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Evaluation of the short circuit current based on the instantaneous values analysis

***Abstract** The paper presents the method of the short-circuit currents evaluation in electrical power systems. The grid parameters are recognized by the current impulse injection. The analytical and numerical results are presented. The results prove the effectiveness of the proposed method.*

***Streszczenie** Artykuł opisuje metodę oceny prądów zwarciowych w systemach energetycznych. Parametry sieci są identyfikowane na podstawie wstrzykniętego impulsu prądu. Wyniki analityczne oraz numeryczne są prezentowane. Wyniki potwierdzają efektywność zaproponowanej metody (**Ocena prądu zwarciowego na podstawie analizy wartości chwilowych**).*

Keywords: short-circuit current, current injection, system identification, transient analysis.
Słowa kluczowe: prąd zwarciowy, wstrzykiwanie prądu, identyfikacja parametrów systemu, analiza stanów nieustalonych.

Introduction

The equivalent circuit parameters of a power supply system such as short circuit impedance are important data for both power supply authorities and industrial customers. The parameters have several applications. They are used to calculate the short circuit currents and to verify models of power system networks.

Several methods have been proposed to calculate the power system impedance parameters. They can be classified into two groups: invasive and noninvasive. The noninvasive approaches use the existing load current and voltage variations to identify the network equivalent impedance [1,2]. The invasive approaches impose intentional disturbances to the system and use the voltage and current response for estimation. These experiments can be oriented on the short-circuit current evaluation. One of such methods consists of short time short-circuit execution [3].

Estimation of a short-circuit current in electrical power systems is the important problem. Methods of such current's evaluation have been elaborated from the beginning of electrical grid development. The analysis of numerical grid models is one of the important approaches for short-circuit current evaluation, this is valuable approach [4,5,6]. Numerical models comprise whole electrical systems and should be permanently bring up to date. Independently on numerical model analysis the measurements of real system can append the numerical methods. These experiments are oriented on the short-circuit current evaluation. The reactive current injection presented in also can be treated as the injection or disturbance method. The method based on reactive current injection allows to recognize steady state component of short circuit current. The phasor estimation is essential while steady state component of the short circuit current is evaluated.

The analysis presented in the presented paper is oriented on transient component evaluation. The grid parameters are recognized by injection of current impulse.

Transient response

The steady state response is the sinusoidal function depending on voltage sources acting in the system. Each source has its share, and this response does not depend on the time instant when short circuiting occurred. The transient response has different nature. It depends on the time instant when short-circuit arises, because it depends on the system state at this time instant [4]. If a circuit model of system is used the state is determined by inductor currents and capacitor voltages. Let the chosen port of the grid be short circuited at time instant $t=0$.

The current of this port is equal to zero $i(t)=0$ for $t<0$ and $i \neq 0$ for $t>0$. This current is discontinuous at $t=0$. The aim of the investigation is to estimate the right side limit of the current after the short circuit occurrence. For convenience this current limit will be denoted as $i(0^+)$ or $i(0)$. Current $i(0)$ is determined by state of grid at $t=0$.

The state vector is composed of inductor currents and capacitor voltages. These values are continuous so, the state for $t=0$ has been reached before short circuit occurred.

The state equation can be written as follows

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{e}(t) \tag{1}$$

and response equation

$$i(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{e}(t) \tag{2}$$

Vector $\mathbf{e}(t)$ contains source voltages operated in the grid. Current $i(t)$ means the time varying short-circuit current. Numerical matrices $\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}$ depend on grid parameters. State equation (1) is valid for $t \geq 0$, response equation (2) is valid for $t>0$.

The aim of the considerations presented below is to find the value of the short circuit current at $t=0^+$. This value will be denoted as $i(0)$.

Solution of equation (1) has two components: steady state component $\mathbf{x}_u(t)$ and transient component $\mathbf{x}_p(t)$

$$\mathbf{x}(t) = \mathbf{x}_u(t) + \mathbf{x}_p(t) \tag{3}$$

As the result, according to (2), short circuit current also has two components

$$i(t) = i_u(t) + i_p(t) \tag{4}$$

The paper [7] presents the procedure for estimation of steady state current $i_u(t)$. The present paper is concentrating on the recognition of the transient current component value $i_p(0)$ at the beginning of the short circuit process.

For $t=0$ equation (40) takes the algebraic form

$$i(0) = \mathbf{Cx}(0) + \mathbf{De}(0) \tag{5}$$

and

$$i(0) = i_u(0) + i_p(0) \tag{6}$$

If currents $i(0)$ and $i_u(0)$ are predicted then beginning value of transient $i_p(0)$ can be estimated according to (6). Transient component is decaying function starting from value $i_p(0)$.

The process described by state equations (1) and (2) can be analysed by using electrical circuit properties. On the base of the compensation principle the inductor for given current value $i_L(0)$ can be substituted for current source, similarly the capacitor for given voltage $u_C(0)$ can be substituted for voltage source. These properties are shown in Fig. 1. The presented equivalence is valid only for distinct instantaneous values of current and voltage. Especially for $I_L = i_L(0)$ and $U_C = u_C(0)$.

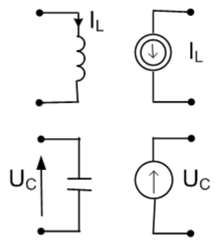

Fig. 1. Substitution of the inductor current and the capacitor voltage.

Models shown in Fig. 1, considered for time instant $t=0$, can be expanded to the multiport, which contains all inductors, capacitors, and sources appearing in the grid.

The algebraic equation for such circuit can be written as

$$i_{z1}(0) = \mathbf{H}\begin{bmatrix} \mathbf{i}_L(0) \\ \mathbf{u}_C(0) \\ \mathbf{e}(0) \end{bmatrix} \tag{7}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_L & \mathbf{g}_C & \mathbf{g}_E \end{bmatrix} \tag{8}$$

is row real number matrix. This matrix is rare, most of its elements are equal to zero. It follows from the circuit properties illustrated in Fig. 2.

a) ***b)***

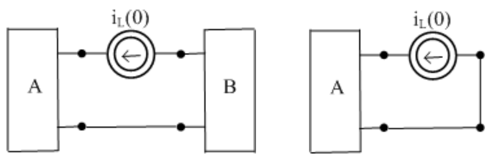

Fig. 2. Two 1-ports connection through current source. A) 1-port B is present, b) 1-port B is omitted.

In Fig. 2a two 1-ports A and B are connected through current source $i_L(0)$. For 1-port A the conditions remain non changed when 1-port B is omitted as shown in Fig. 2b.

Similarly in Fig. 3a two 1-ports A and B are in parallel. Between these ports voltage source $u_C(0)$ is connected in parallel. For 1-port A the conditions remain non changed when 1-port B is omitted as shown in Fig. 3b.

a) **b)**

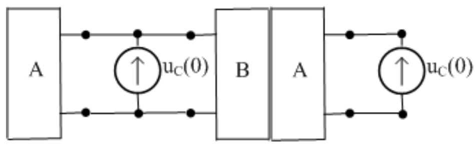

Fig. 3. Two 1-ports connection with parallel voltage source. a) 1-port B is present, b) 1-port B is omitted.

The circuit configurations presented in Fig. 2 and 5 cause that grid matrix $\mathbf{H} = \begin{bmatrix} \mathbf{h}_L & \mathbf{g}_C & \mathbf{g}_E \end{bmatrix}$ in (8) contains many zero elements and the port structure shown in Fig. 2 contains only the limited circuit fragment placed in the neighbour of the short- circuit port.

Short-circuit Current Estimation

Circuit shown in Fig. 4 is composed of *Recognized 2-port, Optional 1-port* and connecting them inductor L_s. *Optional 1-port* substitutes whole grid except for the neighbor of short-circuit port composed of *Recognized 2-port* and connecting inductor. The rectangular current pulse $i_p(t)$ is injected to port 11' while the grid is examined. The circuit is stimulated by current pulse i_p in order to recognize its properties while the short circuit happens between terminals 11'.

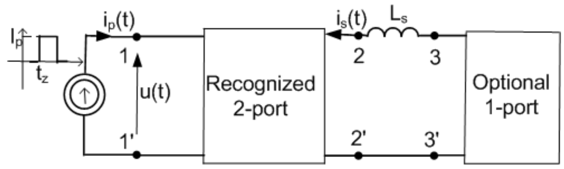

Fig. 4. Structure of the analyzed grid

Recognized 2-port representing the part of circuit placed in the neighbour of the short circuited port is searched as one of two alternative circuit models shown in Fig. 5. The first circuit is capacitive, and the second circuit is inductive.

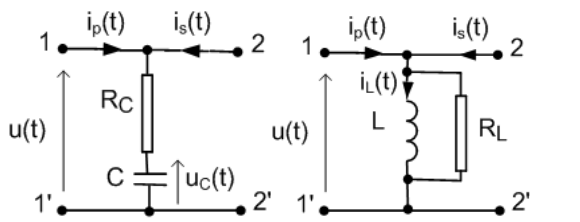

Fig. 5. Recognized 2-port. a) capacitive circuit, b) inductive circuit

The proceeding considerations show that the value of inductance current $i_s(t)$ at time instant $t=t_z$ is not required for prediction of short circuit current appearing between terminals 11' at time $t=t_z$. Only voltage waveform $u(t)$ marked in Fig. 4 should be observed while testing current pulse $i_p(t)$ is injected.

Capacitive 2-port

Assume that recognized 2-port seen in Fig. 4 has the capacitor form. It means that the circuit shown in Fig. 5a should be farther analysed.

Capacitor voltage $u_C(t)$ in Fig. 7a, inductor current $i_L(t)$ in Fig.7b and inductor current $i_s(t)$ in Fig. 4 are continues at $t = t_z$. Current pulse $i_p(t)$ and voltage $u(t)$ between terminals 11' are discontinues. These variables have different values at the left side time limit t_z^- and right side time limit t_z^+. Impulse current $i_p(t_z^-) = 0$, $i_p(t_z^+) = I_p$. The steep front of the current impulse is important for the proposed measurement procedure. Voltage $u(t)$ is discontinues $u(t_z^-) \neq u(t_z^+)$.

The following relations at time instant $t = t_z$ holds good for the analysed circuit structure.

$$u(t_z^-) = u_C(t_z) + R_C i_s(t_z) \tag{9}$$

$$u(t_z^+) = u_C(t_z) + R_C i_s(t_z) + R_C i_p(t_z^+) \tag{10}$$

From (9) and (10)

$$R_C = \frac{u(t_z^+) - u(t_z^-)}{i_p(t_z^+)} \tag{11}$$

As resistance R_C is known, capacitor voltage $u_C(t_z)$ can be computed from (9)

$$u_C(t_z) = u(t_z^-) - R_C i_s(t_z) \tag{12}$$

Capacitor voltage $u_C(t_z)$ and the grid current $i_s(t_z)$ determine the short circuit current $i_z(t_z^+)$

$$i_z(t_z^+) = \frac{u_C(t_z)}{R_C} + i_s(t_z) \tag{13}$$

Putting the right side of (12) into (13) gives

$$i_z(t_z^+) = \frac{u(t_z^-)}{R_C} \tag{14}$$

where R_C is given in (11).

It means that only resistance R_C computed according to (11) and voltage $u(t_z^-)$ are sufficient in order to predict short circuit current at instant $t = t_z$.

Circuit shown in Fig. 5a contains one resistor and one capacitor. This circuit can be replaced for more general structure shown in Fig. 6.

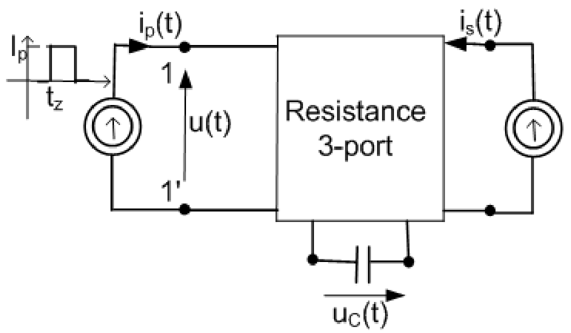

Fig. 6. Resistance 3-port with one capacitive port

Only one equation for circuit shown in Fig. 6 will be considered. This equation concerns the port which be potentially short circuited at time instant $t = t_z$

$$u(t) = r_{pp} i_p + r_{ps} i_s + h_{pC} u_C \tag{15}$$

Current impulse I_p is injected to the 3-port at $t = t_z$. For two time limits t_z^- and t_z^+ two equations can be written

$$u(t_z^-) = r_{ps} i_s(t_z) + h_{pC} u_C(t_z) \tag{16}$$

$$u(t_z^+) = r_{pp} I_p + r_{ps} i_s(t_z) + h_{pC} u_C(t_z) \tag{17}$$

Subtracting equations (17) and (16) side by side can be obtained

$$r_{pp} = \frac{u(t_z^+) - u(t_z^-)}{I_p} \tag{18}$$

Short circuit current is obtained from (17) by putting $u(t_z^+) = 0$, substituting impulse current by short circuit $I_p = i_z(t_z^+)$ and taking into account (16) can be obtained

$$i_z(t_z^+) = \frac{u(t_z^-)}{r_{pp}} \tag{19}$$

The preceding text shows how these parameters can be elaborated from the voltage observed across the considered port while current impulse is injected to the port.

Simulations

Figs. 12-17 show the PLECS simulation results obtained for the grid with the short circuited capacitive port. This port has the structure like this shown in Fig. 9a.

The block diagram of the PLECS program is shown in Fig. 10.

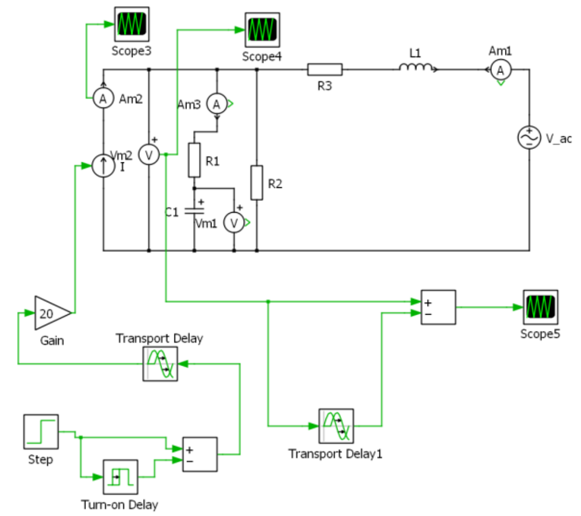

Fig. 7. PLECS block diagram for the grid with capacitive port

The numerous values in the block diagram are chosen as follows $R_1 = 2\Omega$, $R_2 = 1000\Omega$, $C_1 = 500\mu F$, $L_1 = 5mH$, $R_3 = 0.5\Omega$, $Vac = 325V$, $f = 50Hz$, phase=0.

The presented simulation illustrates the testing procedure for the short circuit current prediction. The current source *I* is connected to the examined port. The injected current is shown in Fig. 8. The presented simulation concerns the short circuit expected at time instant $t_z = 0.06$ s.

Inductance L1 (Fig. 7) connects the grid with the examined capacitive 2-port. Current of this inductance is not needed for the proposed prediction method. Voltage across the examined port should be observed. This voltage is shown in Fig. 9.

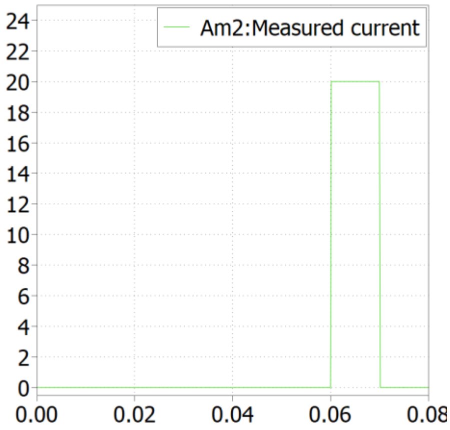

Fig. 8. Current injected to the examined grid.

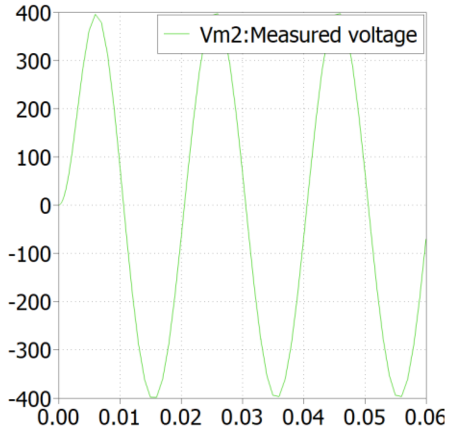

Fig.11. Voltage across the examined port.

Only voltage at instant $t_z = 0.06\,\mathrm{s}$ is essential. This voltage $u(t_z^-) = -72\,\mathrm{V}$.

The voltage jump caused by the injected current pulse should be precisely measured. In order to realize such measurement the procedure shown in the simulation program (Fig. 10) is proposed. Two voltage waveforms are compared. The present and delayed waveforms are subtracted. The time delay should be equal to the integral multiple of the system period. In the presented simulation time delay is equal 0.04s. Difference of two waveforms present and delayed is shown in Fig. 10.

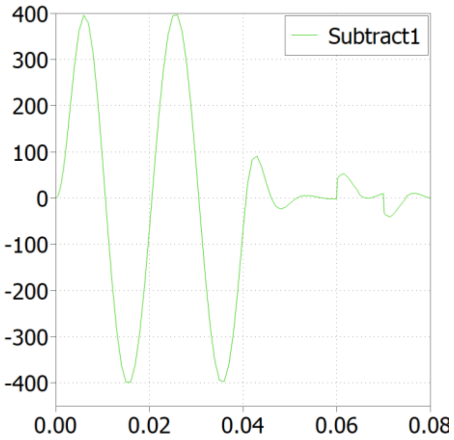

Fig. 10. Two voltage waveforms compared.

Short excerpt of two voltage waveform is shown in Fig. 11. It can be observed that voltage jump $u(t_z^+) - u(t_z^-) = 42\,\mathrm{V}$.

Table 1 contains the results obtained from simulation of the circuit shown in Fig. 7. The PLECS simulation gives the results useful for prediction of short circuit current. It is assumed that short circuit happens at $t_z = 0.06\,\mathrm{s}$.

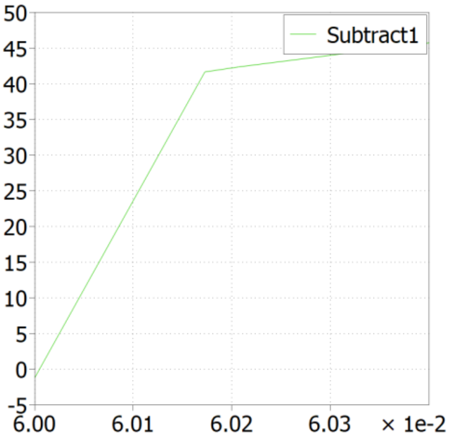

Fig. 11. Excerpt of the voltage difference for the capacitive port.

Table 1. System parameters obtained from the scope diagrams – capacitive port.

I_p	20A	Scope 3
$u(t_z^-)$	-72V	Scope 4
$u(t_z^+) - u(t_z^-)$	42V	Scope 5
r_{pp}	2.1 Ω	
$i_z(t_z^+)$	-34.3A	

The readings obtained from the simulation waveforms enable one to calculate the final parameters R and $i_z(t_z^+)$, according to (18) and (19): $R = 2.1\Omega$ and $i_z(t_z^+) = -34.3\,\mathrm{A}$.

Inductive 2-port

Assume that input 2-port seen in Fig. 4 has the inductor form shown in Fig.7b. The tested grid with rectangular current impulse injected to the grid is presented in Fig. 12.

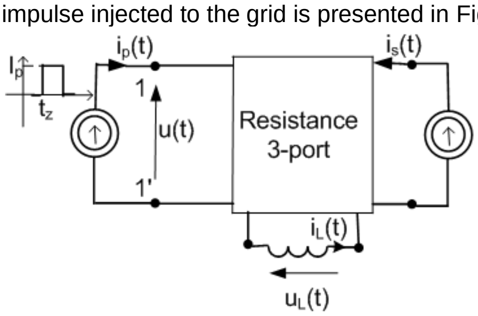

Fig. 12. Resistance 3-port with one inductive port

Short circuit current can be evaluated as

$$i_z(t_z^+) = \frac{u(t_z^-)}{r_{pp}} \tag{20}$$

where

$$r_{pp} = \frac{u(t_z^+) - u(t_z^-)}{I_p} \tag{21}$$

Simulation of the structure shown in Fig. 12 delivers the voltage waveform shown in Fig. 13.

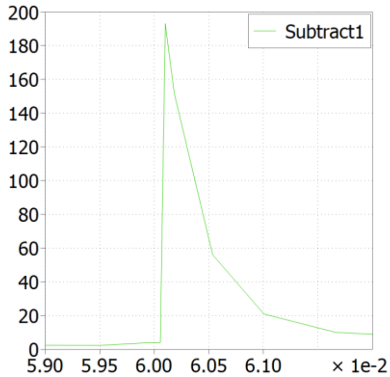

Fig. 13. Voltage waveforms while current impulse is injected to the inductive port.

The voltage responses of the capacitive and inductive circuit on the current pulse injected to the system are different. In capacitive circuit the port voltage after the big jump at the beginning is slightly increasing (Fig. 11). In inductive circuit the port voltage after the big jump at the beginning is rapidly decreasing (Fig. 13).

Conclusions

Assuming that the circuit model representing the power grid is linear the short-circuit current is equal to the sum of two components - steady state component and transient component. Steady state response is the sinusoidal function depending on voltage sources acting in the system. Each source has its share and this response does not depend on the time instant when short circuiting occurred. The steady state component can be computed using symbolic complex numbers method.

The transient component has a different nature. It depends on the time instant when short circuit arises, it depends on the initial phase of this switching.

The starting value of short circuit current can be expressed as linear algebraic function of the capacitor voltages and inductor currents at the switching instant. The set of all capacitor voltages an inductor currents forms the system state. But for the chosen port only limited set of state variables influences on the port current while short circuit happens. Exists such port neighbourhood that only capacitors and inductors placed in this neighbourhood influence on the port current at the switching instant.

The rectangular current pulse is injected to port while the grid is examined. Recognized 2-port representing the part of circuit placed at the neighbour of the short circuited port is searched as one of two alternative circuits models: capacitive and inductive circuit. The parameters of these models can be elaborated from the measurements of the port voltage. As the result the short circuit current can be predicted from the data obtained while the experiment with pulse current injection is done.

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