation of the plant In such a case, the energy security in a given region or, in some cases, the energy security in the country depends on such pumping systems.

An example would be the power plant located in Kozienice. Pump drive systems with a long shaft were used there. The power plant is located on the Vistula River, where there is a large difference in water levels throughout the year in winter and summer. In the central Vistula, changes in water flow range from 97.7 m^3/s to 7550 m^3/s. For this difference in water flow, the change in the water height in the river is 6.18 m. The presented data were recorded for the section between Pulawy and Magnuszew [5]. In that cases, the use of pump systems with a long shaft is crucial. Due to such a difference in water level, the engine driving the pump system must be located above the flood level.

The disadvantage of a long shaft is the fact that there are no ideal materials that are resistant to torsional forces. More importantly is fact, have their own mechanical strength. Incorrect selection of materials to build a long shaft that will not withstand high stresses will result in its damage or, in exceptional cases, damage to the pump blades and the electric motor.

Based on data from the flow and changes in water level in the Vistula River and the lack of ideal materials, research of pump drive systems with long shaft is crucial to ensuring the safe operation of the power plant.

In order to fully describe the physical processes in systems, it is necessary to take into account the oscillating processes in drive systems. Calculations of oscillating vibration processes in electric drives can be carried out in two ways depending on the length of the drive shafts. From the developing a model of the system point of view, the mentioned ways are diametrically opposed. In the case when the drive shafts are not long, the connection of the motor with the working machine is made through a elastic clutch, which is a spring-elastic connection. Such a system is considered as a system with concentrated parameters and is described on the basis of the Euler-Lagrange theory. If the results obtained are not reliable, the Euler-Poisson theory should be applied. The analyzed system is described as a system with distributed parameters. From the point of view of mathematics, in the case where the shaft is short, the drive system is described by ordinary differential equations. In the case of long shafts, on the other hand, the drive system is described by equations with partial derivatives, which boil down to determining boundary or mixed tasks.

By creating complex models of the drive engine, clutch and driven working machine, the obtained system model is becoming complicated and the description of the components of the drive system belongs to various fields of science. Depending on the type of load on electric motors, transient processes have different forms. This fact requires thoroughly analyze drive systems of us, which forces us to use various fields of science for the analysis of systems. In order for a specialist in electrical engineering to be able to solve the tasks set, he should also have extensive knowledge in other fields of science: mechanics, hydraulics, in order to ultimately develop a model of a consistent system.

In order to avoid separate electrical, mechanical, pump analyses, it is proposed to use interdisciplinary approaches to modelling. The basis for such an analysis is the application of the modified integral variational Hamilton–Ostrogradsky principle, by extending the conservative Lagrangian function. It should be emphasized that this approach is valid for both systems with focused and distributed parameters [6], [7], [8].

Multiple energy conversion complicates the analysis of electro-mechanical-hydraulic processes for a number of reasons. The process itself is complicated, and moreover there are no developed mathematical models of vertical pumps integrated in the electromechanical part of the drive system [9], [10], [11], [12].

Mathematical model of the system

Figure 1 shows the basic system of motion transmission and the calculation diagram of the pump drive.

The long transmission shaft is on both sides connected to the motor shaft and vertical pump by additional elastic clutches, which significantly reduces shock torques on the motor and pump shafts, as well as reduces torsional torques in the long shaft - Fig.1.

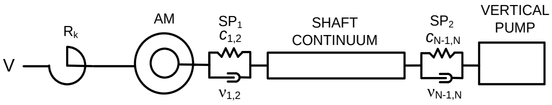

Fig.1. Basic scheme of the pumping system

The solution of the basic task is complicated by the fact that the parameters of the transmission system are distributed, and this leads to the determination, in addition to the energetic components of the extended lagrangian, of the linear density of the functions of the lagrangian components. The modified Hamilton–Ostrogradsky principle is defined by the following equation. [13]:

$$S=\int_{t_1}^{t_2}\left(L^*+\int_l L_l dl\right)dt,\ \ I=\int_l L_l dl\ , \tag{1}$$

where S – operations according to Hamilton-Ostogradsky, L^* – modified Lagrangian function, L_l – density of modified Lagrangian function, I – internal energy function. The extended function and its linear density are defined by dependency:

$$L^*=\tilde{T}^*-P^*+\Phi^*-D^*\ ,\ \ L_l=T_l-P_l+\Phi_l-D_l\ , \tag{2}$$

where $\tilde{T}^*$ – kinetic coenergy, P^* – potential energy, F^* – energy of dissipative forces, D^* – energy of external and internal inpotential forces, T_l, P_l, F_l, D_l – relevant linear densities of energy functions.

To obtain a mathematical model of an object, one should determine the variation of the functional and compare it to zero [8], [13].

$$\delta S=\delta\int_0^{t_1}\left(L^*+\int_l Ldl\right)dt=\int_0^{t_1}\delta L^* dt+\int_0^{t_1}\int_l \delta Ldldt\ . \tag{3}$$

The equations obtained on this basis are Euler-Lagrange (Euler-Poisson) equations, in other words – a mathematical model of the pump unit under consideration.

The first step to develop a model of the proposed method is to create a calculation scheme of the analyzed object. In this case, a motion transmission system for all drives in discrete space is introduced [14].

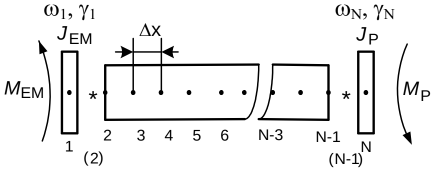

Fig.2. Calculation scheme of the shaft continuum

Mathematical model of a vertical pump

When creating a model of a mathematical hydraulic subsystem containing a vertical pump type OB 16-87, the results of numerical modeling is used.

Figure 2 shows the results of calculations for the pump OB 16-87 and of the selected pipeline. The coordinates of the operating points (Q, H) for each calculated speed n have been numerically calculated [15, 16].

The characteristics of the pump lift height OB 16-87 have been approximated by parabolic dependencies:

$$H_p=a+bQ+cQ^2\ , \tag{4}$$

where: a, b, c − factors that depend on speed n, are determined using the least squares method [16].
For a rotational speed of 585 rpm, the factors are:

$$H_{p585}=4.949+5.268Q-1.56Q^2\ , \tag{5}$$

And for the speed of 300 rpm:

$$H_{p300}=1.473+2.702Q-1.56Q^2\ , \tag{6}$$

The relationship between efficiency, lifting height of the pipeline and a pump as a function of efficiency is shown in Figure 3.

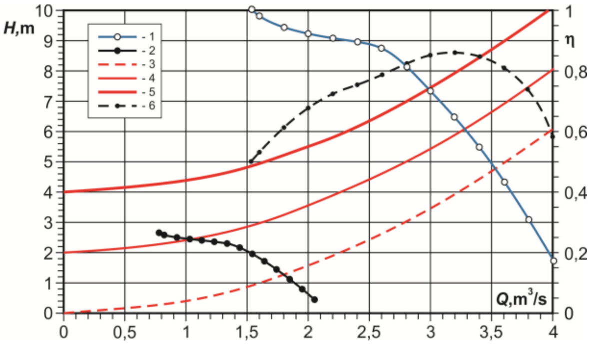

Fig.3. Characteristics of efficiency, lifting height: pipeline and a vertical pump as a function of flow Q

Characteristics 1, 2 were made for rotational speeds: 585 and 300 rpm; characteristics 3, 4, 5 represent the lifting height of the pipeline for respectively: Hg = 0 m; Hg = 2 m; Hg = 4 m; and characteristic 6 represents the efficiency of the pump. The coordinates of the operational points (Q, H)

for each calculated speed n have been numerically calculated [15, 17].

The efficiency of the pump η' for speed $n < n_o$ when $H_g > 0$ differs from the value n_o. The conversion of the efficiency value was performed on the basis of the similarity of states method:

$$H_i = H_o (Q_i / Q_o)^2 , \tag{7}$$

Solving equations (4) and (7) relative to Q_i for the coefficients a_i, b_i, c_i, which corresponds to the velocities n_i the calculated load moment of the drive motor is found:

$$M = \frac{30\rho g Q H}{\pi n \eta'} . \tag{8}$$

Figure 4 shows the dependence of the moment as a function of angular velocity determined on the basis of similarity theory using the method of least squares for the height of water lift: 1 − H_g=0 m; 2 − H_g =2 m; 3 − H_g =4 m; H_g =6 m [15]. The approximation of waveforms in Figure 4 was made with accuracy R^2.

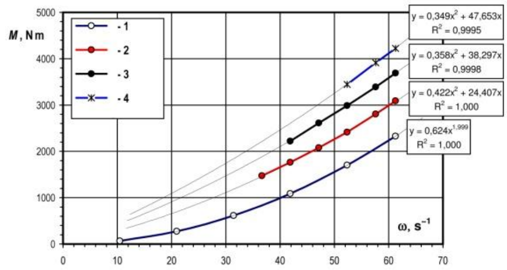

Fig.4. Dependence of torque on the pump shaft as a function of angular velocity

The torque characteristics of the vertical pump are approximated with a parabolic curve, which simplifies the mathematical modeling theory, because there are no complicated formulas with conventional operators in the model.

Computer simulation results

A transient states analysis was carried out for the drive train as shown in Figure 1. The 12-52-8A engine ratings are as follows: P_N = 320 kW; U_N = 6 kV; I_N = 39 A; ω_N = 740, s^{-1}, p = 4, J_1 = 49 kg·m^2, J_2 = 50 kg·m^2. The magnetization curve of the motor is approximated by the equation: Ψ_m = 12.4 arctg(0,066i_m). The parameters of the air coil have values: R_O = 1.2 Ω, L_O = 30 mH. The tests were carried out for the water lifting height H_g = 4m as follows. At the moment of t = 0s, the pump system shown in Figure 1 is supplied from the rigid network. In the steady state, the rotor overrun is led after 5 seconds. There were two experiments lasting 16 seconds each conducted. In the analysis of oscillating processes in the system, elastic clutches in the transmission of drive motion were changed. The simulation was performed for two experiments with the following parameters of elastic couplings:

1. – $c_{1,2}$ = 3.09 MN·m, $\nu_{1,2}$ = 70 N·m·s, f_0 = 48.5 Hz;

2. – $c_{1,2}$ = 3.75 MN·m, $\nu_{1,2}$ = 85 N·m·s, f_0 = 53.4 Hz;

where: $c_{1,2}$ – clutch stiffness factor, $\nu_{1,2}$ – clutch stiffness factor, f_0 – fundamental frequency of the pump system.

Elastic clutches have been selected in such a way as to represent the different mechanical states of the pump system in the sense of oscillating processes.

The results of the computer simulation are presented in the form of drawings.

When analysing Figure 5 and the clutch parameters during resonance, it should be explained why the natural resonant frequency is 48.5 Hz when the motor is supplied with 50 Hz. The asynchronous machine cannot spin at synchronous speed and the resonant frequency is reduced by the slip value s of the asynchronous motor. The torque moment oscillation in Figure 5 occurs at a frequency of about 48.5 Hz.

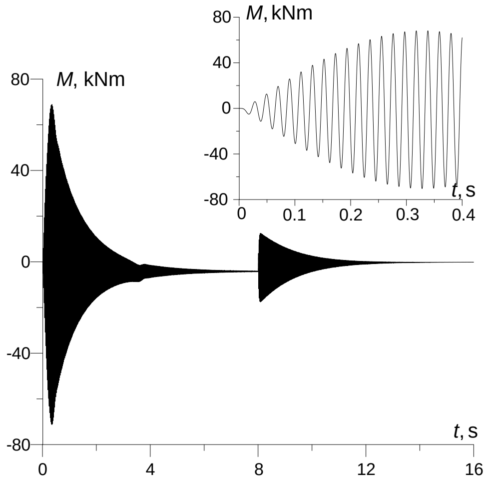

Fig.5. Instantaneous moment of elasticity in the elastic clutch for the first clutch

In order to more accurately visualize the vibrational frequency of the elasticity moment in work, most of the drawings contain additional mini drawings with a small time range, in which you can clearly see the own resonant frequency of the dual-mass system.

Figures 6 – 7 show the instantaneous waveforms: the stator phase A voltage and the voltage drop on the air coil windings in the first experiment in the resonant state.

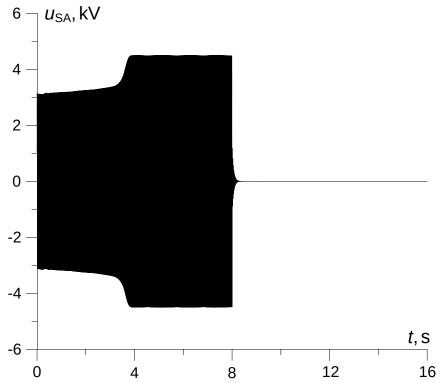

Fig.6. Instantaneous phase supply voltage of phase A of the motor for the first experiment.

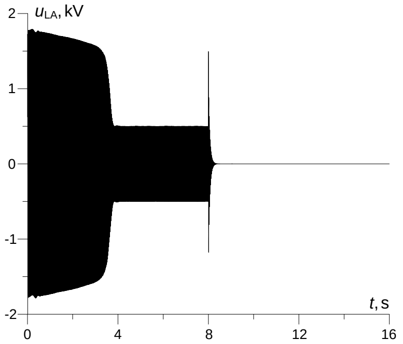

Fig.7. A momentary voltage drop of phase A on the air coil for the first experiment

Figures 8, 9 show the waveforms: the instantaneous values of the starting electromagnetic moment of the asynchronous motor, the elastic moment in the elastic.

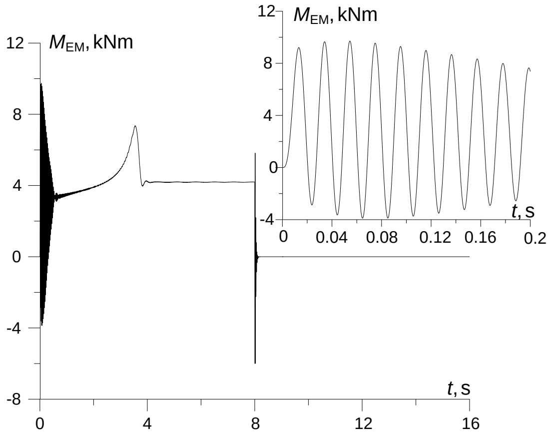

Fig.8. Instantaneous machine starting torque for the second experiment

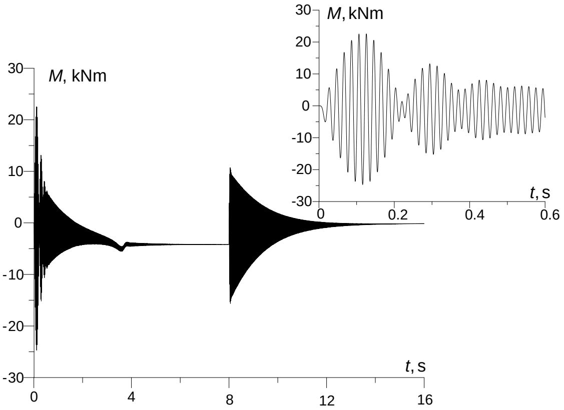

Fig.9. Instantaneous moment of elasticity in the elastic clutch for the second experiment

Conclusion

Consideration of interdisciplinary variational approaches to modeling pump systems that work under complex and harsh operating conditions simplifies the creation of a mathematical model of a single integrated electro-mechanical-hydraulic system.

There is also a very important question of the reliability of motion transmission of asynchronous drives, controlled by frequency converters. Such drive trains are more susceptible to operate in resonant and near-resonant states (rumbling)

An important criterion for choosing a elastic clutch in motion transmission is its main purpose: various types of parasitic vibrations damping. The selection of clutches should be made after prior analysis of the entire drive system. Experimental research is expensive and sometimes dangerous, and for the effective solving of the tasks set, the mathematical modeling apparatus should be used.

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