**Sliding mode controls induction motor drive under unknown load torque through a super twisting algorithm**

*Abstract*: In this paper, a reduced induction motor (IM) model is used to control the mechanical speed and flux in the field oriented control based on sliding mode controller (SMC), associated with a super twisting algorithm second order sliding mode observer (STASMO). The proposed observer is used to enhance the performance of the controller under uncertainties; thus, the observer is used to estimate the rotor flux, and the load torque which is considered as unknown disturbance to the machine. This combination ensures the convergence of the controlled speed and rotor flux to their desired input references in finite time. The results show the robustness of the controller in the presence of unknown disturbance.

**Słowa kluczowe**: Sliding mode, super-twisting algorithm, induction motors, field orientation control, unknown load torque.

**Keywords**: Sliding mode, super-twisting algorithm, induction motors, field orientation control, unknown load torque.

**I. Introduction**

The induction motor is an essential element in the industry among other motors, due to its robustness and low maintenance [1]. The mathematical model of this machine is highly nonlinear and coupled multivariable system, this complex mathematical representation makes its control a challenging problem especially in the case of parameter and external disturbance (load torque) variations. Hence the control and observation of the states of the induction machine have been the subject of many researchers. 

In the literature, there are several methods presents such as Backstepping control, input-output feedback linearization, neural network control and passivity based controllers [2,3,4, 5], and sliding mode control [6]. Sliding mode controller is considered the used most solution for the complex system uncertainties [7] and provide the best performance for the control of an IM even in the presence of an internal and external perturbation [8] to name just few. The control of an IM is known by its difficulty of measuring the rotor flux and load torque which need sensors to be get they values. In the last decade, several observers have been designed to overcome these difficulties such as sliding mode observer (first and second order) [9], high gain observer [10], MRAS adaptive observer [11,12,13], adaptive neural network nonlinear observer [14]. Super twisting is tool for chattering attenuation and is used to observe the state of the nonlinear uncertain system [15,16] and in [17], the super twisting high order was used to estimate the rotor position and the mechanical speed, as well as tracking the parameter variations with an online algorithm. The main advantage of the super twisting observer is to reduce the chattering attenuation.

Due to the complexity of the dynamical model of the induction motor (IM), we have reduced it from five to four states by describing the motor model in the synchronous frame with the rotor flux aligned on the d-axis and, therefore a nonlinear sliding model controller is applied to the reduced model. An observer based on super-twisting theory [18] has been designed to estimate the rotor flux and the external disturbance (load torque) to improve the performance of the proposed controller under an external disturbance which is the load torque of the machine. The main advantage behind using second order sliding mode observer, is to reduce the shattering on the estimated states and therefore reducing the time cost. The important features of this combination are the convergence of the controlled states to the desired value in a finite time with less chattering as showing in the results under a variation of unknown load torque. We begin this by presenting the dynamical model for the machine in the α-β and then convert it into d-q frame of reference with the proposition of rotor flux aligned at the d-axis (Field Oriented Theory).

In section (2) A sliding model controller was designed and applied to the filed oriented control induction model. In section (3) describes the second order sliding mode observer based on super twisting algorithm. Finally, the proposed solution was applied to the different operating modes of the induction machine, the results were discussed through several simulations on MATLAB/Simulink and a conclusion of this work with future perspectives.

**II. Induction motor model**

The mathematical model of an IM in the stationary frame of reference (α-β) is described by the following equations [18]:

\[
\begin{align*}
\frac{d\varphi_a}{dt} &= -k_{i} i_a + \frac{k_{r}}{T_f} \varphi_b + k_{d} \varphi_d + k_{i} k_{d} i_d \\
\frac{d\varphi_b}{dt} &= -k_{i} i_b + \frac{k_{r}}{T_f} \varphi_a + k_{d} \varphi_d + k_{i} k_{d} i_d \\
\frac{d\varphi_a}{dt} &= k_{i} i_b - \frac{l}{T_f} \varphi_a + k_{d} \varphi_d \\
\frac{d\varphi_b}{dt} &= k_{i} i_a - \frac{l}{T_f} \varphi_b + k_{d} \varphi_d \\
\frac{d\varphi_d}{dt} &= \frac{-C_{r} + l}{T_f} (\varphi_b i_a - \varphi_a i_b) \\
\frac{d\omega}{dt} &= \frac{C_{r}}{J} (T_{e} - T_{r}) 
\end{align*}
\]
Where
\[ k_1 = \frac{R_s}{\sigma L_s}, \quad k_2 = \frac{M}{\sigma L_s}, \quad k_3 = \frac{M}{T_r} \]
\[ k_4 = \frac{1}{\sigma L_s}, \quad \sigma = 1 - M/(L_s L_r) \]
\[ i_{sw} = i_d, u_d, u_q, \quad \omega, \quad \phi_{sw}, \quad \phi_{sd}, \quad \phi_{sq} \]
are stator currents, stator voltages and rotor fluxes, respectively. \( \omega \) is the mechanic angular velocity. \( L_s, L_r, M \) and \( \sigma \) are stator, rotor and mutual inductance. \( R \), and \( R_r \) are stator and rotor resistance. \( \sigma \) is the leakage coefficient, \( T_r \) is rotor time constant.

In order to reduce the order of the IM model from five to four states, we have used Park’s transformation and applied the rotor flux orientation method [23]. In (1) is written in the \( d-q \) synchronous frame is:
\[
\begin{cases}
\dot{x} = f(x) + gu \\
y = h(x)
\end{cases}
\]

With
\[ \dot{x} = \begin{bmatrix} x_1, x_2, x_3, x_4 \end{bmatrix}^T = \begin{bmatrix} \frac{d i_d}{dt}, \frac{d i_q}{dt}, \frac{d \phi_{sd}}{dt}, \frac{d \phi_{sq}}{dt} \end{bmatrix}^T \]

\[ h(x) = \begin{bmatrix} x_3, x_4 \end{bmatrix}^T, \quad u = \begin{bmatrix} u_{sd}, u_{sq} \end{bmatrix}^T, \quad g = \begin{bmatrix} k_4, 0, 0, 0 \end{bmatrix}^T \]

\[ f(x) = \begin{bmatrix} -k_1 x_1 + x_2 x_3 + k_3 x_2^2 + \frac{k_2}{T_r} x_3, \\
-k_1 x_2 + x_3 x_4 - k_3 x_2 x_3 + \frac{k_2}{T_r} x_4 x_3, \\
k_3 x_1 - \frac{1}{T_r} x_3, \\
\frac{p^2 M}{J L_r} x_3 x_2 + \frac{p}{J} C_r \frac{f_r}{J} x_4 x_3 \end{bmatrix} \]

The output states which will be controlled are given by:
\[ y = h(x_3, x_4) \]
which are the rotor flux and the mechanical speed respectively.

III. Sliding mode control of IM

The sliding mode control (SMC) is a nonlinear control law characterized by its choice of sliding surface which forces the state trajectories to move to this sliding surface [20], despite the uncertain, the system is stable during the tracking of desired output along this surface [21]. The time varying sliding surface defines as
\[ S(x, t) = \begin{bmatrix} d \frac{d x_3}{dt} + \delta_3 \end{bmatrix}^{n-1} e_i \]
while, \( e_i \) is the tracking errors, \( n \) is the system order or the relative degree of the system and \( \delta_3 \) is the strictly positive constants.

Filippov’s [22] construction of the equivalent dynamics while in sliding mode can be written as
\[ dS(x, t)/dt = 0 \]
when the state variables reach the stability of the system by moving along of the surface, then it should be satisfied \( S(x, t) = 0 \) and the condition \( S(x, t) \dot{S}(x, t) < 0 \) define by the Lyapunov stability [23] function of the system (IM) that guaranty a sufficient and necessary condition of the requested stability.

The designed controller based on sliding mode (SM) is given by:
\[
\begin{cases}
\dot{u}_{sd} = u_e - u_a \\
\dot{u}_{sq} = u_e - u_q
\end{cases}
\]
Where: \( u_e \) and \( u_a \) are the equivalent and the nominal controllers respectively. The nominal controller is given by the following equation:
\[ \dot{S}(x, t) = -k_1 \text{sign}(S(x, t)) \]
where \( k_1 > 0 \) is a positive constant that ensures the convergence of the overall controller.

Applying the equation of the surface to the rotor flux \( (x_3) \) and the mechanical speed \( (x_4) \) of the IM model (2), the surfaces are written as following:
\[
\begin{cases}
S(x_3, t) = \frac{d}{dt} + \delta_3. e_{x_3} \\
S(x_4, t) = \frac{d}{dt} + \delta_4. e_{x_4}
\end{cases}
\]

where: \( e_{x_3} = \varphi - \varphi_{ref} \) and \( e_{x_4} = \omega - \omega_{ref} \)

Then, the dynamic of these surfaces defines as
\[
\begin{align*}
S(x_3, t) &= \delta_3 (x_3 - x_{3_{\text{ref}}}) + k_3 x_1 - \frac{1}{T_r} x_3 - x_{3_{\text{ref}}} \\
S(x_4, t) &= \delta_4 (x_4 - x_{4_{\text{ref}}}) + \frac{p^2 M}{J L_r} x_3 x_2 + \frac{p}{J} C_r \frac{d}{dt} x_4 x_3 - x_{4_{\text{ref}}}
\end{align*}
\]

From (4) and (5) the derivative of the sliding surface has the following form:
\[
\dot{S}(x_3, t) = \ddot{e}_{x_3} + \dot{\delta}_3 e_{x_3} = -k_3 \text{sign}(S(x_3, t))
\]
\[ \dot{S}(x_4, t) = \ddot{e}_{x_4} + \dot{\delta}_4 e_{x_4} = -k_4 \text{sign}(S(x_4, t)) \]

Where:
\[ \ddot{e}_{x_3} = k_3 x_1 - \frac{1}{T_r} x_3 - \dot{\phi}_{ref} \]
\[ \ddot{e}_{x_4} = \frac{p^2 M}{J L_r} x_3 x_2 + \frac{p}{J} C_r \frac{d}{dt} x_4 x_3 - \dot{\phi}_{ref} \]
After derivative of the surfaces of above equations and substituting the \((x_3, x_2, x_3, x_4)\) into it, and using (3) to find the sliding model voltage controllers, we can easily find the control voltage of the IM \((u_{sd}, u_{sq})\). So, to slide the system to the stability in finite time, we have to choses the right values of the positive constants \(\delta_3, \delta_4, k_3, k_4\).

Into these equations of the control calculated, there is the rotor flux and the load torque which are difficulties to measured. In this work we considering the load torque as uncertainly in our sliding mode controller and the rotor flux is not measured, they will be observed via the STASMO with finite time convergence [24,25].

IV. Super twisting sliding mode observer

The super twisting algorithm has basic form [23, 26] as following:
\[
\begin{cases}
\dot{\tilde{x}}_1 = f(\tilde{x}_2) + \lambda_1 |\tilde{x}_1 - \tilde{x}_3|^{\alpha_3} \cdot \text{sign}(\tilde{x}_1 - \tilde{x}_3) + \rho_1 \\
\dot{\tilde{x}}_2 = \lambda_2 \cdot \text{sign}(\tilde{x}_1 - \tilde{x}_3) + \rho_2
\end{cases}
\]
Based on super-twisting algorithm (7), The observer used to estimate the state variables and disturbance (unknown disturbance from the current measurements and velocity) has the following form:

\[
\begin{aligned}
\dot{x}_1 &= f_1(t, \dot{x}_1, \dot{x}_2, u) - \lambda_1 \left| x_1 - \hat{x}_1 \right|^3 \cdot \text{sign}(x_1 - \hat{x}_1) + \rho_1 \\
\dot{x}_2 &= f_2(t, \dot{x}_1, \dot{x}_2) - \lambda_2 \cdot \text{sign}(x_1 - \hat{x}_1) + \rho_2
\end{aligned}
\]

Where: \([x_1, x_2]^T \in \mathbb{R}^2\) is the state estimations vector, \(\lambda_1\) and \(\lambda_2\) are the observer gains, \(f_1\) and \(f_2\) are continuous nonlinear functions and are bounded. \(\rho_i; \; i=1,2\) is the external unknown disturbance or the perturbation due of the parameter’s variation.

**Assumption 1:** The external disturbance (uncertainties and/or parameters variation) is bounded and satisfies the slowly time varying condition.

**Assumption 2:** the \(f_1\) and \(f_2\) are known continuous nonlinear functions

**V. Rotor flux and load torque estimation**

Since the stator flux and load torque are not measured and should be measured using sensors to compensate them, and since flux and load torque sensors are more expensive than a super twisting observer is proposed in (7) which will be applied to the following flux and mechanical speed dynamics:

The following assumption is used to construct the flux observer

**Assumption 3:** suppose that the external disturbance \(\rho=0\) for the flux due the robustness of the proposed controller views the parameters variation, so \(f_1,2\) \((x, t)\) are continuous and bounded functions.

\[
\begin{aligned}
\frac{d\hat{x}_1}{dt} &= -k_1 \hat{x}_1 + k_2 \dot{x}_1 + k_3 \text{sign}(i_{sa} - \hat{i}_m) + k_4 u_{sa} + \\
\frac{d\hat{x}_2}{dt} &= k_1 \hat{x}_2 - \frac{1}{T_r} \dot{x}_2 - \omega \hat{x}_2 + \lambda_2 \cdot \text{sign}(i_{sa} - \hat{i}_m)
\end{aligned}
\]

\[
\begin{aligned}
\frac{d\hat{\phi}_r}{dt} &= -k_1 \hat{\phi}_r + k_2 \omega \hat{\phi}_r + k_3 u_{sq} + \\
\frac{d\hat{\phi}_s}{dt} &= k_1 \hat{\phi}_s - \frac{1}{T_r} \dot{\phi}_s + \omega \hat{\phi}_r + \lambda_4 \cdot \text{sign}(i_{sb} - \hat{i}_{sb})
\end{aligned}
\]

As the load torque proposed as disturbance to a system and based on the assumption below. The speed dynamic equation can be written according to currents measured and the flux estimated as:

Thus, the load torque which is supposed to be an unknown input (or disturbance) to the induction machine will be estimated using the following dynamical equations:

\[
\begin{aligned}
\frac{d\phi}{dt} &= \frac{p^2 M}{J_L} \left( \phi_{r_0} - \phi_{s_0} \right) - \frac{P}{f} C_r - \frac{L}{f} \omega \\
\frac{dC_r}{dt} &= 0
\end{aligned}
\]

**Assumption 4:** the dynamic of the load torque is varying too slowly to impact the electrical parameters; it can be assumed that \(C_r = 0\).

We define the noise on the measured velocity as \(k_6 (\omega - \dot{\omega})\), applying the STASM on the above equation and define the observation error function from the velocity \((\epsilon = \omega - \dot{\omega})\), then the designed observer of the disturbance gives as

\[
\begin{aligned}
\frac{d\hat{\omega}}{dt} &= \frac{p^2 M}{J_L} \left( \phi_{r_0} - \phi_{s_0} \right) - \frac{P}{f} C_r - \frac{L}{f} \omega + \\
\frac{d\hat{C}_r}{dt} &= -\lambda_5 \cdot \text{sign}(\epsilon) - k_6 (\omega - \dot{\omega})
\end{aligned}
\]

Where: \(\lambda_5, \lambda_6, k_6\) are positive constants values

**VI. Results discussion**

The validation of the proposed controller observer designed have been achieved using MATALB/Simulink. The using induction motor has Rated power 1.5kW, Rated voltage 220V, Rated speed 1428rpm, two poles (\(p=2\)) and Nominal frequency 50Hz. The electrical and mechanical parameters of this motor was calculated using offline identification methods in real benchmark [28], \(R_s=9.65\Omega\), \(R_r=4.304\Omega\), \(L_s=0.4718H\), \(L_r=0.4718H\), \(M=0.4475H\), \(J=0.0293kg/m2\), \(f_r=0.0038 Nm.s/rd\). The controller parameters values are: \(\delta_1=3000, \delta_2=4000, k_1=900, k_2=3470, \) and the flux observer parameters values are: \(\lambda_1=4000, \lambda_2=2, \lambda_3=4000, \lambda_4=2, \lambda_5=7000, \lambda_6=500, k_6=90\). All these parameters are chosen to achieve the desired performance of the proposed controller and observer.

The simulation results have been performed using the benchmarks of the speed and the load torque which describe the Motor operation in different modes (motor and generator modes) and motor performance at low speed as well.
Controller performance analysis:
To show the performance of the controller, the reference of the speed has been chosen to cover the entire speed range used by the IM (low speed, motor and generator). This controller is tested also under external disturbance variation. The Fig 1 shows the reference and the actual speed and the Fig 2 shows the tracking speed error, these results shown that the actual speed tracks the desired reference under external disturbance variation. There is a small peaks speed error of ±0.1 rad/s at the time 1.5s, 2.5s and 5s because at these times points the external disturbance was changed. However, off these times points, the error speed is eliminated, so perfect tracking response is obtained with presence of the external disturbance, so the controller is converging in finite time and the error is very small compared with the work presented in [4, 12]. The Fig 3 shows the reference and the actual rotor flux and the Fig 4 shows the tracking flux error, these results shown that the actual flux tracks the desired reference under external disturbance variation. The error is mush less, in the range of 0.002Wb, so the controller of the flux converging in finite time. The results show that the proposed controller is robust to external perturbation and with very fast convergence to desired reference. However, there is the chattering phenomenon in the results due the high frequency switching of a sliding mode.

The Fig 2 (b) shown the results of the reference and desired flux and the tracking error of its. The tracking error for both speed and flux shown that the system is converging in finite time and the error is very small compared with the work presented in [4,12].
Observer performance analysis:

In this part, we are going to discuss the results of the designed observer of the rotor flux and the load torque. The Fig 5 and 6 show the estimated value of rotor flux and the error respectively, in these figures, it has been observed that the error is very less and the chattering on the estimated value is very less due the super-twisting performance to reduce this chattering.

The Fig 7 shows the actual load torque apply on the IM and the estimated load torque. The estimated error of the load torque shows on the Fig 8, there is peaks of 7 Nm appear at the time 1.5s, 2.5s and 5s. This error is appearing due the change of the value of the load torque, after this time the observer converge in finite time to the actual value of the load torque with accepted error. Therefore, the controller designed is robust to this estimated error of the external disturbance and the tracking performance of the speed and the rotor flux don’t affect.

VII. Conclusion

In this paper a sliding mode controller is proposed to control the speed and rotor flux of the induction machine based on the rotor flux orientation, associated with a super twisting observer to estimate the flux and load torque (unknown disturbance). Simulation results shows a better performance of the proposed controller-observer in different machine operating modes (motor, generator modes) as well as in the presence of an external disturbance. the estimation accuracy of the rotor flux and load torque is provided as well reduced the chattering phenomenon on the machine operating modes (motor, generator modes) as well as in the presence of an external disturbance. The Fig 5 and 6 show the estimated value of rotor flux and load torque don't affect. The Fig 7 shows the actual load torque apply on the IM and the estimated load torque. The estimated error of the load torque shows on the Fig 8, there is peaks of 7 Nm appear at the time 1.5s, 2.5s and 5s. This error is appearing due the change of the value of the load torque, after this time the observer converge in finite time to the actual value of the load torque with accepted error. Therefore, the controller designed is robust to this estimated error of the external disturbance and the tracking performance of the speed and the rotor flux don’t affect.

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