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Orthogonal power of electrical networks with periodic voltages and currents – a follow-up on Quade's geometrical power concept

Moc ortogonalna sieci elektrycznych o okresowych napięciach i prądach – kontynuacja koncepcji mocy geometrycznej Quade'a

Abstract: *The central aspect of this essay, which is a follow-up on my previous article that reviewed the Quade concept for the geometric summation of non-active powers, is the introduction of the term „orthogonal power". This new term for the useless power precisely reflects that its real root cause is orthogonality. The term also distinguishes the fundamental Quade concept from other technical definitions. The focus is further on a clear understanding of the meaning of total orthogonal and total apparent power of poly-phase networks, thus a reflection on the Quade concept. A new graphical representation of powers in vector space is introduced. For the first time we have measured the total orthogonal power of an electric arc furnace (EAF) which is a non-linear and dynamic three-phase load. Simulation results are discussed as well. Eventually the geometric summation of orthogonal power components is analysed.*

Streszczenie: *Głównym aspektem niniejszej pracy, która stanowi kontynuację mojego poprzedniego artykułu, w którym dokonano przeglądu koncepcji Quade'a z zakresie geometrycznego sumowania nieaktywnej energii, jest wprowadzenie terminu „energia ortogonalna". Ten nowy termin dla bezużytecznej energii pokazuje, że jej rzeczywistą podstawową przyczyną jest ortogonalność. Termin ten także czyni rozróżnienie pomiędzy podstawową koncepcją Quade'a a innymi technicznymi definicjami. Uwaga jest następnie skoncentrowana na jasnym zrozumieniu znaczenia całkowitej ortogonalnej i całkowitej pozornej energii wielofazowych sieci, a więc rozważań dotyczących koncepcji Quade'a. Wprowadzono nowe graficzne przedstawienie energii w przestrzeni wektorowej. Po raz pierwszy dokonano pomiarów ogólnej (czy całkowitej?) energii ortogonalnej w piecu łukowym (EAF), która ma charakter nieliniowy i dynamiczne trzy-fazowe obciążenie. Omówiono także wyniki badań symulacyjnych. Dokonano analizy geometrycznego zsumowania komponentów energii ortogonalnej.*

Keywords: Quade concept, orthogonality, orthogonal power, vector space, reactive power, non-active power, aggregate power, power measurement, electric arc furnace (EAF), power components
Słowa kluczowe: koncepcja Quade'a, ortogonalność, przestrzeń wektorowa, energia reaktywna, energia nieaktywna, energia scalona, pomiar energii, piec łukowy (EAF), składniki energii

Introduction

There is a vast number of publications on power theory. Most relevant for the author are the references [1 – 7]. Concepts are time based [2, 3, 4], frequency based [1, 5] or apply instantaneous quantities [6]. But to date it has not been commonly recognized that immediately measurable useless power of electrical networks with periodic voltages and currents is not an algebraic scalar but a geometric vector quantity. The time domain geometric concept developed by *Quade* [12 – 15] is general. It contains the sinusoidal complex power $\underline{S}$ = P + jQ and the aggregate power $S_{\Sigma-agg}$ defined in the German standard DIN 40110-2:2002-11 (similarly IEEE 1459-2010) as limit cases.

The fundamental result of the *Quade* concept should have a proper name. That is why the term „orthogonal power" for useless power is introduced here to distinguish it from other (technical) non-active power definitions, especially in regards poly-phase networks. Note that at the two-pole the terms *orthogonal* and *non-active* (or *reactive* in the sinusoidal case) have identic meaning and are interchangeable.

In the following the geometric concept that was reviewed in [8] is briefly summarized, graphical representations are introduced, the meaning is analysed and a first application is presented.

Orthogonality at the two-pole

Orthogonality is the real root cause of useless power in electrical networks. Vector space and orthogonality are closely linked. *Quade's* method applies vector space to determine orthogonal power.

The periodic current i(t) of a two-pole with periodic voltage u(t) and active power P can always be split into the (in principle) useful proportional current component

(1) $$i_{prop} = P/U^2 \cdot u$$

and the useless orthogonal current component

(2) $$i_{orth} = i - i_{prop}$$

Due to the orthogonality the RMS are related by

(3) $$I^2 = I_{prop}^2 + I_{orth}^2$$

Multiplied by the squared two-pole RMS voltage U², the generally valid orthogonal power relation of the root mean squares results, that defines the apparent power S:

(4) $$U^2 \cdot I^2 = U^2 \cdot I_{prop}^2 + U^2 \cdot I_{orth}^2$$
$$S^2 = P^2 + Q_{orth}^2$$

The active (or proportional) power P is the average of the extensive quantity energy and sums *algebraically*. The orthogonal power Q_{orth} is not an average value because the oscillation $p_{orth}(t) = u(t) \cdot i_{orth}(t)$ has average zero due to the orthogonality of u(t) and i_{orth}(t). The resolution of a sum $\sum p_{orth-\mu}(t)$ requires the superposition of these oscillations that interfere typically. The summation problem is solved by the *geometric* summation of orthogonal power vectors and is based on a correspondence of periodic voltages and currents of equal period but arbitrary curve forms in time domain with their vector representations in Euclidean vector space [8]. The *Quade* concept bases further on the fact that every active energy can be expressed as the scalar product of two vectors [11], concretely: $P = \vec{U} \cdot \vec{I}$. Orthogonality is defined by this scalar product, illustrated in figure 1.

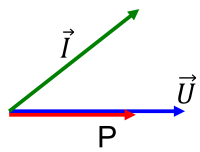

Fig. 1: Illustration of the scalar product

P is zero if $\vec{U}$ and $\vec{I}$ are perpendicular and maximal if $\vec{U}$ and $\vec{I}$ are collinear. Then $|\vec{U}|\cdot|\vec{I}| = |\vec{U}\cdot\vec{I}|$. In Euclidean $\mathbb{R}^3$ vector space the vector product $\vec{U}\times\vec{I}$ is defined and the vectorial orthogonality relation holds:

(5) $$|\vec{U}|^2\cdot|\vec{I}|^2 = (\vec{U}\cdot\vec{I})^2 + |\vec{U}\times\vec{I}|^2$$

Because the scalar product represents the active power and the product of the magnitudes the apparent power, the vector product (more general: wedge product) obviously represents the orthogonal power: $\vec{Q} = \vec{U}\times\vec{I}$ (resp. $\vec{Q} = \vec{U}\wedge\vec{I}$), illustrated in figure 2.

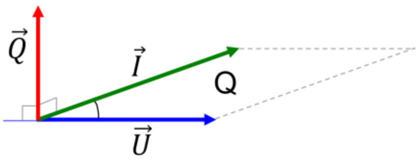

Fig. 2: Illustration of the vector product

The general geometric concept in multidimensional vector space is presented in [9], [10] and shows that orthogonal power $\vec{Q}$ is a bivector / antisymmetric tensor.

However, for practical measurement purposes three dimensions are sufficient because only sums of scalar products $\vec{Q}_\mu\cdot\vec{Q}_\nu$ need to be evaluated.

Illustration of two-pole powers in vector space

The here introduced figure 3 (by *H. Haase*) illustrates the relations of two-pole voltage, current and powers in vector space. It can be interpreted as follows.

The magnitudes of the vectors of voltage and current are their RMS. The voltage vector lies on the x-axis by convention. The current vector lies in the x-y-plane. The power factor λ is the cosine of the angle between $\vec{U}$ and $\vec{I}$, it's magnitude is $\lambda = |P|/S$.

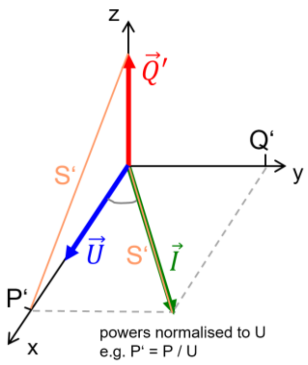

Fig. 3: Two-pole powers in vector space

The magnitude of the axial vector $\vec{Q}' = \vec{U}\times\vec{I}$ is Q'. $\vec{Q}'$ is determined by the vector product and by the mutual position of the vectors $\vec{U}$ and $\vec{I}$. If the current vector leads or lags the voltage vector, can be determined from the fundamental 50 or 60 Hz oscillations which are measured by digital PQ-analyzers. With a lagging (inductive) current, the orthogonal power vector points in positive z direction (+Q if $\vec{U}$ is turned towards $\vec{I}$, right turn). With leading (capacitive) current, the orthogonal power vector points in negative z direction (-Q if $\vec{U}$ is turned towards $\vec{I}$, left turn). In case the network is ohmic, the orthogonal power vector is zero, P=S and voltage and current vectors are collinear, lying on the x-axis. In the purely orthogonal case the current vector lies on the ±y-axis and P=0.

The representations of other two-poles (or gates) are generally rotated around the x-axis, the orientation of the vectors $\vec{Q}'_\mu$ lying in the y-z-plane.

How the magnitudes of active and orthogonal powers result geometrically, is depicted in the here introduced figure 4 (by *H. Haase*).

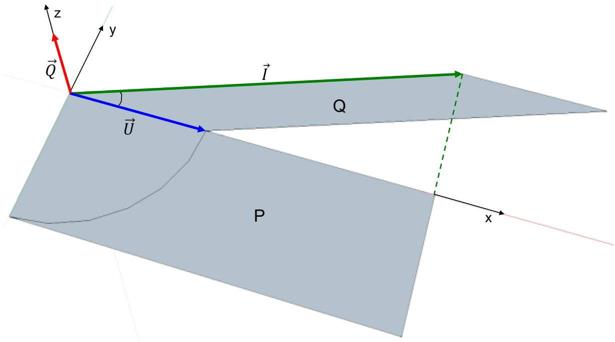

Fig. 4: Power magnitudes in vector space

The areas (grey) are the magnitudes of P and $\vec{Q}$. $\vec{U}$ and $\vec{I}$ lie in the x-y-plane. If $\vec{I}$ is collinear to $\vec{U}$ then Q = 0 and P is maximal. If on the other hand $\vec{I}$ is perpendicular to $\vec{U}$ then Q is maximal and P = 0. Note that other $\vec{U}$ and $\vec{I}$ pairs lie in different planes generally.

Total orthogonal power

Due to its vector character, the total orthogonal power $\vec{Q}_{orth-tot}$ of any electrical network in any periodic condition equals the geometric sum of the orthogonal power vectors $\vec{Q}_\mu$ of all two-poles or gates (similar to the summation of mechanical forces):

(6) $$\vec{Q}_{orth-tot} = \vec{Q}_1 + \vec{Q}_2 + .. + \vec{Q}_n$$

This abstract sum of vectors is solved by

(7) $$\vec{Q}^2_{orth-tot} = (\vec{Q}_1 + \vec{Q}_2 + .. + \vec{Q}_n)^2$$
$$= Q_1^2 + Q_2^2 + .. + Q_n^2 + \sum \vec{Q}_\mu\cdot\vec{Q}_\nu$$

that contains the squared magnitudes which are calculated with (4) and scalar products that are the interference terms (phase information). Remarkably, the scalar products are resolved with only immediately measurable voltages and currents. Total orthogonal power is measured by [9], [10]:

(8) $$Q^2_{orth-tot} = \sum_{\mu=1}^{n-1}\sum_{\nu=1}^{n-1}\left(\overline{u_\mu u_\nu}\cdot\overline{\iota_\mu\iota_\nu} - \overline{u_\mu\iota_\nu}\cdot\overline{\iota_\mu u_\nu}\right)$$

where (n-1) is the number of independent two-poles (gates), μ, ν the conductor numbers 1 to n, i_μ, i_ν the conductor currents, u_μ, u_ν measured against the reference conductor (e.g. n) and the forms $\overline{x_\mu y_\nu} = \frac{1}{T}\int_0^T x_\mu\cdot y_\nu\,dt$ are averages resp. squared RMS. Equation (8) is particularly well suited for the measurement of the total orthogonal power at the terminals of poly-phase networks. Upon reduction into a single-phase condition the result of (8) reduces continuously into the two-pole orthogonal power equation (4). A poly-phase network in purely proportional condition (e.g. ohmic resistor network), where $\vec{Q}_{orth-tot} = 0$, can be asymmetric. There is no real „asymmetry non-active power". Upon transition from non-sinusoidal to sinusoidal condition the vector quantity $\vec{Q}$ reduces into the imaginary value ±jQ that is summed algebraically.

P_Σ and $Q_{orth-tot}$ are orthogonal, so the orthogonality relation of the total powers follows naturally:

(9) $$S^2_{tot} = P^2_\Sigma + Q^2_{orth-tot}$$

Total apparent power is determined by total orthogonal power and not vice versa.

Concluding, the fundamental orthogonal power concept of *Quade* is characterized by its generality ("↔" means continuous transition):

- poly-phase ↔ single-phase
- periodic ↔ sinusoidal
- vector space ↔ complex plane

Contrarily, other power definitions are characterized by:

- Aggregate power is a result of the definitions of DIN 40110-2 and is valid only in one limit condition (IEEE 1459 similarly). It is the power of the ideally compensated poly-phase load.
- Algebraic summation of non-active power magnitudes is as well restricted to limit cases.

Meaning of the total orthogonal power $\vec{Q}_{orth-tot}$

With his finding that orthogonal power is a geometric quantity, *Quade* has solved the summation problem elegantly and generally in time domain. In every electrical network with periodic voltages and currents that are proportional in every branch, only active power is transformed and $\vec{Q}_{orth-tot} = 0$. With fixed total apparent power the total active power is reduced by orthogonality (per branch). The total orthogonal power $\vec{Q}_{orth-tot}$ measured at the terminals is a measure for the orthogonality of any electrical network with periodic voltages and currents that results from

- time shifted zero crossings of voltage u(t) and current i(t) and/or
- non-sinusoidality (harmonics) and/or
- discontinuity

Meaning of the total apparent power S_{tot}

In all electrical networks with periodic voltages and currents, single-phase or poly-phase, in symmetric or asymmetric, sinusoidal or non-sinusoidal condition, the total apparent power S_{tot} measured at the terminals only states that the network is in some orthogonal condition if $S_{tot} > P_\Sigma$, so if $\vec{Q}_{orth-tot} > 0$. If every branch voltage and current of the network are proportional then $\lambda_{tot} = |P_\Sigma|/S_{tot} = 1$. $\lambda_{tot} = 1$ is also measured for the system network + perfect compensation. If the network is in a total orthogonal condition then $\lambda_{tot} = 0$. Apparent power is not an independent quantity but always a consequence of orthogonality, determined via (9). Orthogonality does not depend on asymmetry.

The optimal utilization of the usual energy supply network with sinusoidal and symmetric supply voltages results if the load is symmetric and proportional (the goal of compensation). Then $S_{\Sigma-agg} = S_{tot} = |P_\Sigma|$ [8]. Only for this limit condition the aggregate power equation can be determined with the *Lagrange Multiplier* method (evaluated already by *Quade* [13]). The German term „Rechtleistung" for aggregate power, literally „right power", indicates the „right" condition of a poly-phase load connected to a normal (the ideal) energy supply network. In asymmetric and/or non-sinusoidal cases the aggregate power and thus aggregate non-active power have no proper meaning, both are generally no measurands.

Exemplarily, the deviations of aggregate and total fictitious powers from total apparent power are depicted in figure 5 for symmetric sinusoidal supply voltages at asymmetric star connected ohmic resistors. The fictitious (or virtual) starpoint concept and fictitious powers (index 0) are explained in [8] and are part of DIN40110-2 [17].

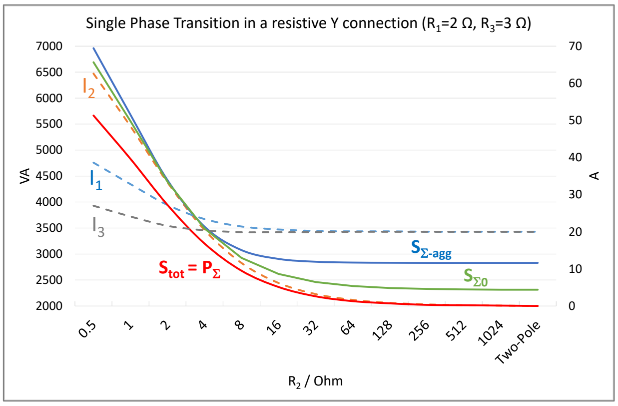

Fig. 5: Transition from three-phase to single-phase condition in an asymmetric ohmic Y connection

The courses of aggregate power $S_{\Sigma-agg}$ and fictitious total apparent power $S_{\Sigma 0}$ differ significantly from total apparent power S_{tot} and from each other due to the asymmetry and due to the ohmic condition. Note that for all R2 values $S_{tot} = P_\Sigma$ and $Q_{orth-tot} = \pm jQ_{12} \pm jQ_{23} = 0$.

$S_{\Sigma-agg}$ is composed of the „fictitious aggregate orthogonal power" and the „fictitious aggregate proportional asymmetry power" according to the rules of DIN40110-2. In this ohmic case these components equal $S_{\Sigma-agg}/\sqrt{2}$ for all R2 values and would have to be generated by a compensation in order to let the network (the load) appear symmetric to the supply side.

$S_{\Sigma 0}$ differs from S_{tot} because it results from the sum of magnitudes $Q_{10} + Q_{20} + Q_{30}$ only.

Power measurement at an electric arc furnace

One example for a dynamic and non-sinusoidal three-phase three-conductor load is the electric arc furnace. Its simplified equivalent circuit with primary and secondary side voltage measurements is depicted in figure 6 (R is a series reactor, T the furnace transformer). The high current system of the EAF is always a star connection, the furnace transformer's vector group is typically Dd0.

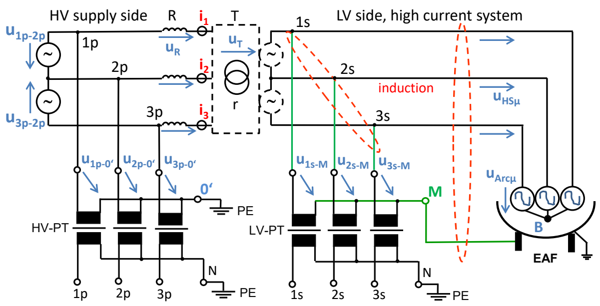

Fig. 6: Simplified equivalent circuit of AC-EAF

On the HV side the (almost) symmetric supply voltages are measured against the „artificial" starpoint 0' of a PT or against the fictitious starpoint 0. The difference between 0' and 0 is small for quite symmetric conditions. Thus on the HV side an asymmetric condition in the furnace is reflected only by the line currents. On the LV side the voltages are measured against the „reference" ground „M" of the furnace vessel that is closest to the inaccessible real starpoint „B" in the steel scrap charge. Thus the powers per phase (per electrode) are correctly determined in principle. But the LV side voltage measurement is subject to induction by the large electrode currents [21]. The real phase voltages $u_{HS\mu}$ remain unknown and are only approximately measured by the „electrode" voltages $u_{\mu s-M}$ with unavoidable variable errors in magnitude and phase shift which result in up to ≈10%

deviation from the correct primary side total active power and energy. The real arc voltages are not measurable.

On 12th July 2024 we carried out the first measurement of the total orthogonal power since *Quade* developed his concept 1933-1937. The measurement is implemented in our *BSE-Elarc* electrode regulation system for EAF. This system measures the LV side „electrode“ voltages $u_{\mu s\text{-}M}$ and typically the HV side currents i_{μ}. It regulates the arc parameters such that an impedance setpoint and a current setpoint (per phase) are kept on average by moving the graphite electrodes hydraulically up and down depending on the conditions inside the furnace. The typical electrode regulation on the market only measures the LV side powers and total energy. As mentioned, these are always more or less faulty by induction. Therefore our goal was to be the first to implement the accurate HV side power measurement. The goal is twofold: 1. accurate MW and kWh measurement because these are important parameters of each EAF and important for the reporting of the regulation system and 2. the first measurement of the total orthogonal power, i.e. the first total useless power measurement that determines the degree of active energy transformation in the load. The implementation in our system also shows that it is easy to do. We as well intend to be an example for manufacturers of power- and PQ-measurement devices.

The 150 t furnace designed by BSE, where we carried out the measurement, figure 7, is equipped with a furnace transformer of 156 MVA rated power.

Fig. 7: EAF where the total orthogonal power was first measured

Connected in series on the HV side is a reactor with onload tap changer. Its additional inductance is required for arc stability in scrap melting. In regards arc stability „stable melting“ means more sinusoidal condition, „unstable melting“ means more non-sinusoidal condition. At the 34.5 kV substation the EAF is compensated with a Static Var Compensation (SVC). The high current system geometry is electrically symmetric, having three equal short circuit impedances:

$$\underline{Z}_{SC1} = (0.32 + j\,3.12)\ \mathrm{m\Omega}$$
$$\underline{Z}_{SC2} = (0.30 + j\,3.10)\ \mathrm{m\Omega}$$
$$\underline{Z}_{SC3} = (0.32 + j\,3.11)\ \mathrm{m\Omega}$$

This design is achieved applying the *BSE-FNM* (Farschtschi Network Method) [19] that is especially required to properly dimension the high current loop (right upper corner in figures 7, 8). This sophisticated and unique field simulation is necessary because significant current displacements (eddy currents, skin+proximity effects) in the large conductors need to be considered accurately to get realistic results. Figure 8 depicts the EAF high current system of figure 7 simulated with *BSE-FNM*.

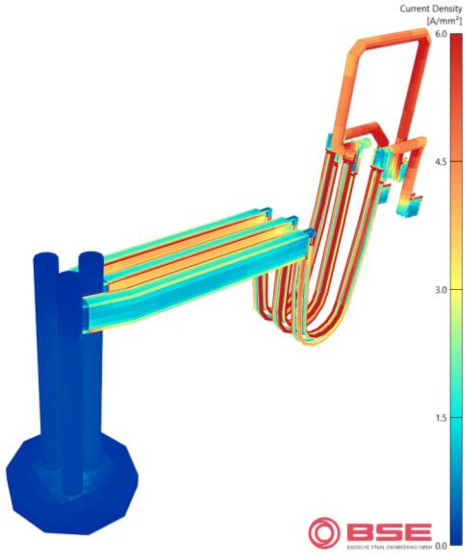

Fig. 8: High current system simulated with *BSE-FNM* at 67 kA. Colours: current density, linear scale

Peculiarities of electric arc furnaces are described in [20, 21]. The following figure 9 depicts the power trends of the LV side of a typical heat consisting of two scrap charges showing the dynamical melting behavior with significant variations in scrap melting (sample time 2 seconds).

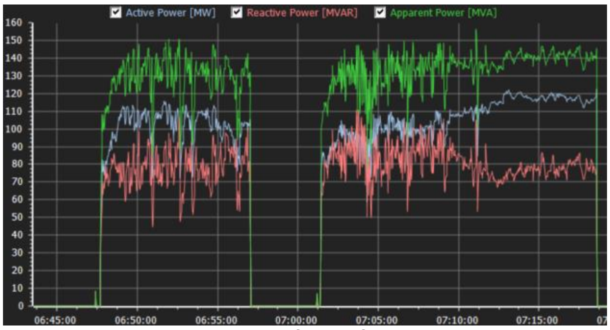

Fig. 9: Measured power trends of one of the heats evaluated

The second half of the second charge is the low variation liquid period with molten steel and slag.

Evaluated are scrap melting periods operated with a transformer tap voltage of 1318 V and 68 kA electrode currents at reactor tap 1.19 Ω, in total 33 minutes. These unsteady periods with partly very non-sinusoidal arc voltages and currents result in differences between the fictitious / aggregate and the orthogonal power values.

Exemplarily figure 10 depicts the measured „electrode“ voltage $u_{2s\text{-}M}$ and a typical scrap melting current form.

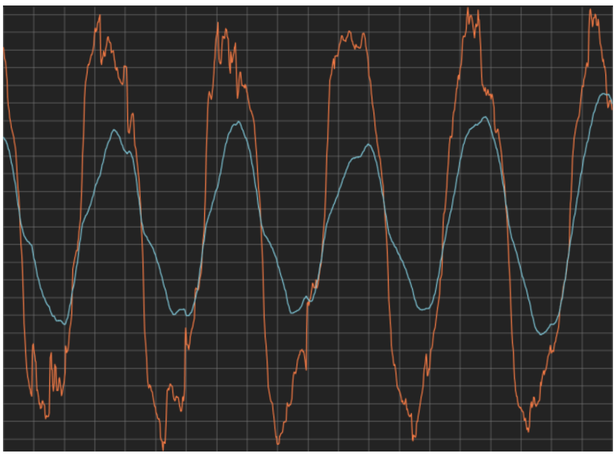

Fig. 10: Illustrative oscillogram of 50 Hz electrode voltage $u_{2s\text{-}M}$ (red) and line (arc) current i_2 (blue) in scrap melting

The „electrode“ voltage is very distorted because it includes the arc voltage.

Figure 11 depicts measured typical arc current and primary voltage wave forms during a stable melting period. On the primary side the supply voltages (measured against the artificial starpoint 0‘) do not contain the arc voltages.

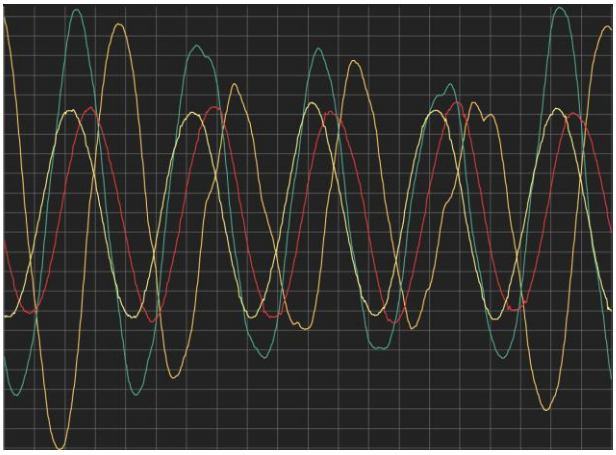

Fig. 11: Illustrative oscillogram of 50 Hz primary voltages $u_{1p\text{-}2p}$, $u_{3p\text{-}2p}$ (red, yellow) and line (arc) currents i_1, i_3 (dark yellow, green)

Note the variability of the currents and the stability and sinusoidality of the compensated supply voltages.

The total orthogonal power was measured by implementing the specialization of equation (8)

(10) $$Q_{orth-tot}^2 = U_{1p-2p}^2 \cdot I_1^2 - P_{12}^2 + U_{3p-2p}^2 \cdot I_3^2 - P_{32}^2$$
$$+2 \cdot \frac{1}{T}\int_0^T u_{1p-2p} \cdot u_{3p-2p} dt \cdot \frac{1}{T}\int_0^T i_1 \cdot i_3 dt$$
$$-2 \cdot \frac{1}{T}\int_0^T u_{1p-2p} \cdot i_3 dt \cdot \frac{1}{T}\int_0^T u_{3p-2p} \cdot i_1 dt$$

for the three-phase three-conductor system with phase 2 as the reference conductor. The evaluation results are:

Total active power

$$P_\Sigma = P_{12} + P_{32} = 108.5 \text{ MW}$$

Total orthogonal power

$$Q_{\text{orth-tot}} = 107.1 \text{ MVAr}$$

Total apparent power

$$S_{\text{tot}} = \sqrt{P_\Sigma^2 + Q_{\text{orth-tot}}^2} = 152.46 \text{ MVA}$$

At the electric arc furnace the total degree of active energy transformation is stated by the total power factor

$$\lambda_{\text{tot}} = 108.5/152.46 = 0.712$$

Fictitious total non-active power

$$Q_{\Sigma 0'} = Q_{10'} + Q_{20'} + Q_{30'} = 111.7 \text{ MVAr}$$

Fictitious total apparent power

$$S_{\Sigma 0'} = \sqrt{P_\Sigma^2 + Q_{\Sigma 0'}^2} = 155.72 \text{ MVA}$$

Supply voltages $U_{1p\text{-}2p}$, $U_{2p\text{-}3p}$, $U_{3p\text{-}1p}$

34.39 kV, 34.30 kV, 34.33 kV

Line currents I_1, I_2, I_3

2627 A, 2621 A, 2608 A

Aggregate power

$$S_{\Sigma-agg} = \sqrt{{}^1\!/_3 \left(U_{1p-2p}^2 + U_{2p-3p}^2 + U_{3p-1p}^2\right)}$$
$$\cdot \sqrt{\left(I_1^2 + I_2^2 + I_3^2\right)} = 155.76 \text{ MVA}$$

Aggregate non-active power

$$Q_{\Sigma-\text{agg}} = \sqrt{S_{\Sigma-\text{agg}}^2 - P_\Sigma^2} = 111.75 \text{ MVAr}$$

The total orthogonal and apparent powers are smaller than the total fictitious powers due to the interference terms that in this case result mainly from harmonics of the line currents. Under these symmetrical conditions the fictitious total apparent power almost equals the aggregate power. Note that always $S_{tot} \le S_{\Sigma 0'} \le S_{\Sigma\text{-agg}}$ [8].

Computation of arc furnace power

EAF are electrically complex aggregates. To determine their operating behavior in terms of optimal power input, measurements are mandatory. There are so many operational influences that all EAF “behave” individually. On the other hand some real properties of the system can only be found by simulation because not all parameters can be measured. Proper assessments require measurement and simulation. In particular cases like the determination of the short circuit reactances of EAF, a proper simulation is superior to a measurement.

BSE applies a program that computes the EAF powers for given setpoints. This is very useful to determine projections based on measurements. The program uses an empirically proven non-linear phenomenological model for the arc voltages that approximates reality sufficiently well. Applied are rectangular functions generated with *Fourier* series. The model needs to consider that arc voltage and arc current are always in phase. The arcs are non-linear resistors with typically asymmetric half waves in one period (→ the graph u=f(i) shows hysteresis). The level of harmonics generated depends on the arc voltage and current. Larger arc voltage and/or less arc current generate more harmonic content. Similarly melting on scrap creates more harmonics than melting in liquid steel / slag. The EAF high current system with arcs is a non-linear, inductively coupled, resistive-inductive three-phase load. The electrical characteristic of the furnace is dominated by the variable high current system reactances (→ harmonics, geometry) and by the variable arc resistances.

The arcs are stable on average (in terms of active power input) in scrap melting if the total system reactance (that includes a series reactor, the transformer and the high current system) is greater than a minimal value. The stability characteristic can be simulated with our model. Figure 12 depicts an example of the computed characteristic course of the arc voltage as a function of current. The real arc voltage increases until the arc is extinguished when raising the electrode up steadily. The voltage P_{arc}/I has got a maximum at a certain small current (long arc), depending on the total system reactance. The larger harmonic content at smaller current indicates arc instability by rapidly decreasing arc active power.

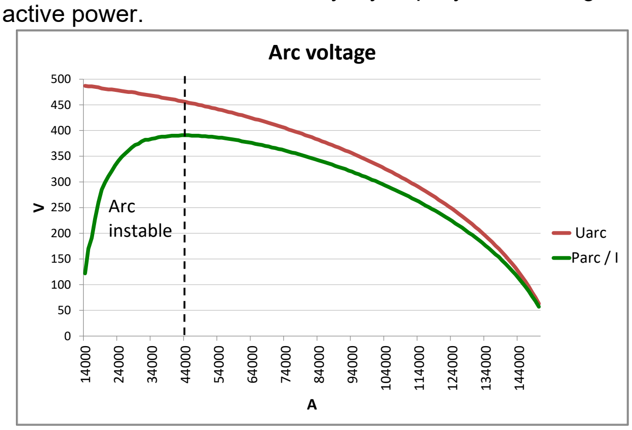

Fig. 12: Computed arc stability characteristic

In the following example computation results are presented for another EAF with asymmetric high current system short circuit reactances and the equipment parameters

Transformer: 21000 / 950 V, 2488 A

Serial reactor: 0.7 Ω

Short circuit impedances of the high current system:

$\underline{Z}_{SC1}$ = (0.33 +j 2.14) mΩ

$\underline{Z}_{SC2}$ = (0.33 +j 1.80) mΩ

$\underline{Z}_{SC3}$ = (0.33 +j 2.16) mΩ

The figures 13 – 16 show the behavior of the total powers (including reactor, transformer, high current system, arcs) under the condition that the symmetric and sinusoidal supply voltages are fixed at 21 kV and the electrode currents 1 and 2 are fix at 55 kA and current 3 varies from 35 kA to 55 kA. Differences between total orthogonal, total fictitious and aggregate powers become especially obvious. A measurement would give very similar results. For large current asymmetry and large harmonic content the differences of the powers are greatest (I_3 smaller than 50 kA). The total power factor PF_{tot} indicates that the degree of energy transformation is less with greater harmonic content and asymmetry. On the right side of the maximum the power factors decrease again due to the increasing total inductive load at larger currents (short arcs). The load of circuit loop 12 remains constant at

S_{12} = 21000 V · 2488 kA = 52.248 MVA.

The load of circuit loop 32 increases linearly with the current from S_{32} = 33.33 MVA to S_{32} = 52.248 MVA.

Only for the symmetric and sinusoidal system a meaningful total (three-phase) nominal load can be calculated with the rated values:

S_{rated} = 1.732 · 21000 V · 2488 A = 90.494 MVA.

The actual total apparent power S_{tot} is 90.34 MVA at I_1=I_2=I_3=55 kA, indicating the remaining short circuit reactance asymmetry and harmonic influence.

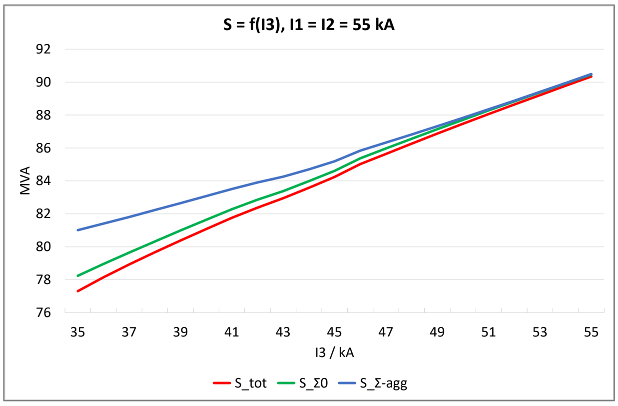

Fig. 13: Computed total apparent powers

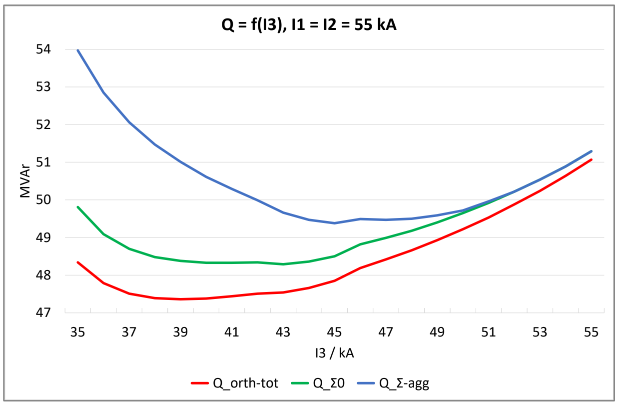

Fig. 14: Computed total orthogonal and non-active powers

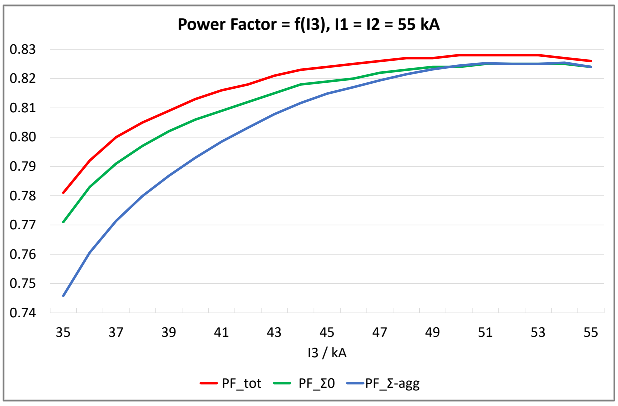

Fig. 15: Computed total power factors

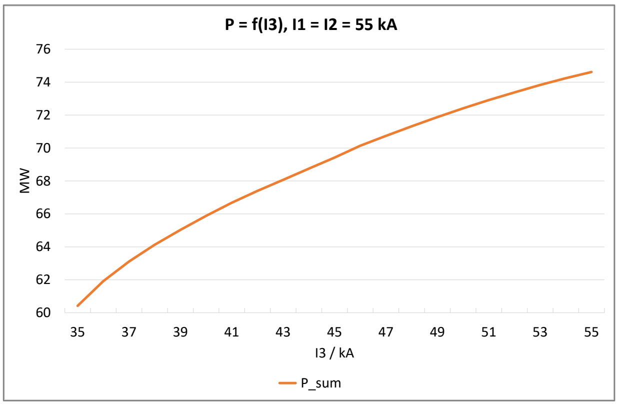

Fig. 16: Computed total active power

There are other interesting system parameters that indicate the effect of harmonic content per phase, listed in tables 1 and 2.

Table 1: Computed EAF parameters at I_3 = 35 kA, I_1 = I_2 = 55 kA

I_3 = 35 kA	Ph.1	Ph.2	Ph.3
U_{arc} [V]	481	386	627
P_{arc} / I [V]	428	347	426
U_{arc} / (P_{arc} / I)	1.12	1.11	1.47
X_p / X_{sc}	1.54	1.36	3.44
X_{p0} / X_{sc}	1.80	1.16	2.42
PF_p	0.779	0.783	0.647
PF_{p0}	0.635	0.890	0.784

Table 2: Computed EAF parameters at I_1 = I_2 = I_3 = 55 kA

I_3 = 55 kA	Ph.1	Ph.2	Ph.3
U_{arc} [V]	491	493	479
P_{arc} / I [V]	433	434	424
U_{arc} / (P_{arc} / I)	1.13	1.14	1.13
X_p / X_{sc}	1.51	1.59	1.48
X_{p0} / X_{sc}	1.26	1.47	1.35
PF_p	0.789	0.798	0.786
PF_{p0}	0.841	0.815	0.815

Index “p” means primary (HV side), index “0” means fictitious, index “sc” means short circuit (no arcs). Note that all relevant system impedances need to be included in an assessment. The ratio $U_{arc}/(P_{arc}/I)$ indicates the degree of non-linearity. For a long arc at 35 kA this is significant with a ratio of 1.47. For 55 kA the ratio reduces to 1.13.

The tables also indicate the difference between the real phase values that can only be calculated and the fictitious phase values that can be measured. $X_{p\mu}/X_{sc\mu}$ is the ratio of the total system reactance $X_{p\mu} = Q_{p\mu}/I_{\mu}^2$ and the short circuit reactance of the high current system per phase. The ratio is large for large harmonic content. A value greater than 2 indicates arc instability (reduced active power). Note also, that the measurable fictitious phase reactance ratios $X_{p0\mu}/X_{sc\mu}$ and fictitious phase power factors $PF_{p0\mu}$ differ significantly from the real phase values. This can lead to wrong assessments and needs to be considered. At 55 kA where the system is almost completely symmetric, the $PF_{p0\mu}$ values are greater than the real $PF_{p\mu}$ values, indicating that the fictitious supply voltages are sinusoidal and that the $PF_{p0\mu}$ do not represent the non-sinusoidal load.

General power analysis

Obviously, the digital power measurement devices used today are based on incomplete power definitions because only aggregate power and „per phase" power magnitudes are used. The concept of fictitious (virtual) starpoint voltages is not implemented. However, only small programming effort would be required to improve the situation by implementing the total orthogonal and apparent powers for poly-phase systems and the fictitious starpoint concept.

A comprehensive power analysis of the condition of poly-phase systems is achieved by measuring the

- Total active power
- Total orthogonal (and apparent) power: degree of orthogonality
- Active, orthogonal and apparent powers of the (n-1) gates: loads, asymmetry
- Decomposition of the gate apparent powers: harmonic power contents of each gate circuit

The fictitious powers are used to determine "per phase" values measured against the fictitious starpoint but these differ from the real phase values at the load impedances except for limit conditions (symmetry, sinusoidality). However, the fictitious phase power factors $\lambda_{\mu 0} = P_{\mu 0}/(U_{\mu 0} \cdot I_{\mu})$ indicate the unnecessary and useless load of the conductors of the energy supply with the related orthogonal current components $i_{orth-\mu 0}$ acc. to equation (2). The aggregate power is used (only) for comparison to determine if a poly-phase network connected to the (ideal) energy supply grid is optimally utilized. Then
$S_{\Sigma-agg} = S_{\mathrm{tot}} = |P_{\Sigma}|$.

Summation of orthogonal power components

Generally, at the two-pole and thus at each gate of a poly-phase system a power decomposition is possible. Already *Quade* discussed this [15]. The fundamental (50 or 60 Hz) powers are of special interest. The vector character of the orthogonal power is universal. It is generally wrong to just sum the squares of non-active power components algebraically. A comparison of the results of algebraic summation with the total two-pole orthogonal power illustrates this clearly.

The two-pole or gate apparent power is decomposed as follows [18]:

(11) $$S^2 = U^2 \cdot I^2 = (U_1^2 + U_H^2) \cdot (I_1^2 + I_H^2)$$
$$S^2 = U_1^2 I_1^2 + U_1^2 I_H^2 + I_1^2 U_H^2 + U_H^2 I_H^2$$
$$S^2 = S_1^2 + D_I^2 + D_U^2 + S_H^2$$

D is an apparent power because only unequal order harmonics are contained in voltage and current components. What does this decomposition mean?

$U_1^2 I_1^2$ is the squared fundamental (50 or 60 Hz) apparent power. This can contain fundamental active and reactive power P_1 and Q_1:

(12) $$S_1^2 = P_1^2 + Q_1^2$$

$U_1^2 I_H^2$ is the squared current distortion power:

(13) $$D_I^2 = U_1^2 \cdot \sum_{k=2}^{N} I_k^2,\ \ Q_I = D_I,\ \ P_I = 0$$

$I_1^2 U_H^2$ is the squared voltage distortion power:

(14) $$D_U^2 = I_1^2 \cdot \sum_{k=2}^{N} U_k^2,\ \ Q_U = D_U,\ \ P_U = 0$$

$I_H^2 U_H^2$ is the squared harmonic apparent power that contains all other harmonics of equal and unequal harmonic order:

(15) $$S_{\mathrm{H}}^2 = P_{\mathrm{H}}^2 + Q_{\mathrm{H}}^2$$

The total active power (or proportional power) of the two-pole is

(16) $$P = P_1 + P_{\mathrm{H}}$$

The two-pole orthogonal power is

(17) $$Q^2 = U^2 I^2 - P^2 \leq Q_{sum}^2 = Q_1^2 + D_I^2 + D_U^2 + Q_H^2$$

All non-active power components can be computed separately applying the power orthogonality relation (4). However, it is wrong to assume that the squares of the non-active power components generally sum algebraically like the apparent power components S_μ, D_μ. The (mathematically) correct summation considers the vector character of the orthogonal power components and contains the interference terms (scalar products):

(18) $$Q^2 = (\vec{Q}_1 + D_I + D_U + \vec{Q}_H)^2$$
$$= Q_1^2 + D_I^2 + D_U^2 + Q_H^2 + \sum \vec{Q}_\mu \cdot \vec{Q}_\nu$$

The interference terms solved are (the apparent power terms D_I and D_U are treated as orthogonal power vectors):

(19) $$\sum \vec{Q}_\mu \cdot \vec{Q}_\nu = 2(\vec{Q}_1 \cdot \vec{Q}_I + \vec{Q}_1 \cdot \vec{Q}_U + \vec{Q}_1 \cdot \vec{Q}_H)$$
$$+2(\vec{Q}_I \cdot \vec{Q}_U + \vec{Q}_I \cdot \vec{Q}_H) + 2(\vec{Q}_U \cdot \vec{Q}_H)$$

In general, all interference terms need to be evaluated. This is the reason why a power tetrahedron can only be generated for special conditions. It does not include the interference terms. The power tetrahedron of DIN40110-1 is such a special case, where the voltage is rigidly sinusoidal. In this special case the summation of the squares of the orthogonal resp. apparent power components is correct because no equal order harmonics are contained in voltage and in harmonic current.

(20) $$S^2 = U_1^2 \cdot I^2 = U_1^2 I_1^2 + U_1^2 I_H^2 = S_1^2 + D_I^2$$

(21) $$Q^2 = U_1^2 \cdot I^2 - P_1^2 = Q_1^2 + D_I^2$$

The following power tetrahedron results, figure 17.

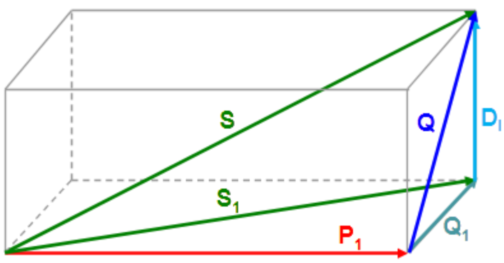

Fig. 17: Power tetrahedron of DIN 40110-1

It can be determined for each gate of a poly-phase network if the gate voltages are purely sinusoidal. Generally this is not the case and then this power tetrahedron is meaningless. At a dynamically compensated arc furnace it is approximately valid on the high voltage supply side (figure 11 exemplarily).

Care is to be taken with current and voltage distortion powers. Even if the total orthogonal power is zero, these quantities can have values.

The following simple example computed with a spreadsheet, figure 18 and table 3, proves the validity of *Quade's* concept. Voltage and current are distorted as depicted in figure 18.

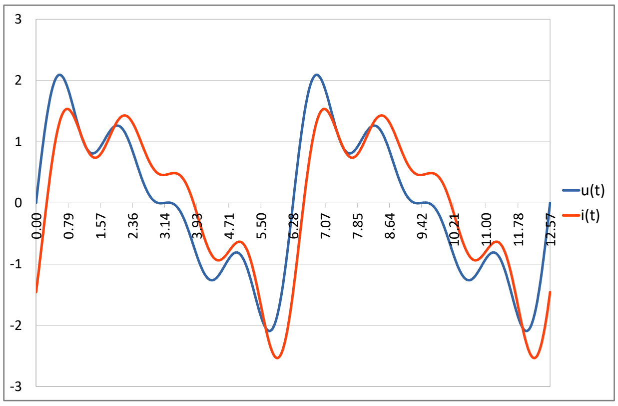

Fig. 18: Distorted two-pole voltage and current, two periods, illustrative

The correct orthogonal power from the two-pole power orthogonality relation is

$$Q = \sqrt{U^2 \cdot I^2 - P^2} = 0.6893 \text{ VAr}$$

The result from the wrong sum of the squared orthogonal resp. apparent power components is

$$Q_{sum} = \sqrt{Q_1^2 + D_I^2 + D_U^2 + Q_H^2} = 1.017 \text{ VAr}$$

This is too large by a factor 1.475.

Considering the interference terms in the summation gives the correct result:

$$Q_{orth-tot} = \sqrt{Q_1^2 + D_I^2 + D_U^2 + Q_H^2 - 0.5593} = 0.6893 \text{ VAr}$$

Conclusion

The term „orthogonal power“ was introduced to point out that orthogonality is the real root cause of useless power of single-phase or poly-phase networks (generally of arbitrary networks) with periodic voltages and currents. The term also distinguishes the result of the *Quade* concept from other technical non-active power definitions and indicates the physical relevance and fundamental meaning for every electrical network. At the two-pole (single phase system or a gate of a poly-phase system) the terms orthogonal power and non-active (or reactive) power have identical meaning. The measurement and the simulation results discussed point out the significant differences between the fundamental orthogonal power concept and the other „means to an end“ definitions.

Table 3: Results of the computed IEEE two-pole power decomposition for figure 18

$P_{two\text{-}pole}$	1.1893	W
$P_1+P_H+P_I+P_U$	1.1893	W
$U_{two\text{-}pole}$	1.1719	V
$I_{two\text{-}pole}$	1.1730	A
$S_{two\text{-}pole}$	1.3746	VA
$Q_{two\text{-}pole}$	0.6893	VAr
$\lambda_{two\text{-}pole}$	0.8652	
Interference terms	– 0.5593	VAr
U_1	0.9994	V
I_1	0.9997	A
U_H	0.6120	V
I_H	0.6126	A
S_1	0.9990	VA
P_1	0.8649	W
Q_1	0.5000	VAr
S_H	0.3749	VA
P_H	0.3244	W
Q_H	0.1880	VAr
D_I	0.6122	VA
D_U	0.6118	VA
$D_I^2 + D_U^2$	0.7490	(VA)²
P_I	0	W
P_U	0	W

The author wishes to thank H. Haase for his support and his colleagues A. Vogel and S. Garcia for implementing the total orthogonal power measurement in the BSE-Elarc regulating system.

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Correction to [8]

The equation $\vec{f}^2 \cdot \vec{g}^2 - \vec{f} \cdot \vec{g} = \begin{vmatrix} \vec{f}^2 & \vec{f} \cdot \vec{g} \\ \vec{g} \cdot \vec{f} & \vec{g}^2 \end{vmatrix} \geq 0$

on page 2, second column, must read:

$$\vec{f}^2 \cdot \vec{g}^2 - (\vec{f} \cdot \vec{g})^2 = \begin{vmatrix} \vec{f}^2 & \vec{f} \cdot \vec{g} \\ \vec{g} \cdot \vec{f} & \vec{g}^2 \end{vmatrix} \geq 0$$

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