1. Valeriy OBORZHYTSKYY[1], 2. Stefan PASZEK[2], 3. Anna PIWOWAR[2]

Lviv Polytechnic National University (1), Silesian University of Technology (2)
ORCID: 1.0000-0002-4829-7451; 2.0000-0001-8483-7032; 3.0000-0002-5578-3757

doi:10.15199/48.2025.02.33

Application of the symmetric and antisymmetric excitation technique at the symmetrical two-port networks design

***Abstract**. The application of the symmetric-antisymmetric excitation method, which decomposes a two-port network with a symmetric structure into two separate two-pole half-circuits, was considered. For the input impedances of these half-circuits, analytical expressions were written, which relate them with the two-port device parameters being designed based on a two-pole network. The values of the input impedances calculated by these expressions allowed to determine the parameters of the elements of the circuit, which had been chosen for the implementation of this device.*

***Streszczenie.** Rozważono zastosowanie metody wzbudzenia symetryczno-antysymetrycznego, która dekomponuje czwórnik o symetrycznej strukturze na dwa oddzielne dwubiegunowe półobwody. Dla impedancji wejściowych tych półobwodów zapisano wyrażenia analityczne, które wiążą je z parametrami czwórnika projektowanego na podstawie układu dwójników. Wartości impedancji wejściowych obliczone za pomocą tych wyrażeń pozwalają określić parametry elementów obwodu, który został wybrany do realizacji tego urządzenia (**Zastosowanie techniki symetrycznego i antysymetrycznego wzbudzenia w projektowaniu czwórników symetrycznych**)*

Keywords: symmetrical two-port network, even-odd excitation, input impedance, Z-parameters.
Słowa kluczowe: czwórnik symetryczny, wzbudzenie parzysto-nieparzyste, impedancja wejściowa, parametry Z.

Introduction

Two-port networks have a wide range of applications in electronics, transmission and communication systems, automatic control systems, etc. The mathematical representation of such networks is based on relations that link input and output voltages and currents. In general, such ratios include four coefficients, which are complex parameters of a two-port network - complex elements of the parameter matrix of network that determine its operational characteristics and are calculated in accordance with the possibility of ensuring them. Then, from the values of these four complex parameters, the transition is made to the values of the element's parameters of the circuit, which is selected for the implementation of the two-port network. Such a transition is often associated with using of optimization methods or circuit synthesis methods, such as filters.

In many cases, one or another function of a two-port network is performed by circuits with electrical symmetry. It is known that a network with two ports is called symmetrical if the input and output ports can be swapped without changing the voltage and current on the ports. For such symmetrical circuits, only two complex parameters out of four will be different. This greatly simplifies the process of finding the values of the parameters of the circuit elements that provide the desired characteristics. In contrast to the asymmetric case, where complex circuit parameters (or matrix elements) are usually determined using short-circuit, open-circuit, and load tests, the bisection method has been used for symmetric circuits. The mentioned method is the basis of Bartlett's bisection theorem, which shows that any symmetric two-port network can be transformed into a lattice network [1]. For such a transformation, a symmetric network is divided into two identical halves with respect to its symmetry plane. The use of two variants of simultaneous excitation of both network ports results in two pairs of identical halves, which are two-poles in structure and are known as half-circuits. The analysis of the original two-port network is based on the analysis of such two half-circuits. Quite often, for example, [2], such a network division is considered as an eigenstate decomposition. The use of this concept allowed [3] to develop a general equivalent circuit for two-port asymmetric mutual networks with losses. In [4], the bisection theorem was extended to the case of symmetric circuits with cross-coupling. The technology of dividing the symmetric network into halves has found wide application in microwave technics for circuits with distributed parameters, where it is better known as the method of even-mode and odd-mode excitation [5] or method of symmetric and antisymmetric excitation. This method makes it possible to determine the parameters of the circuit elements selected for the implementation of a two-port network based on the input impedances of its half-circuits. In the present paper, we consider the use of this approach to the design of two-port networks based on lumped elements.

Bisection of a symmetrical two-port network

For a linear two-port network, the relationship between the voltages and currents at its input and output is most often defined through the four of *Z* or *Y* parameters, also known as impedance or admittance parameters. Since the relationships between the *Z*-parameters and the *Y*-parameters are well known, only the *Z*-parameters will be used in the following. To describe a two-port network with an electrical axis of symmetry, only two independent Z-parameters, namely, Z_{11} and Z_{12}, are sufficient. Using the bisection method, we reduce such a symmetric network to two simpler two-pole circuits using the excitation of its ports by symmetric (even) and antisymmetric (odd) sources.

Under symmetric (even-mode) excitation, when both ports are simultaneously supplied with the same voltage, a two-pole network can be conditionally divided into two identical two-pole circuits. In this case, no current flows along the connection lines and through the elements that cross the plane of symmetry, i.e., no energy transfer occurs through the plane of symmetry, which corresponds to the open-circuit (idle) mode in this plane (Fig. 1a). Each of such two-pole half-circuits characterizes the input impedance Z_e.

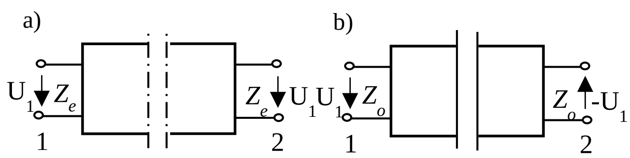

Fig.1. Bisection of a two-port network into half-circuits

Under antisymmetric (odd-mode) excitation, when both ports of a two-port network are simultaneously supplied with the same but opposite voltage, there will be a zero potential along the middle of the network. Such a network can be conditionally divided into two identical two-pole circuits, since the energy will not be transferred across the plane of symmetry, which corresponds to the short circuit mode in this plane (Fig. 1b). The input impedance Z_o is used to characterize these two-pole half-circuits.

It is easy to show that the input impedances of the half-circuits are related to the parameters of a symmetrical two-port network as follows:

(1) $$Z_e = Z_{11} + Z_{12}, \qquad Z_o = Z_{11} - Z_{12}.$$

If such a network division is considered as an eigenstate decomposition, the eigenvalues of the impedance matrix Z are the input impedances of the half-circuits and are determined by (1). Hence, we obtain expressions for the Z-parameters of the two-port network:

(2) $$Z_{11} = \frac{1}{2}\left(Z_e + Z_o\right), \qquad Z_{12} = \frac{1}{2}\left(Z_e - Z_o\right).$$

Such basic parameters of a symmetrical two-pole network as its input impedances for open-circuit and short-circuit at the output are simply defined through the input impedances of the even-odd excitation:

(3) $$Z_{o.c.} = Z_{11} = \frac{1}{2}\left(Z_e + Z_o\right), \qquad Z_{s.c.} = 2\frac{Z_e Z_o}{Z_e + Z_o}.$$

In the design and analysis of electronic networks, especially in the design of filters, a network parameter known as image impedance or its analogue, common in microwave technic, characteristic impedance, is used. For a symmetrical network in terms of input impedances, it is:

(4) $$Z_{c1} = Z_{c2} = Z_c = \sqrt{Z_e Z_o}.$$

Parameters of a loaded two-port network

Important issues of the two-port networks analysis relate to their load mode. First of all, it concerns the possibility of determining the input impedance Z_{in} of the network loaded with impedance Z_L. For a symmetrical two-port network, using the input impedances of the even-odd excitation can be written down:

(5) $$Z_{in} = \frac{Z_L\left(Z_e + Z_o\right) + 2Z_e Z_o}{Z_e + Z_o + 2Z_L}.$$

Given the values of input impedance, load impedance, and image impedance for the network, the values of the input impedances of the half-circuits can be calculated from the expressions:

(6) $$aZ_o^2 + 2Z_o\left(Z_{in}Z_L - Z_c^2\right) + aZ_c^2 = 0, \quad Z_e = Z_c^2 / Z_o,$$

where $a = Z_{in} - Z_L$. Quite often, only reactive elements are used to implement a symmetrical two-port network. In this case, the input impedances of the even-odd excitation will take on imaginary values jX_e and jX_o that can be calculated from Z_L and Z_{in} using the following relations:

(7) $$AX_e^2 + 2BX_e - C = 0,$$

$$X_o = \frac{X_e\left(X_L + X_{in}\right) + 2\left(R_L R_{in} - X_L X_{in}\right)}{X_{in} - X_L - 2X_e},$$

where $Z_{in} = R_{in} + jX_{in}$, $Z_L = R_L + jX_L$, $A = R_{in} - R_L$, $B = R_L X_{in} + R_{in} X_L$, $C = R_L\left|Z_{in}\right|^2 - R_{in}\left|Z_L\right|^2$. The image impedance of such circuit will be $Z_c = \sqrt{-X_e X_o}$.

Important parameters that characterize the operation of a two-port network include the voltage transfer function or voltage transfer ratio $K=U_2/U_1$, where U_1 is the voltage at the network input and U_2 is the voltage at the load impedance. Using the input impedances of the odd-even excitation of a symmetrical network, you can write down:

(8) $$K = \frac{Z_L\left(Z_e - Z_o\right)}{Z_L\left(Z_e + Z_o\right) + 2Z_e Z_o}.$$

It follows from (8) that $K=0$ in the case when $Z_e=Z_o$.

Based on the specified values of the voltage transfer ratio, load impedance, and image impedance, the value of the input impedance of the odd excitation can be determined from the equation:

(9) $$Z_o^2 Z_L\left(K+1\right) + 2Z_o Z_c^2 K + Z_L Z_c^2\left(K-1\right) = 0.$$

The value of Z_e is calculated by Z_c. If a symmetrical two-port network is purely reactive, then $|K|=1$ and it can be used to provide a given phase shift of the voltage at the load relative to the input voltage, i.e., $\varphi=\varphi_{U2} - \varphi_{U1}$. In this case, the value of X_o is determined from the equation:

(10) $$X_o^2 \tan\varphi + 2Z_c X_o - Z_c^2 \tan\varphi = 0.$$

The value of Z_e is also calculated from Z_c.

The above relations make it possible to determine the parameters of the circuit elements selected for the implementation of a symmetrical two-port network with specified values of its operating parameters by applying the technique of symmetric-antisymmetric excitation. This process is carried out in the following sequence:

1. Setting the initial operating parameters of the designed circuit.
2. Calculation of input impedances of symmetric (even) and antisymmetric (odd) excitation according to the specified operating parameters.
3. Selecting a four-pole circuit option and dividing it into even-mode and odd-mode two-pole half-circuits.
4. Calculation of the four-pole circuit impedance elements based on the values of the half-circuits input impedances.
5. Transition from the impedance values of the circuit elements to the values of lumped elements (resistance, inductance, capacitance) that realize them.

Examples of application of the proposed approach

The first simple example that demonstrates the application of the proposed approach concerns the development of a circuit for transforming the load impedance into a given value of the input impedance. In accordance with the above sequence, we have:

1. Initial operating parameters include Z_L= 9,93-j4,35 Ω, and Z_{in}=50 Ω at a frequency of f = 100 MHz.
2. The input impedances of the half-circuits for even and odd excitation calculated by (7) are: X_e=27,841 Ω, X_o=-16,985 Ω.
3. To implement the transformer, we choose a π-type reactive circuit (Fig. 2a), since it must transform the load impedance with an imaginary component.

4. The application of the symmetric-antisymmetric excitation technology to the selected transformer circuit divides it into even-type (Fig. 2b) and odd-type (Fig. 2c) half-circuits. Based on these half-circuits, we obtain formulas for calculating the parameters of their elements through the values of input impedances:

$$\text{(11)} \qquad \mathrm{j}X_1 = \mathrm{j}X_e, \qquad \mathrm{j}X_2 = \mathrm{j}\frac{2X_oX_1}{X_1 - X_o}.$$

Calculations according to (11) give the following values of the element's parameters: X_1=27,841 Ω, X_2=-21,098 Ω.

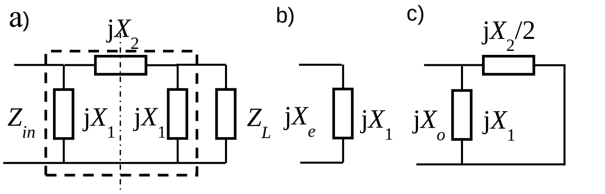

Fig.2. Transformer circuit and its half-circuits of even-odd excitation

5. The reactive resistances X_1 and X_2 of the circuit are realized by inductance L_1=44.31 nH and capacitance C_2=75.43 pF.

As it can be seen from the simulation results shown in Fig. 3, the circuit provides an input impedance of 50 Ohms at a given frequency.

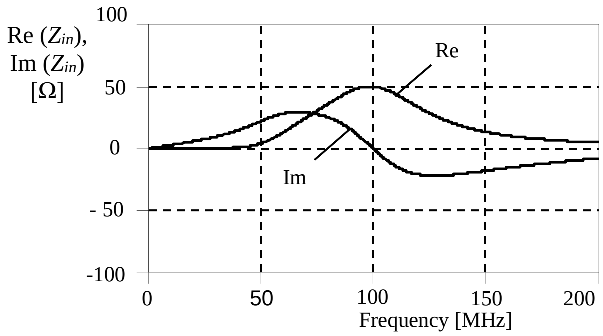

Fig.3. Transformer input impedance characteristic

The next more complicated example concerns the development of a dual-frequency circuit with a different phase delay of the output voltage at different frequencies. For this example, the initial operating parameters are: Z_c=50 Ω, φ_1=45^0 at f_1=100 MHz, φ_2=90^0 at f_2=300 MHz. As a result of calculations according to (10), the input impedances of even and odd excitation were obtained as follows: X_{e1}=120.71 Ω, X_{o1}=-20.71 Ω for f_1, and X_{e2}=50 Ω, X_{o2}=-50 Ω for f_2. To implement the device, a variant of the bridge T-circuit (Fig. 4a) was chosen, which is divided into half-circuits of even (Fig. 4b) and odd (Fig. 4c) excitation.

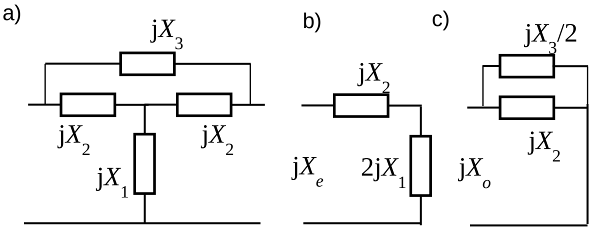

Fig. 4. Phase delay scheme and its half-circuits

The formulas for calculating the parameters of the elements of these half-circuits through the values of the input impedances are as follows:

$$\text{(12)} \qquad \mathrm{j}X_1 = \mathrm{j}\frac{X_e - X_2}{2}, \qquad \mathrm{j}X_3 = \mathrm{j}\frac{2X_oX_2}{X_2 - X_o}.$$

When calculating according to (12), the value of the reactance X_2 must be specified. With the values X_{21}=-8 Ω and X_{22}= 55 Ω selected for the frequencies f_1 and f_2, the result of the calculations for these frequencies is as follows: X_{11}=64.36 Ω, X_{12}=-2.5 Ω, and X_{31}=26.07 Ω, X_{32}=-52.38 Ω.

To implement reactive circuit elements that must take on different calculated values at different frequencies, parallel or series resonant circuits can be used. In this case, the inductance and capacitance values of the series resonant circuit are calculated using the following formulas:

$$\text{(13)} \qquad L_s = \frac{\left(k_f^2 - 1\right)X_{i1}X_{i2}}{\omega_1 k_f\left(k_f X_{i2} - X_{i1}\right)}, \quad C_s = \frac{X_{i2} - k_f X_{i1}}{\omega_1 X_{i1}X_{i2}\left(k_f^2 - 1\right)},$$

where k_f=f_2/f_1, and ω_1=$2\pi f_1$. The formulas for calculating the elements of a parallel resonant circuit are as follows:

$$\text{(14)} \qquad L_p = \frac{k_f X_{i2} - X_{i1}}{\omega_1\left(k_f^2 - 1\right)}, \quad C_p = \frac{k_f^2 - 1}{\omega_1 k_f\left(X_{i2} - k_f X_{i1}\right)}.$$

As a result of the calculations according to (13) and (14), the reactance of X_1 and X_3 are realized by parallel circuits with the parameters of elements L_{p1}=9,5 nH, C_{p1}=241,8 pF and L_{p3}=31,6 nH, C_{p3}=19,03 pF, and the reactance of X_2 are realized by a series circuit with the parameters of elements L_{s2}=34,4 nH, C_{s2}=53,7 pF. The simulation results of the designed circuit are shown in Fig. 5. It can be seen that at operating frequencies of 100 MHz and 300 MHz it provides the specified values of the phase shift. The magnitude of voltage transfer ratio at these frequencies is equal to 1. The narrow operating frequency bands characteristic of dual-frequency devices are explained by the use of resonant circuits.

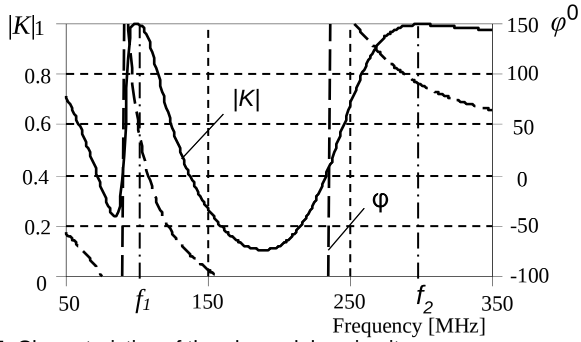

Fig.5. Characteristics of the phase delay circuit

The third example concerns the development of a circuit that ensures a zero value of the voltage transfer ratio at a given frequency. As mentioned above, the value of K=0 is achieved if Z_e=Z_o. To implement such a transmission rejection device, a complex variant of a two-port network formed by parallel connection of non-dissipative π-type and T-type circuits (Fig. 6a) was chosen. For the input impedances of the half-circuits of even (Fig. 6b) and odd (Fig. 6c) excitation, the value of Z_e=Z_o=∞ is chosen. In this case, the values of the parameters of two elements, for example, X_1 and X_3, must be set, and the parameters of other elements are calculated by the formulas:

(15) $$X_2 = \frac{X_1 X_3}{2}, \qquad X_4 = -\frac{X_1 + X_3}{2} \ .$$

Calculations according to (15) for X_1=60 Ω, X_3=140 Ω give the following values of the circuit parameters: X_2=-84 Ω, X_4=-100 Ω. At a frequency of 100 MHz, these reactances are realized by lumped elements L_1=95.49 nH, C_2=18.95 pF, L_3=222.82 nH, C_4=15.92 pF.

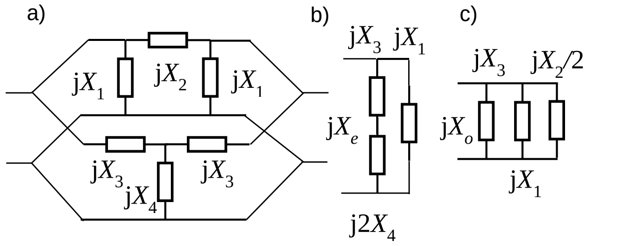

Fig. 6. Transmission rejection circuit and its half-circuits.

The simulation results of the developed scheme are shown in Fig. 7. Curve 1 corresponds to the initial values of the elements X_1 and X_3, the other graphs refer to the values of these elements given in Table 1.

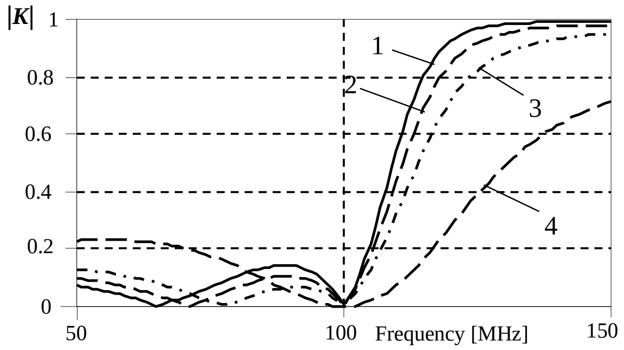

Fig.7. Characteristics of the transmission rejection circuit

Table 1. The parameters of elements

Schedule number	Value of X_1 [Ω]	Value of X_3 [Ω]
1	60	140
2	60	120
3	60	100
4	60	60

As you can see from the graphs, such circuit operates in the high-pass filter mode with a specified cutoff frequency. By selecting the values of the input impedances Z_e=Z_o and the free elements of the circuit, you can influence the appearance of its frequency response.

Conclusion

The use of the symmetric-antisymmetric excitation method makes it possible to simplify the design process of devices based on a two-port network, reducing it to determining the parameters of the elements of two two-pole circuits. The output parameters for calculating of these two-pole circuits are their input impedances, the values of which are determined based on the operating parameters of the device that it must provide.

***Authors**: dr. sc. Valeriy Oborzhytskyy, Lviv Polytechnic National University Department of Radioelectronic Devices and Systems, Professorska str. 2, Lviv, Ukraine, E-mail: valerii.i.oborzhytskyi@lpnu.ua; prof. dr hab. inż. Stefan Paszek, dr inż. Anna Piwowar, Silesian University of Technology, Wydział Elektryczny, ul. B. Krzywoustego 2, 44-100 Gliwice, Emails: Stefan.Paszek@polsl.pl, Anna.Piwowar@posl.pl.*

REFERENCES

[1] Bartlett, A.C., An extension of a property of artificial lines, *Phil. Mag.*, 4 (1927), p. 902.
[2] Perfetti, R., Symmetrical two-port networks: An eigenspace perspective, *Int. J. Circ. Theor. Appl.,* 40 (2012), 307-312, DOI: 10.1002/cta.716
[3] Hernandez-Escobar A., Abdo-Sanchez E., Mateos-Ruiz P., Esteban J., Martın-Guerrero T. M., and Camacho-Penalosa C., A general equivalent circuit for lossy non-symmetric reciprocal two-ports, *IEEE Journal of Microwaves* (2021), pp. 1-11, DOI: 10.1109/JMW.2021.3085408.
[4] Zghoul F. N., An extension of the bisection theorem to symmetrical circuits with cross-coupling, *International Journal of Advanced Computer Science and Applications*, Vol. 7 (2016), No. 6, pp. 449-455.
[5] Pozar D. M. Microwave engineering, 4th ed. New York: John Wiley & Sons, (2012), ISBN: 978-0-470-63155-3.