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Methods and algorithms for analyzing steady-state modes and characteristics of the wound rotor induction motor

***Abstract**. Mathematical models have been developed to analyze the starting modes of an electric drive based on an induction motor with a phase rotor, which allow determining the values of the resistances of rheostat sections in the rotor circuit and performing mathematical modeling of the starting modes of the electric drive. The calculation is based on the developed mathematical model of the motor, which takes into account the saturation of the magnetic circuit based on the magnetization characteristics of the main magnetic flux and leakage fluxes in the stator and rotor.*

***Streszczenie.** Opracowano modele matematyczne do analizy trybów rozruchu napędu elektrycznego opartego na silniku indukcyjnym z wirnikiem fazowym, które pozwalają na określenie wartości rezystancji sekcji reostatu w obwodzie wirnika i przeprowadzenie modelowania matematycznego trybów rozruchu napędu elektrycznego. Obliczenia oparte są na opracowanym modelu matematycznym silnika, który uwzględnia nasycenie obwodu magnetycznego na podstawie charakterystyk magnesowania głównego strumienia magnetycznego i strumieni rozproszenia w stojanie i wirniku. (**Metody i algorytmy analizy stanów ustalonych i charakterystyk silnika indukcyjnego z wirnikiem uzwojonym**)*

Keywords: induction motor, wound rotor, static characteristics, magnet core saturation.
Słowa kluczowe: silnik indukcyjny, wirnik uzwojony, charakterystyki statyczne, nasycenie rdzenia magnetycznego.

Introduction

The vast majority of electric drives are based on squirrel-cage induction motors (SCIM). Their important disadvantage is the relatively low starting torque, which may be insufficient to drive some mechanisms, in particular, lifting and transporting ones. For such electric drives, wound rotor induction motors (WRIM) are used, in which additional resistors in the rotor circuit not only significantly limit the starting currents but also can increase the starting electromagnetic torque. By selecting the rheostat resistance appropriately, the starting torque of the WRIM can be increased up to the critical torque value. This feature of the mechanical characteristic is used for starting if the load torque exceeds the motor's nameplate starting torque. The inclusion of additional resistances in the rotor circuit can be used to control the rotor speed. This control method is used in electric drives for various lifting and transport mechanisms. Rheostat control makes it possible to adjust the speed downward from the nominal value. Therefore, there is a problem with calculating the rotor speed and electromagnetic torque in steady-state mode at a given value of rheostat resistance.

State of the problem

The main disadvantage of the WRIM is significant power losses in the rotor circuit, which reduce the efficiency of the drive. To date, a large number of WRIMs are in operation, designed to operate in traditional rheostat-controlled crane drives, which have not yet exhausted their resource and can operate until it is completely exhausted. Their efficiency can be increased by optimally selecting the parameters of the rheostat resistances and the programmable law of switching on and off the sections of the starting rheostat. To do this, it is necessary to develop programs for numerical analysis of their operation both during startup and during cargo movement. Such programs should ensure high reliability of the calculation results, not be too complicated, and have high speed. This article is aimed at solving this problem.

The purpose of the article is to develop methods and algorithms for analyzing electromechanical processes in WRIM.

Mathematical model

The main element of the electric drive is the WRIM, and therefore the mathematical description of the processes in it is crucial in solving the formulated problems.

As can be seen from the literature [1-8], most of the developed mathematical models of induction motors, including the WRIM, are based on T-shaped substitute electrical circuits. However, they can be used to calculate the steady-state rated mode of the motor quite accurately, and they are of little use for solving problems of electric drive dynamics. Various improvements to the substitute circuits, which are formed by adding additional elements to the known circuits [8], require experimental verification.

Since simplified mathematical models do not provide reliable results, and field models are unsuitable for use in control devices due to their cumbersome nature and low performance, the mathematical model of the WRIM should be a compromise in terms of accuracy and complexity. An intermediate position is occupied by mathematical models based on circular analysis methods, in which the flux coupling of circuits and their own mutual inductances are determined based on the nonlinear magnetization characteristics of the motor magnetic circuit elements [6].

This paper considers a WRIM powered by a three-phase network with a symmetrical voltage system, the three-phase rotor winding of which is star-connected and connected to slip rings rigidly connected to the rotor. This makes it possible to connect a starting rheostat to the rotor winding, increasing the active resistance of the rotor phases $r_2 = r'_2 + r_p$.

Electromagnetic processes in the orthogonal coordinates x, and y are described by the DE system of electrical equilibrium of circuits [6].

$$\frac{d\Psi_{sx}}{dt} = -\omega_0\Psi_{sy} - r_s i_{sx} + u_{sx};$$

$$\frac{d\Psi_{sy}}{dt} = -\omega_0\Psi_{sx} - r_s i_{sy} + u_{sy};$$

(1) $$\frac{d\Psi_{rx}}{dt} = s\omega_0\Psi_{ry} - (r_r + r_p)i_{rx};$$

$$\frac{d\psi_{ry}}{dt} = -s\omega_0\psi_{rx} - (r_r + r_p)i_{ry},$$

where ψ_{sx}, ψ_{sy}, ψ_{rx}, ψ_{ry}, *f*, i_{sx} i_{sy}, i_{rx}, i_{ry} – flux coupling and currents of transformed stator and rotor circuits; r_s, r_r – active supports of these contours; *ω*0 is the angular frequency of the supply voltage; *s* – rotor slip; r_p – phase resistance of a three-phase starting rheostat, each of which consists of several sections; u_{sx}, u_{sy} – components of the supply voltage vector of the stator phases.

The equation of rotor dynamics is as follows:

$$\frac{ds}{dt} = \frac{p}{J\omega_0}\left(M_c(t) - \frac{3}{2}p_0\left(\psi_{sx}i_{sy} - \psi_{sy}i_{sx}\right)\right) \quad (2)$$

where p_0 – the number of motor pole pairs; *J* is the torque of inertia of the electric drive system; $M_c(t)$ – time dependence of the load torque of WRIM.

Calculation of the steady state mode at a given rotor speed

In the steady-state mode (under the condition of constant rotor slip s), the DEs (1) are reduced to a nonlinear system of algebraic equations. If we combine the image vector of the supply voltage with the *x*-axis, then $u_{sx} = U_m$; $u_{sy} = 0$, this system will have the form

$$\begin{aligned}
&\omega_0\psi_{sy} - r_r i_{sx} = U_m; \\
&-\omega_0\psi_{sx} - r_s i_{sy} = 0; \\
&s\omega_0\psi_{ry} - (r_p + r_r)i_{rx} = 0; \\
&-s\omega_0\psi_{rx} - (r_p + r_r)i_{ry} = 0,
\end{aligned} \quad (3)$$

where *U*m is the amplitude value of the phase voltage, which in the vector form of recording has the following

$$\Omega_{xy}\vec{\Psi}_{xy} + R_{xy}\vec{I}_{xy} = \vec{U}_{xy}, \quad (4)$$

The nonlinearity of equations system (4) is caused by a nonlinear dependence $\vec{\Psi}_{xy} = \vec{\Psi}_{xy}\left(\vec{I}_{xy}\right)$ of the vector of flux linkages from the vector of currents. System (4) makes it possible to investigate the influence of any coordinate ξ, which is included in this system of equations and is taken as an independent variable, i.e. calculate the static characteristics as a function of ξ. For this, it is necessary for each given value ξ determine the remaining coordinates, which is reduced to solving a nonlinear system of finite equations.

One of the most effective methods for solving nonlinear systems of algebraic equations is Newton's iterative method. To solve the problem of convergence of the iterative process, we will apply the method of continuation according to the parameter, which we will introduce into the system by multiplying the vector of applied voltages ε. Changing the parameter ε from zero to one is equivalent to increasing the applied voltage from zero to a given value. At each step of integration, it is necessary to solve the system of equations of the form

$$\boldsymbol{A}\frac{d\vec{I}_{xy}}{d\varepsilon} = \vec{U}_{xy} \quad (5)$$

where

$$\boldsymbol{A} = \begin{bmatrix} x_{sysx} - r_s & x_{sysy} & x_{syrx} & x_{syry} \\ -x_{sxsx} & -x_{sxsy} - r_s & -x_{sxrx} & -x_{sxry} \\ sx_{rysx} & sx_{rysy} & sx_{ryrx} - r_r & sx_{ryry} \\ -sx_{rxsx} & -sx_{rxsy} & -sx_{rxrx} & -sx_{rsry} - r_r \end{bmatrix}$$

is the matrix of the Jacobian system (4), the elements of which are differential inductive resistances $x_{jk} = \omega_0 L_{jk}$ contours (own at *j*=*k* and mutual at *j*≠*k*), as well as active resistances r_s – stator contours and $r_r = r_2' + r_p$ is the total reduced resistance of the winding of the rotor circuits together with the starting rheostat.

To calculate the differential inductances of the WRIM circuits taking into account the saturation of the magnetic circuit, we use the dependences of the main magnetic flux on the modulus of the image vector of the current $\psi_\mu = \psi_\mu(i_\mu)$ where $i_\mu = \sqrt{\left(i_{sx} + i_{rx}\right)^2 + \left(i_{sy} + i_{ry}\right)^2}$, and similar dependences of the leakage fluxes of the stator $\psi_{\sigma s} = \psi_{\sigma s}(i_s)$ and rotor $\psi_{\sigma r} = \psi_{\sigma r}(i_r)$ windings, where i_s, i_r are the modules of the image vectors of the corresponding currents.

To obtain the initial value of the vector $\vec{I}_{xy}$, which is in the area of attraction of the method, it is enough to perform several steps of integration by ε by Euler's method within $0 < \varepsilon \le 1{,}0$ which is equivalent to increasing the applied voltage from zero to a given value. The obtained value of the vector $\vec{I}_{xy}$ is refined by Newton's

$$\vec{I}_{xy}^{(k+1)} = \vec{I}_{xy}^{(k)} + A^{-1}N\left(\vec{I}^{(k)}\right), \quad (6)$$

where $\vec{N}\left(\vec{I}^{(k)}\right) = \Omega_{xy}^{(k)}\vec{\Psi}_{xy}^{(k)} + R_{xy}\vec{I}_{xy}^{(k)} - \vec{U}_{xy}$ is the residual vector of the system (4) for the given values of slip *s* and the voltage vector $\vec{U}_{xy}^{(k)}$.

Equations (5), and (6) have the same Jacobi matrices and differ only in the vector of the right-hand sides. For a given value of the current vector $\vec{I}_{xy}$, we determine the electromagnetic torque.

Calculation of the steady state mode at a given load torque

The task of calculating the steady state mode at a given value of the load torque is somewhat more complicated since the speed of rotation of the rotor (slip) is the desired (unknown) value. To calculate the steady state mode, we will use the system of equations of electromechanical equilibrium consisting of DE (1) and (3.). Under the condition of a constant speed of rotation of the rotor, these equations are reduced to a system of finite equations, which will be written in the form of functional dependencies

$$z_1 = -\omega_0\psi_{sy} + r_s i_{sx} - U_m;$$

$$z_2 = \omega_0\psi_{sx} + r_s i_{sy};$$

$$z_3 = -s\omega_0\psi_{ry} + (r_p + r_r)i_{rx};$$

(7) $$z_4 = s\omega_0\psi_{rx} + (r_p + r_r)i_{ry}$$

$$z_5 = \frac{p_0}{\omega_0 J}M_c - \frac{1{,}5p_0^2}{\omega_0 J}\left(\psi_{sx}i_{sy} - \psi_{sy}i_{sx}\right).$$

The system of equations (7) will be considered as a multidimensional dependence

(8) $$\vec{Z} = \vec{Y}\left(\vec{X}\right),$$

where $\vec{Y} = \begin{bmatrix}\vec{\psi}_{xy}\\ s\end{bmatrix}$; $\vec{X} = \begin{bmatrix}\vec{I}_{xy}\\ s\end{bmatrix}$.

In equation (8) there are two disturbing actions: the vector of applied voltages and the torque M_c load. It is obvious that in order to ensure the convergence of the iterative process, it is necessary to increase them alternately. First, we set the slip value, which corresponds to the operation of the WRIM without load, and increase the applied voltage, and then the load torque, that is, the problem is solved in two stages. We determine the value of the vector coordinates from the equations of electrical equilibrium $\vec{X}$, as well as the electromagnetic torque. The received values of the vector $\vec{X} = \vec{X}^{(0)}$ is the initial value for the calculation at the second stage. By substituting the values of the coordinates of the vector $\vec{X}^{(0)}$ in equation (8), we will get a residual vector

(9) $$\vec{Q}^{(0)} = -\vec{Z}\left(\vec{Y}\left(\vec{X}^{(0)}\right), \vec{X}^{(0)}\right)$$

Let's introduce the scalar parameter λ into the system of algebraic equations (8) according to the scheme [6]

(10) $$\vec{Z} = (1-\lambda)\vec{Q}^{(0)}.$$

A change in the parameter λ from $\lambda = 0$ to $\lambda = 1$ is equivalent to a change in the residual vector $\vec{Q}^{(0)}$ to zero, and the vector $\vec{X}$ while changing from $\vec{X} = \vec{X}^{(0)}$ to the desired value, which corresponds to the solution of the vector equation at the specified load torque. To find it, we apply the differential method of solving nonlinear systems of algebraic equations [9]. As a result of differentiating the vector equation (10) with respect to λ as a scalar parameter, we obtain a DE system of the form

(11) $$\boldsymbol{B}\frac{d\vec{X}}{d\lambda} = -\vec{Q}^{(0)},$$

where ***B*** is the Jacobi matrix of system (10), as a result of integration of which over the parameter λ in the range from λ=1.0 to λ=0.0, we obtain a solution (the value of the vector $\vec{X}$) corresponding to the applied torque.

Calculation of static characteristics

The steady-state calculation algorithm is the basis for calculating static characteristics, which can be obtained as a sequence of steady-state modes. However, the differential method is more efficient. For this purpose, the system (7) of finite equations describing the steady-state mode will be considered as a multidimensional function

(12) $$\vec{F}\left(\vec{\psi}_{xy}, \vec{i}_{xy}, s, r_p, U_{xy}\right) = 0,$$

The multivariate dependence (12) makes it possible to study the effect of any coordinate as a parameter on the operating mode of the WRIM ξ, which is included in it, for example, rheostat resistance, supply voltage, etc. For this, it is necessary, considering the vector of applied voltages to be constant, to differentiate equation (12) with respect to this parameter as a variable. As a result, we will get a DE system of the form

(13) $$\boldsymbol{A}\frac{d\vec{X}}{d\xi} = \frac{\partial\vec{F}}{\partial\xi}.$$

where ***A*** is the Jacobian matrix of the system (12).

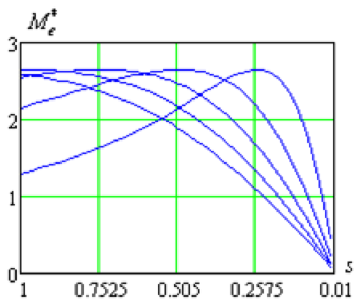
Equations of the form (13) for different coordinates (independent variables) differ only in the vector of the right parts. In particular, to study the influence of resistance r_p we accept the rheostat for the static characteristics of WRIM $\xi = r_p$, and a vector

$$\partial\vec{F}/\partial r_p = \begin{bmatrix}0\\0\\i_{rx}\\i_{ry}\end{bmatrix}.$$

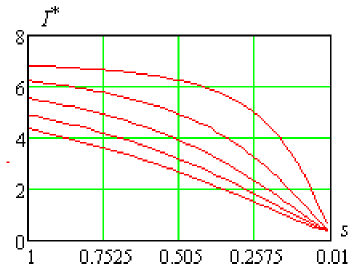
As a result of the integration of the received DE on r_p as a parameter within from $r_p = 0$ to a given value at a given slip (for example $s = 1{,}0$) will obtain the dependence of the electromagnetic torque on the resistance of the starting rheostat. At the same time, for a given value of slip *s* and supply voltage of the stator winding, the independent variable is the resistance of the rheostat r_p.

Calculation of mechanical characteristics $M = M(s)$ for each given value of the resistance of the rheostat are performed similarly, taking in DE (13) the variable magnitude $\xi = s$. At the same time, the supply voltage and resistance r_p are taken as constant, and the slip changes within the given limits.

An example of the results of calculating the mechanical characteristics and the dependence of the consumed current on the slip for five values of the starting rheostat resistance of the WRIM motor (*P* = 250 kW, *U* = 380 V, *I* = 263 A, *p* = 3) is shown in Fig. 1.



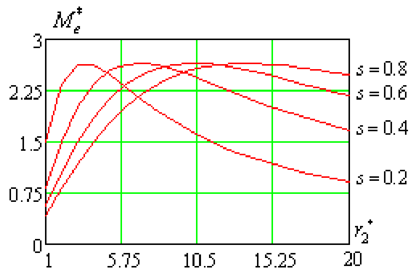
a)



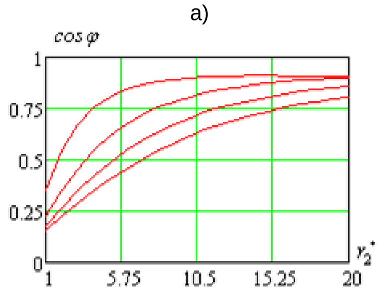
b)

Fig. 1. Starting characteristics of electromagnetic torque (a) and stator current (b) at different values of rheostat resistance in the rotor circuit

In fig. 2 shows example of the results of calculating the dependence of the electromagnetic torque of the WRIM and power factor $cos\varphi$ from the resistance of the starting rheostat for different slips.



a)



b)

Fig. 2. Dependencies of the driving electromagnetic torque of the WRIM (a) and the power factor $cos\varphi$ (b) from the relative value of the resistance of the rotor phases together with the rheostat for different slip values

Verification of the values of the rotor rotation speed in the steady-state mode calculated by the method described above at the given values of the rheostat resistance can be carried out by calculating the transient process (Fig. 3).

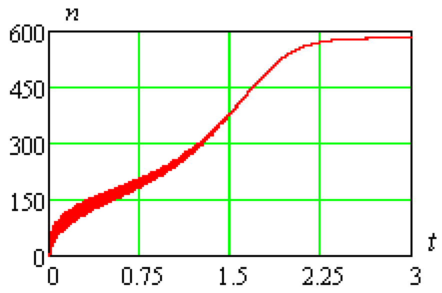


Fig. 3. Transient process to the steady-state speed of the WRIM rotor with the rheostat turned on at the value of the rheostat resistance $r_p = 4r_2$

Conclusions

1.The proposed calculation algorithms make it possible to analyze the static characteristics of WRIMs with regard to magnetic saturation by means of mathematical modeling.

2. The calculation algorithms and the mathematical model of the IM created on their basis use real magnetization curves calculated on the basis of the geometry of the magnetic circuit and the winding data of the motor, which makes it possible to adequately take into account the saturation, thus ensuring the accuracy of the calculation results.

3. The mathematical models created on the basis of the proposed calculation algorithm can be used to design adjustable asynchronous electric drives based on WRIMs.

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