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Approximation of inverse hysteresis loops of transformer steel sheets using *tg* and *arctg* functions

Aproksymacja odwrotnych pętli histerezy blach transformatorowych za pomocą funkcji tg i arctg

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Abstract: A method of approximating inverse hysteresis loops as dependences H=f(B) is presented. This is important for transformer sheets, whose hysteresis loops for directions closer to the transverse direction differ significantly from the "classical" loop shape. Principles of parameter determination of the approximating functions are discussed. The numerically determined hysteresis loops for several selected magnetization directions of a transformer sheet were compared with the loops determined by measurements.

Keywords: approximation, inverse hysteresis loop, magnetic hysteresis, transformer steel sheets

Streszczenie: Przedstawiono metodę aproksymacji odwrotnych, granicznych pętli histerezy jako zależności H=f(B). Jest to istotne dla blach transformatorowych, których pętle histerezy dla kierunków bliższych kierunkowi poprzecznemu różnią się znacząco od „klasycznego" kształtu pętli histerezy. Omówiono zasady wyznaczania parametrów funkcji aproksymujących. Numerycznie wyznaczone pętle histerezy dla kilku zadanych kierunków namagnesowania blachy transformatorowej porównano z pętlami wyznaczonymi pomiarowo.

Słowa kluczowe: blachy transformatorowe, aproksymacja, histereza magnetyczna, odwrotna pętla histerezy

Introduction

In numerical calculations, particularly involving magnetic field distributions in electrical steel sheets, an approximation $H = f(B)$ of the inverse hysteresis loop is frequently applied. This refers to cases of magnetic field distribution analysis, where the quantity sought is the vector potential $\boldsymbol{A}$, which is used to determine the flux density vector $\boldsymbol{B}$ based on the relationship $\boldsymbol{B}=rot(\boldsymbol{A})$. Based on assumptions of the well-known hysteresis models such as the Preisach model and the Jiles-Atherton model, different models of the inverse hysteresis loop have been formulated [1-8]. However, these models concern the "classical" shape of the hysteresis loop.

The hysteresis loops of the transformer steel sheets for magnetization directions close to the rolling direction are similar to the "classical" hysteresis shape. However, the shape of the hysteresis loop changes significantly if the magnetization direction is closer to the transverse direction (Fig. 1).

In the analysis of changes in the field strength it is important to describe the curves of the limiting hysteresis loop in a relatively simple way. However, there are no simple analytical functions that could be used to approximate the curves of the limiting hysteresis loops for magnetization angles greater than 45º. The shape of these hysteresis loops is related to the change in the domain structure; domains which create 180-degree walls are rebuilt into domains which form the 90-degree walls [9].

Approximation principles of the inverse limiting hysteresis loop

The basic issue is to describe the curves of the inverse limiting hysteresis loop; all changes in the field

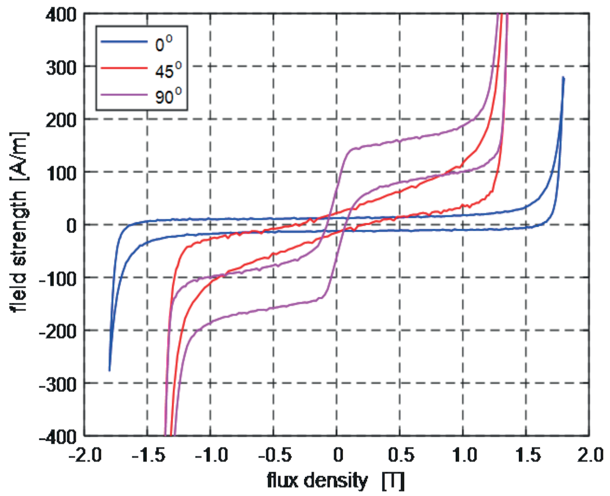

Fig. 1. Inverse loops *H=f(B)* of transformer sheet type M120-27S

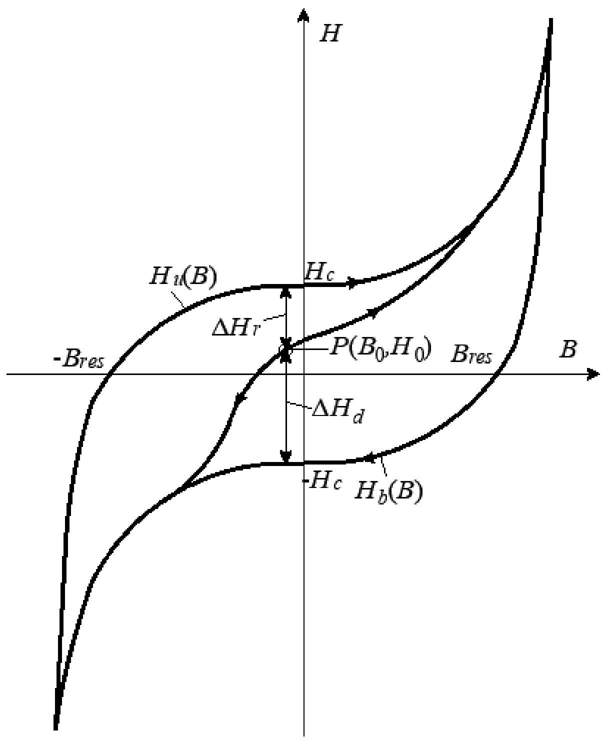

Fig. 2. Inverse hysteresis loop

strength as a dependence on the flux density must be contained in the limiting hysteresis loop. The curves of the limiting hysteresis loop can be often approximated by functions *tg*(*B*), if the approximation concerns the "classical" shape of the hysteresis loop determined for the rolling direction of the transformer sheet [10]. However, for magnetization directions closer to the transverse direction, the shape of hysteresis loops differs significantly from the "classical" shape of magnetic hysteresis (Fig. 1). So far, no relatively simple methods have been proposed for approximating the curves of the inverse limiting hysteresis loop for magnetization directions that differ significantly from the rolling directions.

If the approximation of the inverse limiting hysteresis loops is known, any changes in the field strength can be described in the following way. Any point *P* with coordinates (B_0,H_0) inside the inverse limiting hysteresis loop moves along a certain trajectory to the bottom or upper limiting curve depending on changes in the flux density *B* (Fig. 2).

When the flux density increases, then changes in the field strength can be written as follows [11, 12]:

$$(1) \qquad H_r(B) = H_u(B) - \Delta H_r \; exp[-k_{Hr}(B - B_0)]$$

where:
$H_u(B)$ – the upper curve of the limiting hysteresis loop,
$\Delta H_r = H_u(B_0) - H_0$.

For decreasing values of the flux density *B*, changes in the field strength can be written as:

$$(2) \qquad H_d(B) = H_b(B) + \Delta H_d \; exp[k_{Hd}(B - B_0)]$$

where:
$H_b(B)$ – the lower curve of the limiting hysteresis loop,
$\Delta H_d = H_0 - H_b(B_0)$.

The parameters k_{Hr} and k_{Hd} can be treated as "attenuation" coefficients of the transient components of the field strength. In general case, these coefficients are functions of the magnetic flux density. However, numerical calculations performed for various flux density changes have shown that the coefficients k_{Hr} and k_{Hd} can have a specified, constant value. This significantly simplifies the algorithm for determining the changes in the field strength as a function of the flux density. This is important, if the inverse model of the limiting hysteresis loop is to be included in a large number of elementary segments of the magnetic field area division. The detailed method of determining the parameters of the approximation of changes in the field strength is presented in [11,12].

Approximation of inverse limiting hysteresis loop

The upper and bottom curves $H_u = f(B)$ and $H_b = f(B)$ of the limiting hysteresis loop can be written as the sum of two functions, respectively:

$$(3) \quad H_u(B) = b_H tg(1.57B / B_{sat}) + a_H arctg(w_H B) + H_c$$

$$(4) \quad H_b(B) = b_H tg(1.57B / B_{sat}) + a_H arctg(w_H B) - H_c$$

where: b_H, a_H and w_H – parameters of the approximating functions, B_{sat} – saturation flux density, H_c – coercive force.

Figure 3 shows as an example of approximation functions and curves of the inverse limiting hysteresis loop.

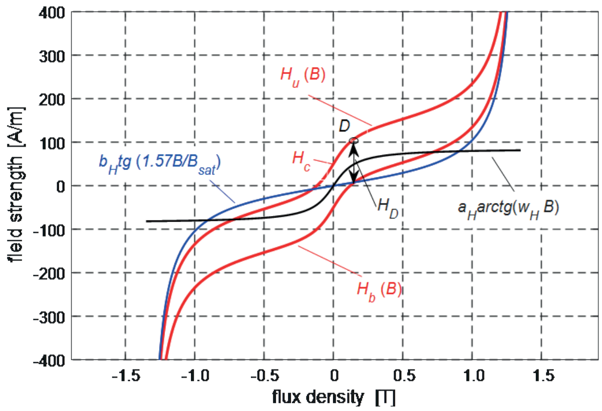

Fig. 3. Approximating functions and inverse limiting hysteresis loop

The saturation flux density B_{sat} and the coercive force H_c for the given magnetization direction are directly determined from measurements. The parameter a_H affects the distance between the curves of the limiting hysteresis loop along the axis of the field strength. The value of this parameter can be estimated by taking into account the distance H_D (Fig. 3) between the axis of the flux density and the point D on the upper curve $H_u(B)$ of the limiting hysteresis loop determined by measurement. At this point the range of rapid changes in the field strength ends for a relatively small change in the flux density, and the function *arctg*(B) tends to a maximum value equaled $1.57a_H$. The function *tg*(B) has values close to zero for small values of the flux density B. So, the value of the parameter a_H can be determined as:

(5) $$a_H \cong \frac{H_D - H_c}{1.57}$$

The second parameter w_H in the *arctg* function affects the rate of changes in the field strength close to zero. The higher the value of w_H, the faster the field strength values change. Approximately, this parameter is related to the derivative dH/dB of the change in the field strength with respect to the flux density, where this derivative is determined based on the measured limiting hysteresis loop. The derivative of the second component of the relationship (3) or (4) for B=0 is equal to $a_H w_H$. This derivative must satisfy the following condition:

(6) $$a_H w_H = \frac{dH}{dB} \quad \text{for} \quad B=0$$

The last relationship allows to determine the parameter w_H for the previously estimated value a_H.

The curves of the limiting hysteresis loop must contain the point $(-B_r,0)$ or $(B_r,0)$ where B_r denotes the remanence flux density. For the bottom curve the following relation can be written:

(7) $$b_H tg(1.57 B_r / B_{sat}) + a_H arctg(w_H B_r) - H_c = 0$$

The analogous relationship is obtained for the upper curve $H_u=f(B)$ and the point $(-B_r,0)$. Knowing a_H and w_H the parameters b_H can be determined as follows:

(8) $$b_H = \frac{H_c - a_H arctg(w_H B_r)}{tg(1.57 B_r / B_{sat})}$$

It should be emphasized that all parameters of the functions approximating the curves of the inverse limiting hysteresis loop depend on the magnetization direction.

Examples of inverse hysteresis loops

The inverse hysteresis loops were determined for the transformer sheet type M120-27S and for several magnetization angles. The parameters of the curves of the limiting hysteresis loop for several magnetization directions are presented in Table 1, and the dependence of the parameter a_H on the magnetization direction is also shown in Figure 4.

Table 1. Parameters of measured hysteresis loops and parameters of approximating functions for several magnetization directions

	B_{sat} [T]	H_c [A/m]	B_r [T]	a_H [A/m]	w_H [1/T]	b_H [A/m]
15°	1.79	14	1.51	1	5.17	3.53
30°	1.63	20	0.58	4	8.05	23.29
45°	1.38	25	0.40	8	11.25	28.98
60°	1.22	30	0.15	25	8.52	37.49
75°	1.36	50	0.12	55	8.47	45.52
90°	1.36	55	0.07	85	10.51	12.94

Figures 5 to 9 show the inverse limiting hysteresis loops for different magnetization directions. The continuous lines represent loops calculated using approximating functions, while the dashed lines refer to loops determined by measurements.

The calculated inverse limiting hysteresis loops are marked in red and blue refers to minor hysteresis loops, that were determined as described in [12]. For the magnetization angles for which the limiting hysteresis loop was not determined by measurements, values of the B_{sat}, B_r and H_c and parameters of the approximating functions should be determined based on an interpolation.

Approximation error analysis

When approximating the limiting hysteresis loops using simple analytical functions as *tg* and *arctg*, certain differences between the loops calculated and those determined by measurement occur, especially for hysteresis loops for magnetization angles greater than 45°.

The most sensitive in terms of selection is the parameter a_H. Figure 10 shows the limiting hysteresis loops for the angle 75° determined for the parameter a_H from Table 1 and the hysteresis loops for a_H increased and decreased by 10 A/m.

Significant discrepancies between hysteresis loops occur in the range where the flux density reaches valuesclose to the saturation flux density. In order to determine the accuracy of the approximation of the measured hysteresis loop, differences between the calculated and measured values were determined using the least squares method. In the considered case, this

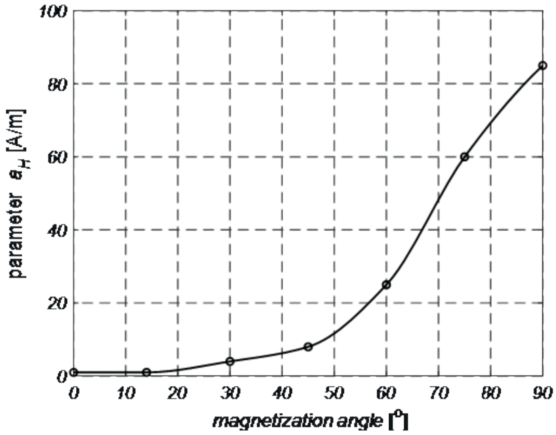

Fig. 4. Dependence of the parameter a_H on the magnetization direction

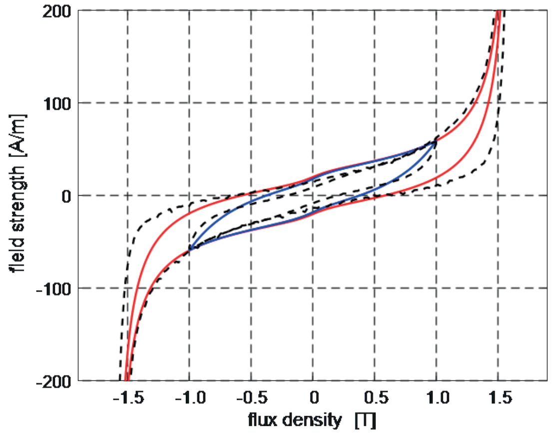

Fig. 5. Inverse hysteresis loops for magnetization angle 30°: continuous lines – calculated loops, dashed lines – hysteresis loops determined by measurement, red color – calculated inverse limiting hysteresis loop, blue color – inverse minor hysteresis loop.

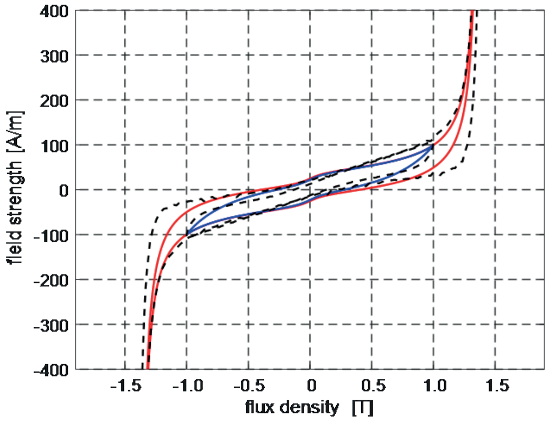

Fig. 6. Inverse hysteresis loops for magnetization angle 45°: loop markings as in Figure 5

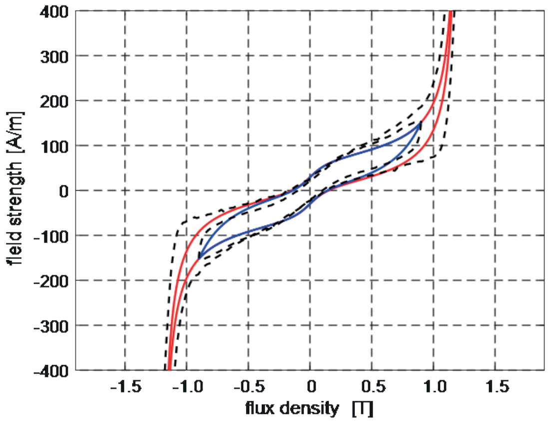

Fig. 7. Inverse hysteresis loops for magnetization angle 60°: loop markings as in Figure 5

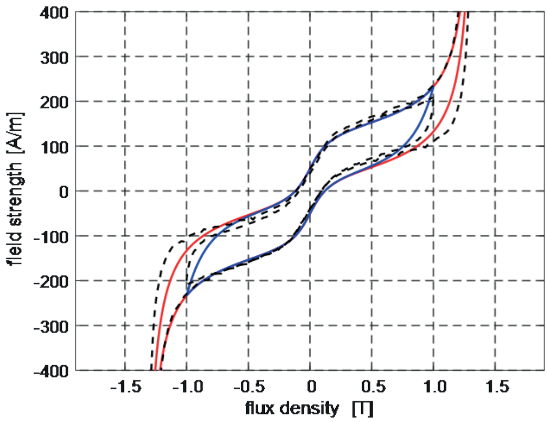

Fig. 8. Inverse hysteresis loops for magnetization angle 75°: loop markings as in Figure 5

error equals 32 A/m, 15 A/m and 20 A/m for a_H = 50, 60 and 70 A/m, respectively.

Conclusions

The inverse limiting hysteresis loops can be approximated by two simple trigonometric functions, that directly relate the magnetic field strength to the

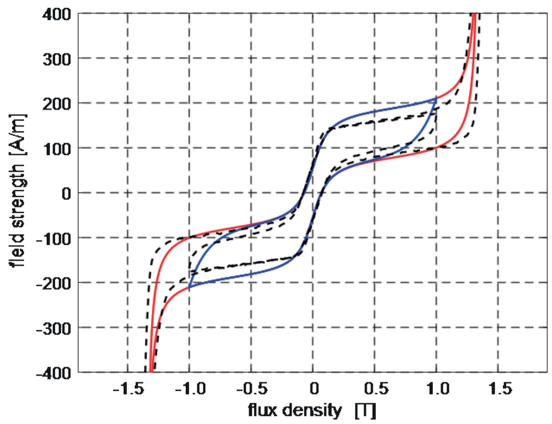

Fig. 9. Inverse hysteresis loops for magnetization angle 90°: loop markings as in Figure 5

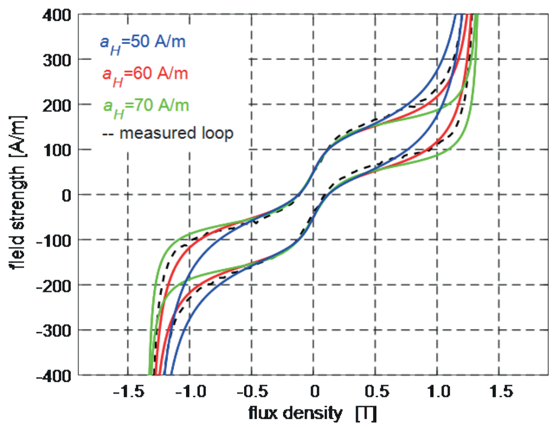

Fig. 10. Measured inverse limiting hysteresis loop (dashed black line) and calculated loops for various values of parameter a_H: 50, 60 and 70 A/m; blue, red and green colors, respectively

flux density. It allows to easily eliminate the field strength in equations of the magnetic field distribution. The parameters of the approximating functions are determined on the basis of measured limiting hysteresis loops for several selected magnetization directions.

It is advisable to perform an error analysis before selecting the a_H parameter value. This is important if changes in the flux density and field strength occur in the range close to the saturation state.

The proposed approximation method of the inverse limiting hysteresis loop can be used in calculations of the magnetic field in the areas of connections between columns and yokes of three-phase transformer cores. In these areas the magnetization direction is often different from the rolling direction.

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