Analysis of transient electromagnetic processes in the ultrahigh voltage transmission line during two-phase short circuits

Introduction

It is necessary to take into consideration the occurrence of emergency modes when designing electrical networks. It is important to do as they usually are accompanied by damage to the elements of electrical networks. The most dangerous and common emergency mode is a short circuit mode. After all, there are significant short-circuit currents in the elements of electrical networks. They cause thermal and electrodynamic action and are accompanied by a sharp voltage decrease in the electrical network.

Short-circuit currents can overheat conductive parts or even melt wires (temperatures can reach up to 20000 °K). So there comes a partial or complete termination of electricity supply to consumers. Also, this damage leads to damage caused by an electric arc that occurs at the point of short circuit and can affect adjacent objects. Short circuits decrease the voltage at the network nodes which affects the technological processes disruption and the stability of the power system as well.

Analysis of recent research

A mathematical models for the calculation of electromagnetic transient processes are included in [1-[5]. The model in three-phase lines has been developed in [6]. But the model can be used at two-phase short circuits as well.

Paper [7,8] presents a mathematical model of the transmission line, which is based on the methods of “lost elements” and “space state of the line”. The proposed method is a simple and practical procedure for modeling the three-phase transmission line directly in the time domain without explicitly using inverse transforms. Satisfactory verification of the obtained results with the performance of the EMTP-RV software suite has been presented.

In [9-11], a mathematical model of a power transmission line with distributed parameters was created, which is based on the equations of a long line of the first order with given boundary conditions of voltage and current along the edges of the line.

In [12], the application of first-order long-line equations for the study of transients processes in the ground wire is considered. The simulation is performed in the frequency domain with the subsequent transition to time.

The article [13] presents a mathematical model of a perfectly transposed three-phase power line. The model allows you to calculate phase currents and voltages along the line as a function of time coordinate. An analysis of the results of a computer simulation that turns on the power line in non-working stroke mode is presented in the paper.

A model of three-phase power lines with distributed parameters has been created in [14] using the software package PSCAD. This model can be used to simulate transient electromagnetic processes during switching, short circuits and other modes of line operation.

In [15] the study of overvoltages in the line of ultrahigh voltage was carried out. The power line here is represented by a series connection of alternate circuit circuits. The computer simulation was performed in the ATP software package.

In [16], with the help of the PSCAD software package, surge simulations in the 500 kV line during a lightning strike were simulated. The coronation phenomenon of wires was neglected during the simulation, i.e. the running phase and interphase active conductivities were not taken into account.

For the analysis of transient processes in a three-phase power line with arbitrary voltage and current distributions in the line, it is proposed to use the numerical inverse Laplace transform algorithm in [17]. Here the Laplace transform methodology is also used to find voltages and currents along the line edges. Phase voltages and currents are obtained as rational functions of their frequency. It is shown how the numerical inverse Laplace transform can be applied to obtain the distribution of electromagnetic waves in a power line.

The transient electromagnetic processes in symmetric and asymmetric short circuits in different places of the high-voltage power line connected to the generating unit busbars. They are analysed in [18]. The equations of voltages and currents of the transmission line and substation buses are written in phase coordinates. It gives a possibility to model easily different asymmetric states. Here, a model for the transient electromagnetic processes analysis has been developed in the MatLab/Simulink software package.

Analysis of the available literature has shown that most studies of transient electromagnetic processes in power
We propose to use a modified Hamilton–Ostrogradsky principle (variational approach) for the analysis of transient electromagnetic processes in the elements of electrical networks [23]. This approach avoids the decomposition of a single dynamic system. The initial equations of the electromagnetic state of the object under study can be obtained solely on the basis of a single energy approach by constructing an extended Lagrange function [23]. This approach is especially relevant for systems with distributed parameters, in particular for long power lines.

The theory of transient electromagnetic processes analysis in long power lines based on variational approaches in a single-line version was developed in [23] and further developed in [24]. To reproduce fully these processes in long power lines, which often operate in asymmetric modes, they are necessary to be modelled in multiphase execution. Therefore, we will build a mathematical model of the line in three-phase execution.

Fig. 1 presents the calculation scheme of the electrical network fragment we study. The key element is a long power line. It is presented in three-phase design as a line with distributed parameters (here are shown only the first and last discrete nodes of the line). A voltage is applied to the beginning of the line. At the end an equivalent three-phase active-inductive load is connected to it.

There are elements both with the concentrated and with the distributed parameters in the fragment of an electric network we investigated.

Fig. 1. Calculation scheme of the studied fragment of the electrical network (for the first and last discrete section of the line)

**Hamilton-Ostrogradsky principle**

Therefore, the Hamilton-Ostrogradsky action functional looks like this [23]:

\[
S = \int L^* \, dt + \int \left( L_1 \right) dlt, \quad I = \int L_1 \, dt, \tag{1}
\]

where \( S \) – action according to Hamilton–Ostrogradsky; \( I \) – energy functional; \( L^* \) – extended Lagrange function, \( L_1 \) – linear density of the modified Lagrange function [23]:

\[
L^* = \tilde{T}^* - P^* + \Phi^* - D^*, \quad L_1 = \tilde{T}_1 - P_1 + \Phi_1 - D_1, \tag{2}
\]

where \( \tilde{T}^* \) – kinetic coenergy, \( P^* \) – potential energy, \( \Phi^* \) – energy dissipation, \( D^* \) – energy of outside nonpotential forces, with index \( l \) the corresponding linear densities of energies.

It is possible to get acquainted with a technique of receiving similar equations of an electromagnetic state. For example, in our works [25-27]. Therefore, in order to reduce the volume of material, we propose ready-made equations for the studied fragment of the electrical network (Fig. 1).

**Mathematical model of a fragment of the electric network**

We present the final equations of the electromagnetic state of the studied fragment of the electrical network in matrix-vector form:

\[
\frac{\partial^2 \mathbf{u}}{\partial t^2} = \left( L_0 C_0 \right)^{-1} \left( \frac{\partial^2 \mathbf{u}}{\partial x^2} - \left( L_0 G_0 + R_0 C_0 \right) \frac{\partial \mathbf{u}}{\partial t} - R_0 G_0 \mathbf{u}, \right) \tag{3}
\]
In (3) and (4) \( L_0, R_0, C_0, G_0 \) – inductance, resistance, capacitance and conductivity matrix.

The problem of solving (3) is to determine the boundary conditions. In our case we know the voltage at the beginning of the line \( u_i = u \big|_{x=0} \), but not at the end of it. Therefore, it is necessary to find only the boundary condition at the end of the line. Note that the line is loaded with an equivalent three-phase active-inductive load.

In [24] symmetrical modes were considered for homogeneous long power lines. That is why the line was modelled in a single-phase (single-line) design. We propose homogeneous long power lines. That is why the line was switched on for an equivalent three-phase active-inductive load.

Therefore, it is necessary to find only the boundary conditions. Let’s write the mentioned equation in matrix-vector form:

\[
\frac{\partial^2 u}{\partial x^2} = R_0 \frac{\partial}{\partial t} u + L_0 \frac{\partial}{\partial t} i,
\]

By discretizing (1) by the method of lines, using the notion of the central derivative, we obtain:

\[
\frac{dN_j}{dt} = (L_0 C_0)^{-1} \left( \frac{1}{\Delta x^2} (u_{j-1} - 2u_j + u_{j+1}) - \right.
\]

\[-(L_0 G_0 + R_0 C_0) v_j - R_0 G_0 u_j \big) , \quad \frac{du_j}{dt} = v_j \]

\[
\frac{dI_i}{dt} = L_0^{-1} \frac{1}{2\Delta x} (u_{j-1} - u_{j+1}) - L_0^{-1} R_0 i_j ,
\]

\[j = 2, \ldots, N.\]

Write (6) and (7) for the last discrete node of the line \( j = N \):

\[
\frac{dN_N}{dt} = (L_0 C_0)^{-1} \left( \frac{1}{\Delta x^2} (u_{N-1} - 2u_N + u_{N+1}) - \right.
\]

\[-(L_0 G_0 + R_0 C_0) v_N - R_0 G_0 u_N \big) , \quad \frac{du_N}{dt} = v_N
\]

\[
\frac{dI_N}{dt} = L_0^{-1} \frac{1}{2\Delta x} (u_{N-1} - u_{N+1}) - L_0^{-1} R_0 i_N ,
\]

Analysing (8) and (9), we see that they have an unknown voltage in the fictitious node \( u_{N+1} \). This makes it impossible to find the voltage at the last discrete node of the line \( u_i \). Also we cannot find the current in the last discrete branch of the line \( i_i \). The voltage \( u_{N+1} \) does not exist in nature and it has no physical meaning. It is a purely fictitious-mathematical quantity.

The value of the current in the first discrete branch of the line or, if necessary, in all discrete line branches, can be calculated by discretizing (5) by the lines method, but now using the concept of the right derivative:

\[
\frac{dI_j}{dt} = L_0^{-1} \frac{1}{\Delta x} (u_j - u_{j+1}) - L_0^{-1} R_0 i_j , \quad j = 1, \ldots, N,
\]

Computer simulation results

We wrote program code in the algorithmic programming language Visual Fortran to perform a computer simulation, based on the developed mathematical model. The code allows you to reproduce transient electromagnetic processes in the studied fragment of the electrical network (Fig. 1).

Description of the order and parameters of the simulation

The simulation was performed as follows in the first experiment. The power line was switched on at time \( t = 0 \) s with asymmetric equivalent three-phase active-inductive load on the normal mode of operation. A two-phase short circuit to ground was simulated after entering the steady state, at time \( t = 0.11 \) s, at the end of the power line (phases \( A \) and \( B \), see Fig. 1).

The power line in the second experiment was switched on for a two-phase short circuit at the end of the power line at time \( t = 0 \) s (as in the previous experiment, the short circuit to ground was simulated in phases \( A \) and \( B \)).

Experiment

Fig. 2 show the spatial distributions of phase voltages at time \( t = 0.001 \) s and phase currents at time \( t = 0.007 \) s, respectively. These figures reflect very well the course of wave electromagnetic processes in the power line, so let’s analyse them.

We can see from Fig. 2 that during the start of the transmission line to normal operation at time \( t = 0.001 \) s, the voltage of phase \( A \) at the beginning of the line has a value of 200 kV. At a distance of 350 km from the beginning of the line, the voltage is still zero.

Fig. 2. Spatial distributions of phase voltages in the line at time \( t = 0.001 \) s.

Figure 3 show the temporal-space voltage and current distributions of phase \( B \). These figures are made in 3D format, they combine both time and space distributions of voltage and current in a line.

Fig. 3. Temporal-spatial voltage distribution of phase \( B \) in line at the time \( t \in (0, 0.03) \) s.

The analyse of the mentioned figures show that when the line is switched on for an equivalent asymmetric active-inductive load. The phase \( B \) voltage has the largest fluctuations at the end of the power line (Fig. 3).

It is advisable to analyze Fig. 3 together with Fig. 2.
Conclusion

The application of Neumann and Robin–Poincare boundary conditions to identify boundary conditions to the differential equation of a long line of the second order makes it possible to effectively solve problems related to the analysis of transient electromagnetic processes in ultrahigh voltage lines, where they must be considered as distributed parameters.

Introduction to the mathematical model of the line, which is based on a discretized equation of a long line. To solve it the boundary conditions of the second and third genera are applied, the parameter “output voltage” (voltage at the end of line) allows making the line model more autonomous and universal on the one hand. On the other – opens wider possibilities for reproduction of emergency states of operation of the line.

Presented in 3D format temporal-spatial distributions of voltages and currents maximally illuminate information about wave processes in the line. They also confirm the physical principles of electrodynamics regarding the flow of wave electromagnetic processes in long power lines. They indicate the high adequacy of the developed mathematical model.

The materials of this work will be used in further research on the joint operation of turbogenerators, unit transformers, switching facilities and ultra-high voltage long transmission lines.

**LITERATURE**


[17] Jung-Chien Li, Transient analysis of three-phase transmission lines with initial voltage and current distributions, Electric power systems research, 35 (1995), No. 3, 177-186


[27] Chaban A., Lis M., Szafrańiec A., Levo niuk V., Mathematical Modelling of Transient Processes in a Three Phase Electric Power System for a Single Phase Short-Circuit, Energies (2022), No. 15, 1126