Introduction

Vortex layer electromagnetic apparatuses with ferromagnetic working elements are intended for intensification of various physical and chemical processes. The apparatuses are hermetic, have no dynamic seals and consist of an electromagnetic device with cooling system, a process chamber and a control panel [1].

Physical and chemical processes in vortex layer electromagnetic apparatuses are intensified due to intensive mixing and dispersion of components being processed, acoustic and electromagnetic processing, high local pressure, etc. [2].

Numerous factors existing in the vortex layer and able to directly affect the raw materials being processed require the developers of devices and technological processes to know the essence of the phenomena and patterns taking place during processing of multicomponent systems [1–2].

Studies conducted in recent years [1-6] show that the apparatuses can be effectively used for: obtaining multicomponent suspensions and emulsions; extraction of protein substances from yeast cells; increasing the microbiological stability of food products and yeast activation in bakery production; improving the quality of semi-finished products and finished meat and fish products; intensification of extraction processes, particularly in cooking of broths, production of berry drinks and pectin; intensification of liquid manure and industrial wastewater treatment processes (disinfection, homogenization, mixing) in the course of their composting, etc.

It follows from analysis of literary sources [7-10] that vortex layer apparatuses are equipped with different types of working bodies and have various working area layouts. In the course of liquid-phase processes, using ferromagnetic elements as working bodies, trap screens or labyrinths designed to keep ferromagnetic particles in the working area may be installed on the ends of non-magnetic insert (or at the outlet only). Grinding and mixing may take place not only with the help of ferromagnetic particles, but also with the help of knives, tubes or rotors. In these cases, sieves act as filters (separators). Ferromagnetic elements are added to the working area using the electromagnetic dispenser.

Analyzing the results of previous research [1-10], one can state that efficiency of the vortex layer apparatus is proposed to be increased due to design improvement and justification of structural and technological parameters of individual systems, without regard to interconnection between them. It should also be noted that most of the results obtained from substantiation of structural and technological parameters are experimental in their nature, having no theoretical basis.

Investigation of the process of electromagnetic field change in the working chamber of the vortex layer apparatus

The first stage of theoretical research lies in determination of the basic parameters of rotating electromagnetic field in the working chamber of the apparatus, which is generated by the inductor coils being alternately switched on/off at respective time intervals: electromagnetic field intensity (H, A/m); magnetic flux density, magnetic induction (V, T) and electromagnetic field’s vector potential (A, T/m).

Given that electromagnetic field rotates, we will determine functional dependences for H, B, and A in the working chamber of the vortex layer apparatus using STAR-CCM+ software package [11–15]. To do this, we will generate the 3D model of the vortex layer apparatus’s main regions, where electromagnetic fields act and originate (fig. 1).

The multifaceted cell and surface grid generator was chosen as the grid model. The basic size of the cell was 0.01 m. General appearance of the resulting grid with geometric dimensions and selected areas is shown in fig. 2.

Separate physical models of the continuum, reference values and starting conditions were selected for each area (fig. 2).

For the inductor coil area, the following was chosen: a three-dimensional model of continuous medium with a non-stationary implicit pattern; an electromagnetism model with finite-volume magnetic vector potential and eddy current suppression [16] and excitation coil model. The coil material is copper with the following properties: magnetic permeability - 1.25663·10⁻⁶ Hn/m; electrical conductivity – 5, 96·10⁷ cm/m. For the coil core region, the following was chosen: a three-dimensional model of continuous medium with non-stationary implicit pattern; an electromagnetism model with finite-volume magnetic vector potential and eddy current suppression. The coil material is steel with the following properties: magnetic permeability – 1.25663·10⁻⁶ Hn/m; electrical conductivity – 3.78·10⁷ cm/m.

Simulating the Process of Operation of Vortex Layer Electromagnetic Apparatus with Ferromagnetic Working Elements

Abstract. The article presents the results of the simulation of the work process of the electromagnetic apparatus of the vortex layer with ferromagnetic working elements on the example of the processing of liquid pig manure. As a result of analytical studies of the process of interaction of cylindrical ferromagnetic elements with a magnetic field in the working chamber of the vortex layer apparatus, the appropriate physical and mathematical apparatus was improved, which is the basis of numerical modeling in the STAR-CCM+ software package.

Keywords: simulation, electromagnetic field, vortex layer apparatus, ferromagnet, processing, liquid manure.

Słowa kluczowe: symulacja, pole elektromagnetyczne, aparat warstwy wirowej, ferromagnes, obróbka, gnojowica.

Streszczenie. W artykule przedstawiono wyniki symulacji procesu pracy aparatury elektromagnetycznej warstwy wirowej z ferromagnetycznymi elementami roboczymi na przykładzie procesu przetwarzania gnojowicy świńskiej. W wyniku badań analitycznych procesu oddziaływania cylindrycznych elementów ferromagnetycznych z polem magnetycznym w komorze robożej aparatury warstwy wirowej utworzono odpowiednią aparaturę fizyko-matematyczną, która jest podstawą modelowania numerycznego w programie STAR-CCM+ pakiet oprogramowania. (Symulacja procesu działania aparatury elektromagnetycznej warstwy wirowej z ferromagnetycznymi elementami roboczymi)

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For the inductor body area, the following was chosen: a three-dimensional model of continuous liquid of constant density with non-stationary implicit pattern; fluid movement is separate and turbulent, which is subject to k-ε turbulence model and Reynolds averaging of Navier-Stokes equation; an electromagnetism model with finite-volume magnetic vector potential and eddy current suppression. The region's environment is transformer oil with the following properties:
density – 895 kg/m³; dynamic viscosity – 198.2 Pa·s;
magnetic permeability – 1.25663·10⁻⁶ Hn/m and electrical conductivity – 1.0·10⁻¹⁰ S/m.

For the working chamber area, the following was chosen: a three-dimensional model of continuous multiphase medium of constant density with non-stationary implicit pattern; the medium's movement is separate and turbulent, which is subject to k-ε turbulence model and Reynolds averaging of Navier-Stokes equation; an electromagnetism model with finite-volume magnetic vector potential and eddy current suppression. The region's environment is liquid manure with the following properties:
density – 997.561 kg/m³; dynamic viscosity – 8.8871·10⁻⁴ Pa·s; magnetic permeability – 1.25663·10⁻⁶ Hn/m; electrical conductivity – 5.5·10⁻⁶ cm/m.

The factors of numerical modeling were chosen as follows: the coil's maximum magnetomotive force ξ max (6000–36000 A·turns; electromagnetic field's rotation frequency n (1000–5000 rpm).

Let us plot the electromagnetic field rotation in STAR-CCM+ software package by changing each coil's magnetomotive force value (numbering the coils clockwise from 1 to 6) to 0 or ξ max according to the algorithm

**(1)**

\[
if \left( \sin \left( \frac{n \cdot \pi \cdot \pi}{60} - \frac{(N-1)\pi}{3} \right) > \sin \frac{\pi}{3} \right) \{ \xi_N = \xi_{max} \} else \{ \xi_N = 0 \}
\]

where **( )** is the condition operator; n is the electromagnetic field rotation frequency, rpm; ξ max is the coil's maximum magnetomotive force, A·turns; t – time, s; N is the clockwise coil number.

When simulating, we assume that the change in the coil's magnetomotive force occurs instantaneously. This causes the electromagnetic field's instantaneous rotation by the angle of 60°.

As a result of numerical modeling, the visualization of electromagnetic field intensity vector **H**, magnetic induction **B** and electromagnetic field vector potential d **A** in the vortex layer apparatus was obtained for the accepted value of the magnetomotive force of only two opposite coils ξ max = 5000 A·turns, which is shown in fig. 3.

For further calculations, distribution of electromagnetic indicators in the working chamber of the vortex layer apparatus is presented in the form of respective tensors stored in the form of a text file with a set of numerical values. The regression equation of electromagnetic parameters in the working chamber of the vortex layer apparatus depending on the coordinates cannot be constructed due to the relationship's complex nature. Therefore, respective regularities are presented in the form of graphs for each plane of the space section: XOY, XOZ, YOZ in fig. 4.

To evaluate the impact of research factors (the coil's maximum magnetomotive force ξ max, A·turns; electromagnetic field rotation frequency n, rpm) we obtain the regularity of change in minimum and maximum values of electromagnetic indicators in the working chamber of the vortex layer apparatus.

**Hence, we obtained the following dependencies of:**

- electromagnetic field strengths:

**(2)**

\[
H_{max} = 88.5855 - 0.0688326 n + 0.00001231 n^2 + 89.0451 \xi_{max} - 0.001832 n \xi_{max} - 0.003642 \xi_{max}^2,
\]

where if ( ) else ( ) is the “if” operator; n is the electromagnetic field rotation frequency, rpm; ξ max is the coil's maximum magnetomotive force, A·turns; t – time, s; N is the clockwise coil number.

Fig. 3. Distribution of electromagnetic indicators in the vortex layer apparatus for accepted value of the magnetomotive force of only two opposite coils ξ max = 5000 A·turns

Fig. 4. Dependencies between electromagnetic indicators in the working chamber of the vortex layer apparatus and coordinates for accepted value of the magnetomotive force of only two opposite coils ξ max = 5000 A·turns
where $D_0$ is the diameter of the base of the ferromagnetic element, $m$; $L_0$ is the length of the ferromagnetic element, $m$.

Under the torque action, the cylindrical ferromagnetic element rotates with angular velocity $\omega_{\text{abs}}$, which equals to the sum of relative $\omega_{\text{rel}}$ and transfer $\omega$ velocities:

$$\omega_{\text{abs}} = \omega_{\text{rel}} + \omega.$$  

Transfer velocity equals to electromagnetic field rotation velocity

$$\omega = 2\pi \frac{n}{D_0}.$$  

Let us determine the ferromagnetic element’s relative velocity. To do this, we write down respective differential equation of the ferromagnetic element’s motion in relation to the stationary vector of electromagnetic field intensity $H$:

$$I_n \phi = \vec{MH} \sin \varphi,$$

where $I_n$ is the inertia moment of the cylindrical ferromagnetic element:

$$I_n = \frac{m L_0^2}{12} \frac{\pi}{4} D_0^2 L_0 \rho_n \frac{L_n}{12} = \frac{\pi}{48} \rho_n D_0^2 L_0^3.$$

Considering (6), (7) and (11), equation (10) takes the following form

$$\phi = \frac{12 I_n H}{\rho_n L_0} \sin \varphi.$$  

Starting conditions of equation (12):

$$\phi_{|t=0} = \varphi_0,$$

$$\phi_{|t=0} = 0.$$  

Differential equation (12) together with (13) was solved by Lyapunov’s approximate method through Jacobi function in [1–2]:

$$\phi = \omega_{\text{rel}} = -\left( \varphi_0 + \frac{1}{192} \varphi_0^3 \right) \frac{3.464 H X}{L_0 Y} \times$$

$$\times \frac{3.464 H X}{L_0 Y} \left( \frac{3}{L_0 Y} - \frac{10.39 H X}{L_0 Y} \right).$$  

Using the demagnetization factor expression in [1–2], the relative speed of a ferromagnetic element is:

$$X = \sqrt{1 + \left( \mu - 1 \right) \frac{\lambda}{\lambda^2 - 1}} \ln \left( \lambda + \sqrt{\lambda^2 - 1} \right) \rho_0,$$

$$\mu = \frac{\varphi_0^2}{16}, \quad \lambda = \frac{L_0}{D_0},$$

where $\mu$ is magnetic permeability.

In the working chamber of the vortex layer apparatus, the following forces act on each cylindrical ferromagnet.

1. Displacement force of the cylindrical ferromagnet under the action of rotating electromagnetic field [2]:

$$F_n = x \vec{A} \vec{H} \vec{F}(\nabla \cdot \vec{H}) = \frac{\pi}{4} D_0^2 L_0 \lambda \vec{A} \vec{H} \vec{F}(\nabla \cdot \vec{H})$$

where $X$ is the magnetic susceptibility of the cylindrical ferromagnet material.

2. Gravity forces [20]:

$$F_g = \frac{\pi}{4} D_0^2 L_0 \rho_0 \vec{g}.$$
where \( g \) is the free fall acceleration, m/s².

3. Archimedes’ force:

\[
F_A = \frac{\pi}{2} D_A^2 L_A \rho g \, R \tag{18}
\]

where \( \rho \) is the liquid density, kg/m³.

4. The force caused by the pressure change in the flow direction due to its acceleration [21]:

\[
F_{ac} = \frac{\pi}{4} D_A^2 \rho \frac{dV}{dt} \tag{19}
\]

where \( dV \) is liquid velocity vector, m/s.

5. The viscous resistance force that occurs when the cylindrical ferromagnet moves with particular relative velocity in the fluid flow:

\[
F_D = \frac{1}{8} \left( \frac{\pi}{2} D_B^2 + 2\pi D_B L_B \right) \rho f_{st} (Re) \left( |V_B| - |V_A| \right), \tag{20}
\]

where \( f_{st} (Re) \) is the viscous resistance factor.

6. The total force of ferromagnets’ contact interaction between themselves and the working chamber wall, which is based on Hertz-Mindlin spring-damper contact model [22]:

\[
\begin{aligned}
F_{cont} & = \frac{F_{cont}}{S_{pa} + S_{pb}} \left( \begin{array}{c}
- K_n (V_B - V_A) - N_t (\tau_B - \tau_A) \\
S_{pa} \end{array} \right), \tag{21}
\end{aligned}
\]

where \( K_n = 4E_{eq} d_A R_{eq}/3 \) is the elastic component’s normal stiffness factor, kg/s²; \( N_t = (5K_p M_{eq})N_{damp} - \) the damping component’s normal damping ratio, kg/s; \( K_t = 8E_{eq} \sqrt{d_i R_{eq}} \) – elastic component’s tangential stiffness ratio, kg/s²; \( N_t = \sqrt{5K_p M_{eq} N_{damp}} - \) the damping component’s tangential damping ratio, kg/s; \( \tau_B \) and \( \tau_A \) are normal and tangential components of the ferromagnet surface’s relative velocity at the point of contact, m/s; \( N_{damp} \) is the attenuation coefficient; \( R_{eq} = (2/D_A + 2/D_B)^{-1} \) is the equivalent radius of two ferromagnets A and B, m; \( E_{eq} = (1 - \nu_A^2)/E_A + (1 - \nu_B^2)/E_B \) – is the equivalent Young’s modulus of two ferromagnets A and B, Pa; \( M_{eq} = (M_A^{-1} + M_B^{-1})^{-1} \) is the equivalent mass of two ferromagnets A and B, kg; \( g_{st} = (2(2 - \nu_A)(1 + \nu_A)E_A + 2(2 - \nu_B)(1 + \nu_B)/E_B)^{-1} \) is the equivalent shear modulus of two ferromagnets A and B, Pa; \( d_i \) is a virtual overlap of ferromagnets A and B in normal and tangential directions, m; \( M_A \) and \( M_B \) are the masses of seeds A and B (for surface \( M_{wall} = \infty \)), kg; \( D_A \) and \( D_B \) are effective diameters of ferromagnets A and B (for surface \( D_{wall} = \infty \)), m; \( E_A \) and \( E_B \) are Young’s moduli of ferromagnets A and B, Pa; \( \nu_A \) and \( \nu_B \) are Poisson ratios of ferromagnets A and B.

Based on the formulas presented above, we obtain the equations of cylindrical ferromagnets movement in a fluid flow under the action of rotating electromagnetic field:

\[
\begin{aligned}
\frac{dV}{dt} & = F_M + F_B + F_A + F_{ac} + F_D + F_{cont}, \tag{22}
\end{aligned}
\]

\[
\frac{dS}{dt} = \tau_M + \frac{d}{dt} + \tau_A + \tau_B. \tag{23}
\]

Substituting (16)–(21) into (22), we finally obtain:

Joint solution of differential equations’ systems (15) and (23) in general is impossible by analytical methods. Investigations [23] proposed the solution of similar systems using the finite element method, which is implemented in STAR-CCM+.

The following physical models are adopted for simulation of ferromagnets in STAR-CCM+: Lagrangian multiphase, discrete element method, existence time, two-way communication, constant density, resistance force and mixed particles. Multiphase interaction between ferromagnets and the wall obey Hertz-Mindlin model.

Ferromagnets’ parameters are as follows: Poisson’s ratio – 0.3; Young’s modulus – 2.0·10¹¹ Pa; density – 7800 kg/m³ base diameter – 2 mm. For ferromagnet-ferromagnet interaction: rest friction ratio – 0.6; normal recovery factor – 0.5; tangent recovery ratio – 0.5. For ferromagnet-wall interaction: rest friction ratio – 0.7; normal recovery factor – 0.5; tangent recovery ratio – 0.5; rolling resistance ratio being 0.001.

The simulation was carried out using a non-stationary implicit pattern with a step of 0.001s per 60 s. Visualization of the result of ferromagnets’ movement simulation under the action of rotating electromagnetic field is shown in fig. 5–7. Fig. 5 shows the distribution of ferromagnets V4 ° and liquid Vl velocities in the working chamber of the vortex layer apparatus. Fig. 6–7 correspond to visualization of the contact interaction force of \( F_{cont} \). Maximum force of ferromagnets’ contact interaction \( F_{cont} \) and the frequency of ferromagnets’ impacts \( \kappa \), per time unit \( \kappa \).

Substituting (16)–(21) into (225), we finally obtain:

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The criteria for process evaluation were chosen as follows: the average speed of ferromagnet movement \( V \), the maximum force of ferromagnets’ contact interaction \( F_{cont} \) and the frequency of ferromagnets’ impacts \( \kappa \), per time unit \( \kappa \).

where \( \mathcal{g} \) is the free fall acceleration, m/s².

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Based on the obtained data and using the Wolfram Cloud, the second-order regression equation was obtained for:

- the average value of the ferromagnets' velocity $V_Ω$:
  $$V_Ω = 2.4802 + 0.000937 H – 93.7 L + 0.001848 n + 2.676 \times 10^{-7} H n – 0.02126 N – 3.152 \times 10^{-6} H N – 6.172 \times 10^{-6} n N + 0.00006602 N^2;$$

- the maximum force of ferromagnets' contact interaction $F_{cont}$:
  $$F_{cont} = -659.787 + 0.0257 H – 1.404 \times 10^{-6} H^2 + 27479.2 L + 0.247917 HL – 431852.0 L^2 + 0.0745 n – 9.949 \times 10^{-6} n^2 + 2.271 N – 0.00378N^2;$$

- the frequency of ferromagnets' impacts per time unit $κ$:
  $$κ = -459.9 – 0.01195 H + 30142.1 L + 0.561 HL – 51092 L^2 + 0.0392 n + 3.44 \times 10^{-6} n^2 + 8.43 \times 10^{-8} n^2 + 0.0000445 HN + 0.000028 n N – 0.000045 N^2;$$

To ensure the highest efficiency of the process of mixture components' grinding, it is required that in the vortex layer apparatus's working chamber, the ferromagnets' average velocity, their contact interaction force and the frequency of ferromagnets' impacts should be maximum:

$$\begin{align*}
&\left\{F_{cont}(H, n, N, L_M) \rightarrow \max, \\
&V_Ω(H, n, N, L_M) \rightarrow \max, \\
&κ(H, n, N, L_M) \rightarrow \max\right\}.
\end{align*}$$

By solving compromise problem (27) together with (24), (25) and (26) in Wolfram Cloud, we obtain rational values of design-operating parameters of the vortex layer apparatus:

- $H = 10000$ A/m, $n = 3825$ rpm, $N = 298$ pcs., $L_M = 0.035$ mm, $κ = 376$ imp./s, $F_{cont} = 457$ N and $V_M = 9.0$ m/s.

**Numerical modeling of the process of components’ mixing in the working chamber of the vortex layer apparatus**

At the third stage of theoretical research, one should investigate the process of components’ mixing in the working chamber of the vortex layer apparatus. The calculation diagram with no regard to starting of rotating electromagnetic field is shown in fig. 9. In the diagram’s upper part of the working chamber, two liquid flows are supplied: a mixture (by the example of pigs’ liquid manure) and additional reagents. The initial velocity of the two components is different: $V_M0$ for the mixture and $V_R0$ for additional reagents.

The components’ physiomechanical properties were assumed as follows. For pigs' liquid manure: density = 1050 kg/m³, dynamic viscosity = 0.80 Pa·s. For additional reagent: density = 1000 kg/m³, dynamic viscosity = 8.8·10⁻⁴ Pa·s. The components’ interaction among themselves obeyed VOF-VOF model. For theoretical research, it was assumed that the holes for reagent supply are concentric.

In the working chamber, there are ferromagnets to the total number of $N=298$ pcs. The size of ferromagnets is assumed in accordance with section 2.3: $L_M = 0.035$ mm, $D_M = 0.002$ mm. The interaction between the components and ferromagnets obeyed VOF-Lagrangian phase model.

As is seen from 9–10, no mixing of component flows practically occurs.

When the rotating electromagnetic field is started, the visualization of the mixing process is obtained, which is shown in fig. 11-13.

It follows from the analysis of fig. 9–11 that, due to the ferromagnets' movement, the components' movement in the working chamber occurs. This causes their redistribution in the cross section, as shown in fig. 9. As the criterion for evaluation of the components' mixing process, we assume the homogeneity coefficient, which was introduced in [12]:

$$\begin{align*}
&\{F_{cont}(H, n, N, L_M) \rightarrow \max, \\
&V_Ω(H, n, N, L_M) \rightarrow \max, \\
&κ(H, n, N, L_M) \rightarrow \max\}.
\end{align*}$$

By solving compromise problem (27) together with (24), (25) and (26) in Wolfram Cloud, we obtain rational values of design-operating parameters of the vortex layer apparatus:

- $H = 10000$ A/m, $n = 3825$ rpm, $N = 298$ pcs., $L_M = 0.035$ mm, $κ = 376$ imp./s, $F_{cont} = 457$ N and $V_M = 9.0$ m/s.
where $\alpha_{Ri}$ is the volume fraction of additional components per volume unit on cross section $S_{out}$; $N_{i}$ is the number of unit volumes on cross section $S_{out}$; $\alpha_{R0}$ is the required proportion of additional components per volume unit on cross section $S_{out}$.

$$\theta = 1 - \frac{1 - \alpha_{R0}}{N_{i} - 1} \sum_{i=1}^{N_{i}} (\alpha_{Ri} - \alpha_{R0})^2$$

Fig. 10. Cross-sections of the working chamber of the vortex layer apparatus without regard to starting of the rotating electromagnetic field

Fig. 11. Cross-sections of the working chamber of the vortex layer apparatus in the rotating electromagnetic field

Fig. 12. Longitudinal sections of the working chamber of the vortex layer apparatus in a rotating electromagnetic field

Fig. 13. Visualization of component flows in the working chamber of the vortex layer apparatus in a rotating electromagnetic field

The required proportion of additional components per volume unit is determined using the following formula:

$$\alpha_{R0} = \frac{V_{R0}S_{R0}}{V_{M0}S_{M0} + V_{R0}S_{R0}}$$

where $V_{R0}$ is the initial flow rate of liquid manure on cross section $S_{in}$, m/s; $V_{M0}$ – the initial speed of the additional component supply on cross section $S_{in}$, m/s; $S_{R0}$ – the area of the opening, through which liquid manure is supplied on cross section $S_{in}$, m²; $S_{M0}$ is the area of the opening, through which the additional component is supplied on cross section $S_{in}$, m².

In order to evaluate the movement of liquid and ferromagnets in the working chamber of the vortex layer apparatus in rotating electromagnetic field, respective velocity distributions were also constructed (fig. 14–15).

14–15 show the liquid’s rotating movement caused by ferromagnets’ movement. According to liquid’s and ferromagnets’ velocity distribution on cross-sections of the working chamber (fig. 12), one can assert the increase in liquid velocity on cross-section $S_{out}$ to the value of 1.5 m/s.

The factors of numerical modeling of the process of liquid manure components’ mixing in the working chamber of the vortex layer apparatus were assumed as follows: initial rates of liquid manure supply $V_{R0}$ (1–5 m/s) and additional component $V_{M0}$ (1–5 m/s) on cross section $S_{in}$.

Areas of the openings for liquid manure and additional component supply are as follows: $S_{M0} = \pi \cdot 0.005^2 = 0.0000785$ m²; $S_{R0} = \pi \cdot 0.045^2 - S_{M0} = 0.0055735$ m².

To quantify the process under research, the volumetric productivity of mixture Q supplied through the working chamber of the vortex layer apparatus, was calculated.

As a result of numerical simulation in STAR-CCM+ according to the complete research plan (5² = 25 numerical experiments), the values of specified criteria were obtained.

Based on the data obtained and using the Wolfram Cloud, the second-order regression equation was obtained – for the mixture’s homogeneity ratio $\theta$ (fig. 16):

$$\theta = -1.07128 - 0.039853 V_{M0} + 0.0035214 V_{R0}^2$$
\[ Q = -0.052935 \, V_R0 + 0.000925 \, V_M0 + 0.0038 \, V_R0^2. \]

- the vortex layer apparatus’s volumetric productivity \( Q \)

\[ Q = -2.0816 \times 10^{-17} + 0.000785 \, V_M0 + 0.0062 \, V_R0. \]

**Fig. 15.** Distribution of fluid velocities and ferromagnets on longitudinal sections of the working chamber of the vortex layer apparatus in rotating electromagnetic field

Dependence diagrams (30) under the condition of maximum \( \theta \) \((V_{R0} = 1 \, m/s, \, V_{M0} = 1 \, m/s) = 0.9867 \) are shown in Fig. 2. 28.

It follows from fig. 16 that the value of the mixture homogeneity ratio \( \theta \) decreases with the increase in the components’ supply velocity. This is explained by the fact that ferromagnets do not provide the mixture with required turbulence at the given parameters of rotating electromagnetic field.

**Fig. 16.** Dependence between the mixture’s homogeneity ratio \( \theta \) and the initial rate of liquid manure supply \( V_{R0} \) and the initial rate of additional component supply \( V_{M0} \)

Dependence diagrams (31) under the condition of maximum \( Q \) \((V_{R0} = 5 \, m/s, \, V_{M0} = 5 \, m/s) = 0.03179 \, m^3/s = 114.4 \, m^3/h \) are shown in fig. 17.

**Fig. 17.** Dependence between the vortex layer apparatus’s volumetric productivity \( Q \) and the initial rate of liquid manure supply \( V_{R0} \) and the initial rate of the additional component supply \( V_{M0} \)

It follows from fig. 17 that with increase in the components’ supply speed, the value of the volume productivity of the vortex layer apparatus \( Q \) increases, which is quite logical.

To ensure the required share of additional components \( \alpha_{R0} \) at the maximum value of productivity of the vortex layer apparatus \( Q \) and mixture homogeneity ratio \( \theta \), one should solve the following system of equations together with (30), (31):

\[
\begin{align*}
\alpha_{R0}(V_{R0}, V_{M0}) &= \alpha_{R0}, \\
\theta(V_{R0}, V_{M0}) &\rightarrow \text{max}, \\
Q(V_{R0}, V_{M0}) &\rightarrow \text{max}.
\end{align*}
\]

By solving problem (32) in Microsoft Excel software package, we get the nomogram for calculating the components’ supply velocities, which is shown in fig. 18.

**Fig. 18.** Nomogram for calculating the initial rate of liquid manure supply \( V_{R0} \) and the initial rate of the additional component supply \( V_{M0} \) depending on the required share of additional components \( \alpha_{R0} \)

In order to determine the initial rate of liquid manure supply \( V_{R0} \) and the initial rate of the additional component supply \( V_{M0} \), one should select the value of additional components’ required share \( \alpha_{R0} \) on the abscissa axis.

**Conclusions**

As a result of analytical studies of rotating electromagnetic field of the vortex layer apparatus, the dependencies between the change in magnetic induction \( B \), intensity \( H \), as well as vector potential \( A \) of the working chamber’s electromagnetic field and the coil’s magnetomotive force \( \xi_{\text{max}} \), as well as the electromagnetic field’s rotation frequency \( n \) were obtained.

As a result of analytical studies of the process of interaction between cylindrical ferromagnetic elements and magnetic field in the working chamber of the vortex layer apparatus, respective physical and mathematical apparatus was modernized, which was taken as the basis for numerical modeling in Star CCM+ software package.

As a result of numerical modeling of the process of interaction between ferromagnetic elements and magnetic field in the working chamber of the vortex layer apparatus, the dependencies between changes in the average value of the ferromagnets’ velocity \( V_\Omega \), the maximum force of their contact interaction \( F_{\text{cont}} \) and the frequency of their impacts per time unit \( \kappa \) and electromagnetic field strength \( H \), its rotation frequency \( n \), ferromagnets’ number \( N \) and their length \( L_M \) were obtained. To ensure the highest efficiency of the process of liquid manure components’ grinding, it is required that, in the working chamber of the vortex layer apparatus, the ferromagnets’ average velocity, the strength of their contact interaction and the frequency of ferromagnets’ impacts should be maximum: \( H = 10000 \, A/m, \, n = 3825 \, \text{rpm}, \, N = 298 \, \text{pcs.}, \, L_M = 0.035 \, \text{mm}, \, \kappa = 376 \, \text{imp./s}, \, F_{\text{cont}} = 457 \, \text{N} \) and \( V_M = 9.0 \, \text{m/s} \).

As a result of numerical modeling of the process of liquid manure components’ mixing in the working chamber of the vortex layer apparatus, the dependencies between mixture
homogeneity ratio \(\theta\) as well as the vortex layer apparatus’s volume productivity \(Q\) and the initial rates of liquid manure \(V_{R0}\) as well as additional component \(V_{M0}\) supply were obtained. The nomogram for calculation of components’ supply rates \(V_{R0}\) and \(V_{M0}\) using the value of additional components’ required share \(\alpha_{R0}\) was obtained.

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